

## 1. Module detail and its structure

Subject Name	Physics
Course Name	Physics 03 (Physics Part-1, Class XII)
Module Name/Title	Unit-04, Module-05: <b>Components of AC circuit</b> Chapter-07: Alternating Current
Module Id	Leph_10702_eContent
Pre-requisites	AC generator, induced emf , induced current , ac, dc, peak value of ac , average value of ac , rms value of ac
Objectives	<p><b>After going through this module learner will be able to</b></p> <ul style="list-style-type: none"> <li>• <b>Differentiate</b> between the response of different components in AC and DC circuits</li> <li>• Understand electrical circuits using pure resistor, inductor and capacitor</li> <li>• <b>Appreciate reactance</b> offered by capacitor and inductor in ac circuits</li> <li>• <b>Represent</b> current and voltage using phasors for pure R,L and C circuits</li> </ul>
Keywords	Instantaneous current , pure resistive circuit, inductive reactance , phase difference between current and voltage, capacitive reactance

## 2. Development team

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## 1. UNIT SYLLABUS

Unit IV: Electromagnetic Induction and Alternating Currents

Chapter-6: **Electromagnetic Induction**

Electromagnetic induction; Faraday's laws, induced emf and current; Lenz's Law, Eddy currents; Self and mutual induction.

Chapter-7: **Alternating Current**

Alternating currents, peak and rms value of alternating current/voltage; reactance and impedance; LC oscillations (qualitative treatment only), LCR series circuit, resonance; power in AC circuits, wattless current; AC generator and transformer.

**2. MODULE WISE DISTRIBUTION****09 Modules**

The above unit is divided into 9 modules for better understanding.

Module 1	<ul style="list-style-type: none"> <li>• Electromagnetic induction</li> <li>• Faraday's laws, induced emf and current;</li> <li>• Change of flux</li> <li>• Rate of change of flux</li> </ul>
Module 2	<ul style="list-style-type: none"> <li>• Lenz's Law,</li> <li>• Conservation of energy</li> <li>• Motional emf</li> </ul>
Module 3	<ul style="list-style-type: none"> <li>• Eddy currents.</li> <li>• Self induction</li> <li>• Mutual induction.</li> <li>• Unit</li> </ul> <p>Numerical</p>
Module 4	<ul style="list-style-type: none"> <li>• AC generator</li> <li>• Alternating currents,</li> <li>• Representing ac</li> </ul>

	<ul style="list-style-type: none"> <li>✓ <i>Formula</i></li> <li>✓ <i>Graph</i></li> <li>✓ <i>Phasor</i></li> <li>• Frequency of ac and what does it depend upon</li> <li>• peak and rms value of alternating current/voltage;</li> </ul>
Module 5	<ul style="list-style-type: none"> <li>• AC circuits</li> <li>• Components in ac circuits</li> <li>• Comparison of circuit component in ac circuit with that if used in dc circuit</li> <li>• Reactance</li> <li>✓ <i>pure R</i></li> <li>✓ <i>pure L</i></li> <li>✓ <i>Pure C</i></li> <li>• Phasor, graphs for each</li> </ul>
Module 6	<ul style="list-style-type: none"> <li>• AC circuits with RL, RC and LC components</li> <li>• Impedance; LC oscillations (qualitative treatment only),</li> <li>• Resonance</li> <li>• Quality factor</li> </ul>

Module 7	<ul style="list-style-type: none"> <li>• Alternating voltage applied to series LCR circuit</li> <li>• Impedance in LCR circuit</li> <li>• Phasor diagram</li> <li>• Resonance</li> <li>• Power in ac circuit</li> <li>• Power factor</li> <li>• Wattles current</li> </ul>
Module 8	<ul style="list-style-type: none"> <li>• Transformer</li> </ul>
Module 9	<ul style="list-style-type: none"> <li>• Advantages of ac over dc</li> <li>• Distribution of electricity to your home</li> </ul>

### **NODULE 4**

### **3. WORDS YOU MUST KNOW**

Let us remember the words we have been using in our study of this physics course:

- **Magnetic field:** The region around a magnet, within which its influence can be felt, denoted by  $B$
- **Magnetic flux:** Intuitive way of describing the magnetic field in terms of field lines crossing a certain area in a magnetic field. Magnetic flux is defined in the same way as electric flux is defined . Magnetic flux through a plane of area  $A$  placed in a uniform magnetic field  $B$  , denoted by  $\phi_B$

- **Electric cell** a simple device to maintain a steady current in an electric circuit is the electrolytic cell
- **Electromotive Force e**: The amount of work done by a cell ( the amount of energy provided by the cell) , to take a unit charge once round the circuit . $\epsilon$  is, actually, a potential difference and *not a force*. The name emf, however, is used because of historical reasons, and was given at a time when the phenomenon was not understood properly.
- **Area vector**: A vector perpendicular to a given area whose magnitude is equal to the given area.
- **Ampere**: It is the unit of current.
- **Volt**: It is the unit of emf and potential difference.
- **Induced emf and Induced current**: The emf developed in a loop when the magnetic flux linked with it changes with time is called induced emf when the conductor is in the form of a closed loop, the current induced in the loop is called an induced current.
- **Weber**: One weber is defined as the amount of magnetic flux, through an area of  $1\text{m}^2$  held normal to a uniform magnetic field of one tesla. The SI unit of magnetic flux is weber (Wb) or tesla meter squared ( $\text{Tm}^2$ ).
- **Faraday's laws of electromagnetic induction**:

**First law**: It states that whenever the amount of magnetic flux linked with the coil changes with time, an emf is induced in the coil. The induced emf lasts in the coil only as long as the change in the magnetic flux continues.

**Second law**: It states that the magnitude of the emf induced in the coil is directly proportional to the time rate of change of the magnetic flux linked with the coil.

- **Fleming's Right Hand rule**: Fleming's right hand rule gives us the direction of induced emf/current in a conductor moving in a magnetic field.

If we stretch the fore-finger, central finger and thumb of our right hand mutually perpendicular to each other such that **fore-finger** is in the direction of the field , **thumb** is in the direction of motion of the conductor, then the central finger would give the direction of the induced current.

- **Induced emf by changing the magnetic field:** The movement of magnet or pressing the key of coil results in changing the magnetic field associated with the coil, this induces the emf.
- **Induced emf by changing the orientation of coil and magnetic field:** When the coil rotates in a magnetic field the angle  $\Theta$  changes and magnetic flux linked with the coil changes and this induces the emf. This is the basis of ac generators.
- **Induced emf by changing the area A: MOTIONAL EMF:** Motional emf is a type of induced emf which occurs when a wire is pulled through the magnetic field. The magnitude of motional emf depends upon the velocity of the wire, strength of magnetic field and the length of the wire. Motional emf arises due to the motion of charges due to a magnetic field.
- **Electric Current:** An electric current equals the rate of flow of electric charge. In electric circuits this charge is often carried by moving electrons in a wire. It can also be carried by ions in an electrolyte, or by both ions and electrons such as in plasma.
- **Voltage:** the difference in electric potential energy between two points per unit electric charge, in an electric circuit
- **Ohm's law:** Electric current through a conductor is directly proportional to the potential difference across the conductor provided the temperature and physical conditions of the conductor remains the same
- **Ohmic conductors:** Conductors that follow ohm's law for reasonable range of physical conditions. conductor wires , conductor plates, strips
- **Non-Ohmic conductors :** Conductors that do not follow Ohm's law example electrolytes, semiconductors

- **Eddy Currents:** Eddy currents are loops of electrical current induced within conductors by a changing magnetic field in the conductor, (as per Faraday's law of induction). Eddy currents flow in closed loops within conductors, in planes perpendicular to the magnetic field. They can be induced within (nearby) stationary conductors by a time-varying magnetic field.
- **Phasors:** In physics and engineering, a phasor, is a complex number representing a sinusoidal function whose amplitude (A), angular frequency ( $\omega$ ), and initial phase ( $\theta$ ) are time-invariant. Basically, Phasors are rotating vectors.
- **Alternating voltage:** The electric mains supply in our homes and work places is a voltage that varies like a function with time. Output from an ac generator.
- **Alternating current:** Current in a circuit driven by ac voltage is called ac current.
- Alternating currents and voltages have **frequency f** and **angular frequency  $2\pi f$**  associated with it
- Two currents, two voltages or currents and voltages may have a phase relation between them. This arises due to electromagnetic induction, self induction or time rate associated with charging and discharging of capacitors.
- Alternating currents and voltages have **instantaneous value** given by:

$$i = i_0 \sin(\omega t + \Phi)$$

$$V = V_0 \sin(\omega t + \Phi)$$

$\Phi$  is the initial phase of the sinusoidal current or voltage

- Alternating currents and voltages have **peak value**  $I_0$  and  $V_0$
- Alternating currents and voltages have **average value over half cycle**

$$V_{avg(T/2)} = \frac{2V_0}{\pi} \cong 0.636 V_0$$



- Alternating currents and voltages have **root mean square values**

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

- **Self-inductance of a coil: L** An electric current can be induced in a coil by flux changes produced by the changing current in it self

$$L = \mu_0 n^2 A l$$

Where  $nl = N$  total number of turns of the coil,  $A$  area of the face of the coil,  $\mu_0$  is the permeability of free space. Its SI unit henry.

Self-inductance is also **called back emf**. It depends upon the geometry of the coil and permeability of the medium inside the coil.

- **Energy required to build up current I:** in a coil of inductance  $L = \frac{1}{2} LI^2$ .
- **Capacitor:** A system of two conductors separated by an insulator .parallel plate capacitors, spherical capacitors are used in circuits. Capacity of parallel plate capacitor is given by

$C = \frac{\mu_0 A}{d}$  . 'A' is the area of the plate,  $d$  separation between the plates,  $\mu_0$  is the permeability of free space.

Capacitors block dc but ac continues as charging and discharging of the capacitor maintains a continuous flow of current.

- **Capacitance:**  $C = \frac{Q}{V}$  S.I. unit farad.
- **Dielectric constant of a material K:** Is the factor by which the capacitance increases from its vacuum value when the dielectric (material) is inserted fully between the plates of a capacitor.

- **Combination of capacitors:** Capacitors may be combined in ways to obtain a value of effective capacitance.

**Series combination:** Capacitances are said to be in series if the effective combined capacitance  $C$  is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

**Parallel combination of capacitors:** capacitances are said to be in series if the effective combined capacitance  $C$  is given by

$$C = C_1 + C_2 + \dots + C_n$$

#### 4. INTRODUCTION

An electrical network is an interconnection of electrical components (e.g. batteries, resistors, inductors, capacitors, switches) or a model of such an interconnection, consisting of electrical elements (e.g. voltage sources, current sources, resistances, inductances, capacitances). An electrical circuit is a network consisting of a closed loop, giving a return path for the current..

AC electricity allows for the use of a resistor, capacitor and inductor within an electrical or electronic circuit. These devices can affect the way the alternating current passes through a circuit in a network. They are only effective with AC electricity.

Today, most of the electrical devices we use require ac voltage. This is mainly because most of the electrical energy sold by power companies is transmitted and distributed as alternating current. The main reason for preferring use of ac voltage over dc voltage is that ac voltages can be easily and efficiently converted from one voltage to the other by means of transformers.

Further, electrical energy can also be transmitted economically over long distances. AC circuits exhibit characteristics which are exploited in many devices of daily use. For example –A combination of a capacitor, inductor and resistor is also used as a tuner in radios and televisions. These devices are very useful for tuning to different broadcasting stations.

Our mobiles connect to desired mobile numbers using electrical circuits that are able to choose the desired ten digit phone number.

We have studied electrical circuits that operated using cells and batteries. The output from a cell or a battery is direct current. We dealt with Ohm's law, Kirchhoff's law, Law of combination of resistors in series and parallel, circuit components like connecting wires, resistances, keys and current, voltage meters to determine current and voltages in the entire or branch of a circuit.

We have learnt about capacitors and inductors.

**Now we will study electrical circuits powered not by cells and batteries but, by alternating voltage source.**

## **5. COMPONENT OF AC CIRCUITS**

In our study of DC electrical circuits, we dealt with resistances in the circuit which made the circuit operate according to required outcome. In AC circuits capacitors and coils (inductors) are capable of changing the value of currents and voltages.

**So, AC circuits will have alternating voltage source, resistances, capacitors and inductors switches and connecting wires.**

Let us start with considering simple AC Circuits that contain only one circuit element (capacitor, or inductor or resistor), connected to an AC source.

## **6. ALTERNATING VOLTAGE APPLIED TO A RESISTOR**

A resistor is a passive two-terminal electrical component that provides a resistance as a circuit element. Resistors act to reduce current flow, and, at the same time, can be used to lower voltage levels within circuits. In electronic circuits, resistors are used to limit current flow, to adjust signal levels, bias active elements, terminate transmission lines, among other uses.

Many of these words are new to you but you will learn their meaning soon.

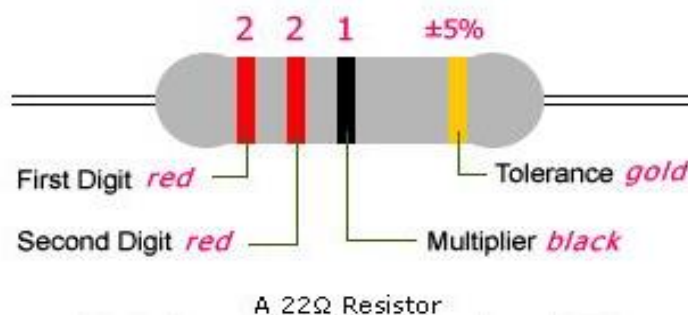
- i) **High-power resistors** that can dissipate many watts of electrical power as heat.

ii) **Fixed resistors** have resistances that only change slightly with temperature, time or operating voltage.

iii) **Variable resistors** can be used as additive circuit elements (such as used for audio volume control or a lamp dimmer, fan regulator), or as sensing devices for heat, light, humidity, force, or chemical activity.



Symbolic carbon resistor



Source image: Wikipedia

You are familiar with Ohm's law connecting voltage, current and resistance

$$V=IR$$



<https://youtu.be/bvhnrL2rO9g>

The ohm (symbol:  $\Omega$ ) is the SI unit of electrical resistance, named after Georg Simon Ohm.

An ohm is equivalent to a volt per ampere.

Since resistors are specified and manufactured over a very large range of values, the derived units of:

**Milliohm ( $1 \text{ m}\Omega = 10^{-3} \Omega$ ),**

**Kilo-ohm ( $1 \text{ k}\Omega = 10^3 \Omega$ ), and**

**Mega-ohm ( $1 \text{ M}\Omega = 10^6 \Omega$ ) are also in commonly used.**

**A resistive circuit is a circuit containing only resistors and ideal current and voltage sources.**

Consider a purely resistive circuit with a resistor  $R$  connected to an AC generator, as shown in Figure given below.

The AC Source produces sinusoidally varying potential difference across its terminals.



**The fig shows a resistor connected to a source  $\epsilon$  of ac voltage.**

We consider a source which produces sinusoidally varying potential difference across its terminals.

Let this potential difference, also called ac voltage, be given by.

$$V(t) = V_0 \sin \omega t$$

where  $V_0$  is the amplitude of the oscillating potential difference and  $\omega$  is its angular frequency.

**To find the value of current through the resistor,**

We apply Kirchhoff's Loop rule,

$\sum \mathcal{E}(t) = 0$  to the given purely resistive circuit

$$V_0 \sin \omega t - I R = 0$$

$$I = \frac{V_0 \sin \omega t}{R}$$

Since  $R$  is a constant, we can write this equation as

$$I = I_0 \sin \omega t$$

Where the current peak value or maximum value will be given by

$$I_0 = \frac{V_0}{R}$$

here  $I_0$  is also called the current amplitude.

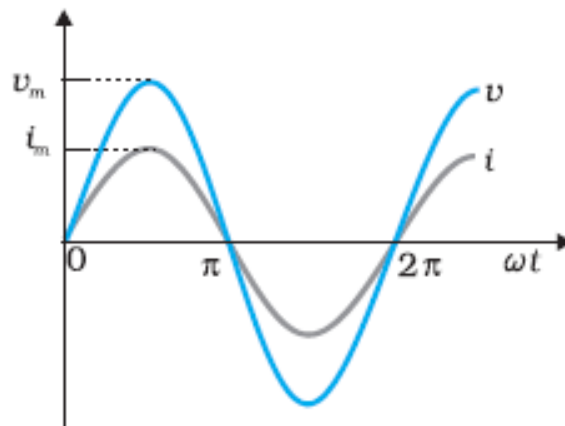
**This shows that Ohm's Law for resistors is applicable to both ac and dc voltages.**

Ohm's law holds for circuits containing only resistive elements (no capacitances or inductances) for all forms of driving voltage or current, regardless of whether the driving voltage or current is constant (DC) or time-varying such as AC.

**At any instant of time Ohm's law is valid for such circuits.**

Resistors which are in series or in parallel may be grouped together into a single "equivalent resistance" in order to apply Ohm's law for analyzing the circuit.

The time dependence of the current and the voltage across the resistor is depicted below:



**In pure resistive circuit the voltage and current are in phase**

From the graph, it is evident that both  $V$  and  $i$  reach zero, minimum (0) and maximum  $V_0$  or  $V_m$  values at the same time.

**Clearly, the voltage and current are IN PHASE with each other.**

We also see that, like the applied voltage, the current varies sinusoidally and has corresponding positive and negative values during each cycle. **Thus, the sum of the instantaneous current values over one complete cycle is zero, and the average current is zero.**

**The fact that the average current is zero, however, does not mean that the average power consumed is zero and that there is no dissipation of electrical energy.**

As you know, Joule heating is given by  $I^2 R t$  and depends on  $I^2$  (which is always positive whether  $I$  is positive or negative) and not on instantaneous value of  $I$ .

**Thus, there is Joule heating and dissipation of electrical energy when an alternating current passes through a resistor.**

The instantaneous power dissipated in the resistor is:

$$p = I^2 R = I_0^2 R \sin^2 \omega t$$

**The average value of  $p$  over a cycle is:**

$$\bar{p} = \langle I^2 R \rangle = \langle I_0^2 R \sin^2 \omega t \rangle$$

$\bar{p}$  denotes its average value and  $\langle \dots \rangle$  denotes taking average of quantity in side the bracket

Since,  $I_0^2$  and  $R$  are constants

$$\bar{p} = I_0^2 R \langle \sin^2 \omega t \rangle$$

Using trigonometric identity  $\sin^2 \omega t = \frac{1}{2} (1 - \cos 2\omega t)$

$$\langle \sin^2 \omega t \rangle \geq \frac{1}{2}$$

$$\bar{p} = I_0^2 R \langle \sin^2 \omega t \rangle = \frac{1}{2} I_0^2 R$$

Or

$$\bar{p} = I_{rms}^2 R$$

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

This shows the advantage of introducing the concept of rms values. In terms of rms values, the equation for power and hence relation between current and voltage in ac circuits are essentially the same as those for the dc case.

It is customary to measure and specify rms values for ac quantities. For example, the household line voltage of 220 V is an rms value with a **peak voltage**

$$V_0 = \sqrt{2} V_{rms}$$

$$= (1.414) (220 \text{ V}) = 311 \text{ V}$$

In fact, the  $I_{rms}$  current is the equivalent dc current that would produce the same average power loss as the alternating current.

**NOTE:**

- When a value is given for ac voltage or current, it is ordinarily the rms value. The voltage across the terminals of an outlet in your room is normally 240 V. This refers to the *rms* value of the voltage. The peak value of voltage or the amplitude of voltage is

$$V_0 = \sqrt{2} V_{rms} = \sqrt{2} (240) = 340 \text{ V}$$

- The power rating of an element used in ac circuits refers to its average power rating.
- The power consumed in an circuit is never negative.
- Both alternating current and direct current are measured in amperes.

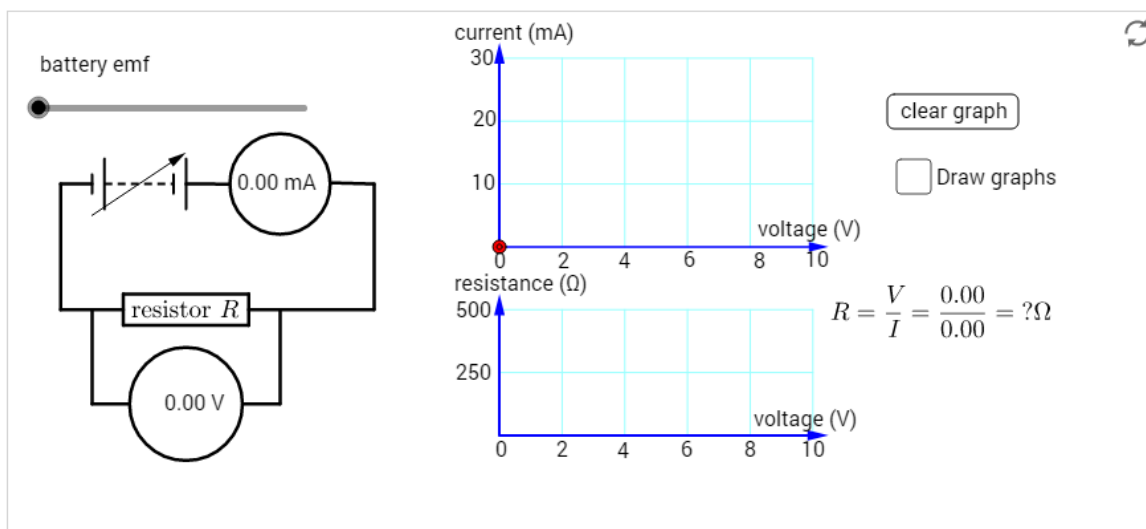
**But how is the ampere defined for an alternating current? It cannot be derived from the mutual attraction of two parallel wires carrying ac currents, as the dc ampere is derived. An ac current changes direction with the source frequency and the attractive force would average to zero.**



**Thus,** the ac ampere must be defined in terms of some property that is independent of the direction of the current. Joule heating is such a property, and there is one ampere of *rms* value of alternating current in a circuit if the current produces the same average heating effect as one ampere of dc current would produce under the same conditions.

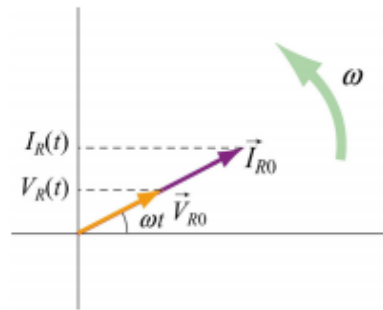
## 7. REPRESENTATION OF ALTERNATING CURRENT AND VOLTAGE BY GRAPHS AND PHASORS

### I-V characteristics (resistor)



Link here: [vr- ggb.ggb](https://www.geogebra.org/m/vr-ggb.ggb)

<https://www.geogebra.org/m/K2gh8kfM?doneurl=%2Fsearch%2Fperform%2Fsearch%2Fac%2Bcircuit%2Baith%2Bresistor%2B%2Fmaterials%2F>

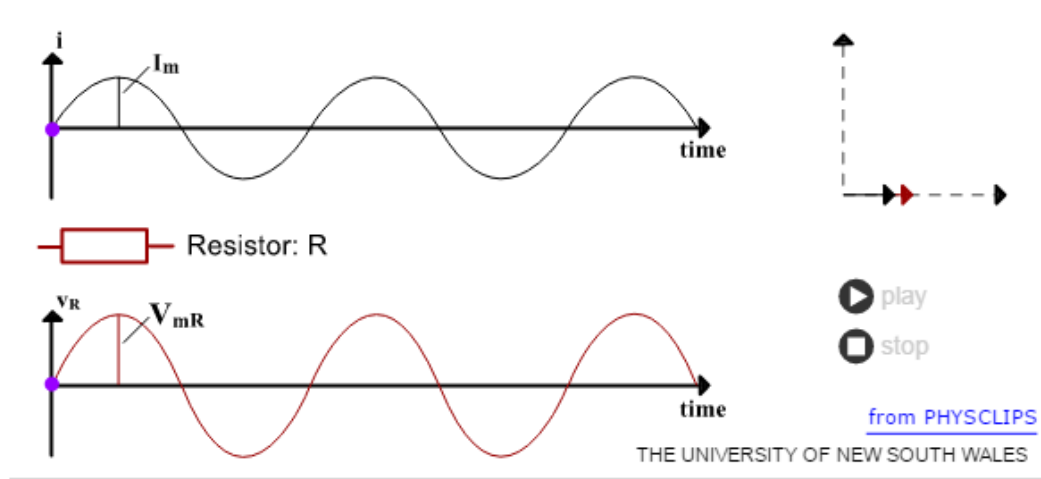


The behavior, of  $I_R(t)$  current through the resistor and  $V_R(t)$  voltage across the resistor, can also be represented with a phasor diagram.

A phasor is a rotating vector having the following properties:

- (i) **Length:** the length corresponds to the amplitude.
- (ii) **Angular speed:** the vector rotates counterclockwise with an angular speed  $\omega$ .
- (iii) **Projection:** the projection of the vector along the vertical axis corresponds to the value of the alternating quantity at time  $t$ .

From the figure we see that phasors  $V$  and  $I$  for the case of a resistor are in the same direction. This is so for all times. This means that the phase angle between the voltage and the current is zero.



[Link here-](#)

<http://www.animations.physics.unsw.edu.au/jw/AC.html#resistors>

THIS LINK ALLOWS YOU TO IMAGINE THE CURRENT AND VOLTAGE THROUGH A RESISTOR CIRCUIT POWERED BY ALTERNATING VOLTAGE SOURCE.

See the video

### EXAMPLE

A light bulb is rated at 100W for a 220 V supply.

Find

- (a) The resistance of the bulb;
- (b) The peak voltage of the source; and
- (c) The rms current through the bulb.

### SOLUTION

- (a) We are given  $P = 100 \text{ W}$  and  $V = 220 \text{ V}$ . The resistance of the bulb is

$$R = \frac{V^2}{P} = \frac{(220\text{V})^2}{100\text{W}} = 484\text{ohms}$$

- b) The peak voltage of the source

$$V_0 = \sqrt{2} V_{\text{rms}} = 311 \text{ V}$$

- c)  $P = I_{\text{rms}} V_{\text{rms}}$

$$I_{\text{rms}} = \frac{P}{V_{\text{rms}}} = \frac{100}{220} = 0.450\text{A}$$

### EXAMPLE

A  $100 \Omega$  resistor is connected to a 220 V, 50 Hz ac supply.

- (a) What is the rms value of current in the circuit?
- (b) What is the net power consumed over a full cycle?

**SOLUTION**

a)

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$$

$$I_{\text{rms}} = \frac{220}{100} = 2.20 \text{ A}$$

b)

$$P = I_{\text{rms}}^2 R = \frac{220 \times 220 \times 100}{100 \times 100} = 484 \text{ W}$$

**EXAMPLE**

- (a) The peak voltage of an ac supply is 300 V. What is the rms voltage?
- (b) The rms value of current in an ac circuit is 10 A. What is the peak current?
- c) Find the resistance in the circuit
- d) Draw a voltage time, current time graph, mark peak and rms values for each
- e) What is the phase relation between current and voltage?

**SOLUTION**

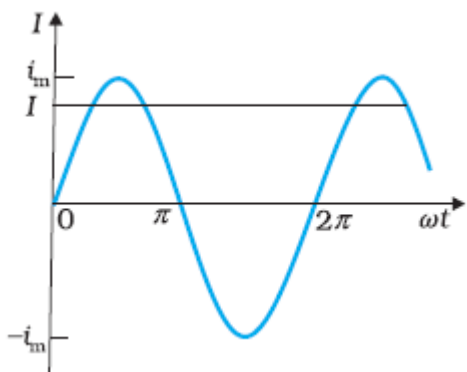
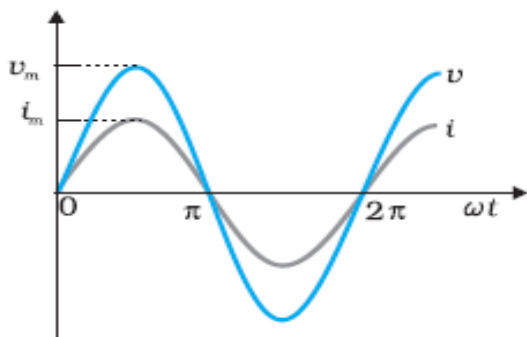
a)

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 0.707 \times 300 = 212.1 \text{ V}$$

$$\text{b) } I_0 = \sqrt{2} I_{\text{rms}} = 1.414 \times 10 = 14.14 \text{ A}$$

$$\text{c) } R = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{V_0}{I_0} = \frac{300}{14.14} = 21.21 \text{ ohms}$$

d)



rms current related to peak current.

### NOTICE

The peak voltage is 300 V and the peak current is 14.14 A the y axis of the graph must have different scales to represent current and voltage

e) The circuit has only a resistance hence current and voltage are in phase

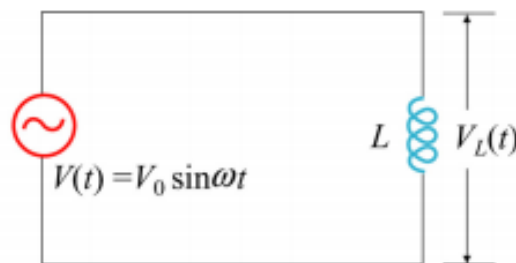
### 8. ALTERNATING VOLTAGE APPLIED TO AN INDUCTOR

Now we are going to study a circuit which has coil of negligible resistance and a source of alternating voltage .the reason to choose a negligible resistance coil is that if it were connected in a dc circuit infinite (very High) current would flow through it, but in alternating voltage circuit due to self inductance of the coil a “resistance” is offered to the current.

This is called **INDUCTIVE REACTANCE**. The coil is also called a inductor, the circuit is known as inductive circuit.

So a coil may not offer resistance in a dc circuit because of low resistance, it offers reactance in an ac circuit. This is a very useful phenomenon.

Consider now a **purely inductive circuit**, with an inductor connected to an AC source, as shown below:



A purely inductive circuit

Usually, inductors have appreciable resistance in their windings, but we shall assume that this inductor has negligible resistance.

Thus, the circuit is a purely inductive ac circuit. Let the voltage across the source be

$$V(t) = V_0 \sin \omega t$$

Using Kirchhoff's rule

$$\sum e(t) = 0$$

This implies,

$$V(t) - L \frac{dI}{dt} = 0$$

**NOTICE**

- The second term is the self induced Faraday emf in the coil or inductor and  $L$  is the self inductance of the coil
- The negative sign follows from Lenz's law
- $V(t)$  implies Instantaneous voltage at instant  $t$
- As voltage changes so does current in the circuit

$$\frac{dI}{dt} = \frac{V(t)}{L} = \frac{V_0 \sin \omega t}{L}$$

The equation for  $I(t)$ , the current as a function of time, must be such that its slope  $dI/dt$  is a sinusoidally varying quantity, with the same phase as the source voltage and an amplitude given by  $V_0/L$ .

To obtain the current, we integrate  $dI/dt$  with respect to time

$$\int \frac{dI}{dt} dt = \frac{V_0}{L} \int \sin \omega t dt$$

We get

$$I(t) = -\frac{V_0}{\omega L} \cos \omega t + \text{constant}$$

The integration constant has the dimension of current and is time independent.

Since the source has an emf which oscillates symmetrically about zero, the current it sustains also oscillates symmetrically about zero, so that no constant or time-independent component of the current exists.

Therefore, the integration constant is zero

Using

$$-\cos \theta = \sin\left(\theta - \frac{\pi}{2}\right)$$

$$I(t) = \frac{V_0}{\omega L} \sin(\theta - \frac{\pi}{2})$$

Here

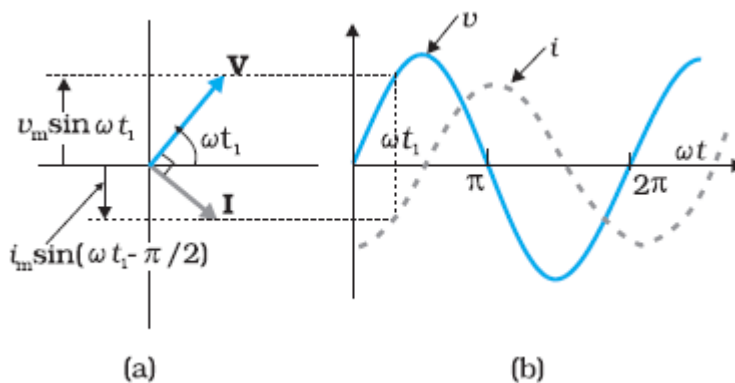
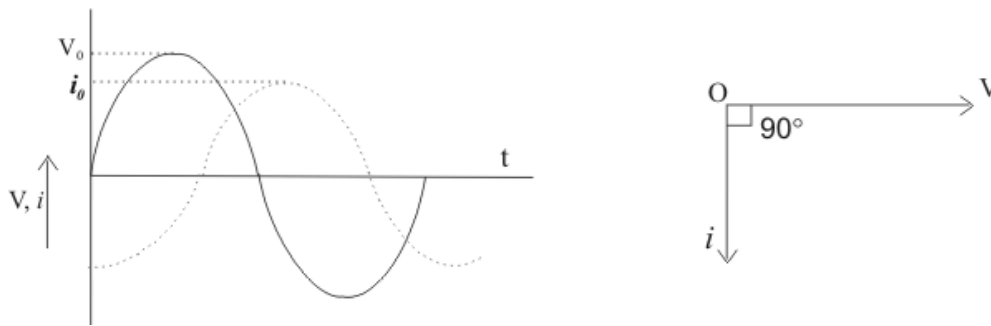
$\frac{V_0}{\omega L} = I_0$  is the peak value of current or amplitude of current

- The **quantity  $\omega L$  is analogous to resistance** from ohm's law.
- It is represented by  **$X_L$**
- Here,  $X_L = \omega L = 2 \pi f L$  is called the **inductive reactance**.

The inductive reactance is directly proportional to the inductance and to the frequency of the current

- It has SI units of **ohms ( $\Omega$ )**, just like resistance.
- The inductive reactance limits the current in a purely inductive circuit in the same way as the resistance limits the current in a purely resistive circuit.
- However, unlike resistance,  $X_L$  depends linearly on the angular frequency  $\omega$ . Resistance does not change with frequency. Thus, the resistance to current flow increases with frequency.
- This is due to the fact that at higher frequencies the current changes more rapidly than it does at lower frequencies. Self inductance will have a larger value due to increased rate of change of flux
- On the other hand, the inductive reactance vanishes as  $\omega$  approaches zero.
- Also, the phase constant is:  $\phi = + \frac{\pi}{2}$
- The current and voltage plots and the corresponding phasor diagram for a purely inductive circuit are given below:





- a) Time dependence of  $I_L(t)$  and  $V_L(t)$  across the inductor.

For the source voltage and the current in an inductor shows that the current lags the voltage by  $\pi/2$  or one-quarter ( $1/4$ ) cycle.

- b) Phasor diagram for the inductive circuit.

As can be seen from the figures, the current  $I_L(t)$  is out of phase with  $V_L(t)$  by  $\phi = \pi/2$ ; it reaches its maximum value after  $V_L(t)$  does by one quarter of a cycle.

Notice: The voltage and the current phasors in the present case at instant  $t_1$ . The current phasor  $I$  is  $\pi/2$  behind the voltage phasor  $V$ . When rotated with frequency  $\omega$  anticlockwise, they generate the voltage

$$V(t) = V_0 \sin \omega t$$

**Current**

$$I(t) = \frac{V_0}{\omega L} \sin\left(\theta - \frac{\pi}{2}\right)$$

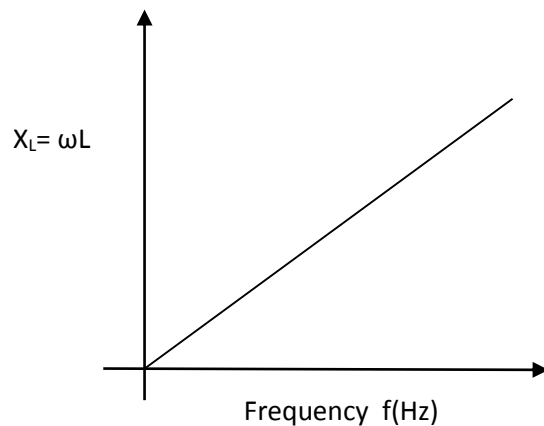
**THUS NOTICE ,**

- **we say that the current lags voltage by  $\pi / 2$  in a purely inductive circuit.**

We see that the current reaches its maximum value later than the voltage by one-fourth of

a period  $\left\{\frac{T}{4} = \frac{\pi}{\omega}\right\}$ ,

- Graph of  $X_L$  versus frequency of voltage



- Why would coils be called ‘choke coil’?
- **Does an inductor consume power like a resistance?**

The instantaneous power supplied to the inductor is

$$\begin{aligned} p &= I(t)V(t) = I(t) = I_0 \sin\left(\theta - \frac{\pi}{2}\right) \times V_0 \sin \omega t \\ &= -I_0 V_0 \cos(\omega t) \sin(\omega t) \\ &= -\frac{I_0 V_0}{2} \sin(2\omega t) \end{aligned}$$

**Average power over one complete cycle = 0**

- **Does the coil store energy?**

As current increases in the coil, it sets up a magnetic field around it and energy is stored in the form of magnetic energy

$$\text{Magnetic energy} = \frac{1}{2} LI^2$$

Maximum value would correspond to maximum value of current

$$\text{Maximum magnetic energy stored} = \frac{1}{2} LI_0^2$$

### THINK ABOUT THESE

- On what factors does  $X_L$  depend upon?
- A connecting wire of negligible resistance offers reactance in ac circuit. Why?
- For the same coil of inductance  $L$  the reactance changes if the frequency is changed .Why?
- What does the slope of the line indicate in the  $X_L$ -  $f$  graph?
- How would you plot a graph such that the slope gives the value of self inductance?
- Two coils of self inductance  $L_1$  and  $L_2$  are given to you.  $L_1 > L_2$ . Draw  $X_L$ -  $f$  for each if they are connected to same frequency source. Which one of the two will have a greater slope? Why?

### EXAMPLE

A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.

### SOLUTION

The inductive reactance,

$$X_L = 2\pi fL = 2 \times 3.14 \times 50 \times 25 \times 10^{-3}$$

$$= 7.85 \text{ ohm}$$

$$I = \frac{V}{X_L} = \frac{220}{7.8} = 28 \text{ A}$$

### TRY THESE

- A 44 mH inductor is connected to 220 V, 50 Hz ac supply.

a) Determine the rms value of the current in the circuit.

#### Would your answer change?

- if frequency is 60 Hz
- If we use dc instead of ac
- If supply voltage is reduced to 110V
- If we consider the resistance of the inductor coil

b) What is the phase relation between current and voltage?

c) What will be the power lost in 2 min /1 hour?

- In a pure inductive circuit the AC frequency is

a) doubled

b) halved.

How does the inductive reactance change in each case?

## 9. ALTERNATING VOLTAGE APPLIED TO A CAPACITOR

A capacitor (initially known as a condenser) is a passive two-terminal electrical component used to store electrical energy temporarily in an electric field.

The forms of practical capacitors vary widely, but all contain at least two electrical conductors (plates) separated by a dielectric (i.e. an insulator that can store energy by becoming polarized).

The conductors can be thin films, foils or sintered beads of metal or conductive electrolyte, etc

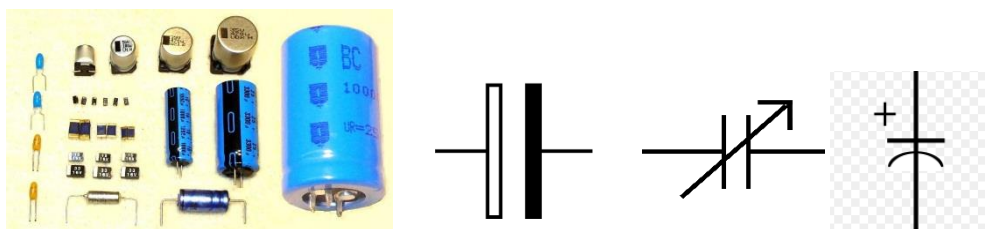
. The non-conducting dielectric acts to increase the capacitor's charge capacity.

Materials commonly used as dielectrics include **glass, ceramic, plastic film, air, vacuum, paper, mica, and oxide layers.**

Capacitors are widely used as parts of electrical circuits in many common electrical devices.

Unlike a resistor, an ideal capacitor does not dissipate energy.

Instead, a capacitor stores energy in the form of an electrostatic field between its plates.



Source image: Wikipedia

### In a DC circuit

When a capacitor is connected to a voltage source in a dc circuit, current will flow for the short time required to charge the capacitor. As charge accumulates on the capacitor plates, the voltage across them increases, opposing the current. That is, a capacitor in a dc circuit will limit or oppose the current as it charges.

**When the capacitor is fully charged, the current in the circuit falls to zero.**

### In an AC circuit

When the capacitor is connected to an ac source, as in Fig. 7.8, it limits or regulates the current, but does not completely prevent the flow of charge. The capacitor is alternately charged and discharged as the current reverses each half cycle.

The figure shows an ac source connected to a capacitor



Let  $q$  be the charge on the capacitor at any time  $t$ . The instantaneous voltage  $v$  is

$$V = \frac{q}{C}$$

If the supply alternating voltage is

$$V = V_0 \sin \omega t$$

Using Kirchhoff's rule

$$V = V_0 \sin \omega t = \frac{q}{C}$$

Current would be time rate of change of charge or  $I = \frac{dq}{dt}$

$$I = \frac{d}{dt} (V_0 C \sin \omega t) = \omega C V_0 \cos \omega t$$

$$\text{But } \cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

So we can write

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Where

$$I_0 = \omega C V_0 = \frac{V_0}{\frac{1}{\omega C}}$$

Comparing it to  $I_0 = \frac{V_0}{R}$  for a purely resistive circuit, we find that  $(1/\omega C)$  plays the role of resistance. It is **called capacitive reactance** and is denoted by  $X_C$ ,

- Here,  $X_C = 1/\omega C$ , is called the capacitance reactance.
- It also has SI units of ohms
- Represents the effective resistance for a purely capacitive circuit.

The capacitive reactance limits the amplitude of the current in a purely capacitive circuit in the same way as the resistance limits the current in a purely resistive circuit. But it is inversely proportional to the frequency and the capacitance.

- Note that  $X_C$  is inversely proportional to both  $C$  and  $\omega$ ,
- The phase constant is given by:  $\Phi = -\frac{\pi}{2}$

### Current and Voltage phase relationship in a pure capacitive ac circuit

$$V = V_0 \sin \omega t$$

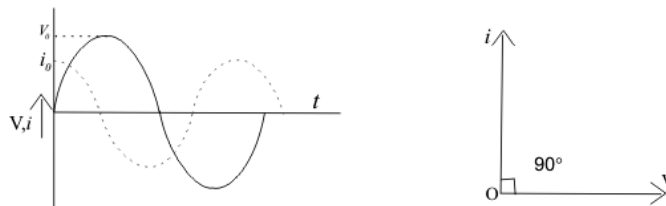
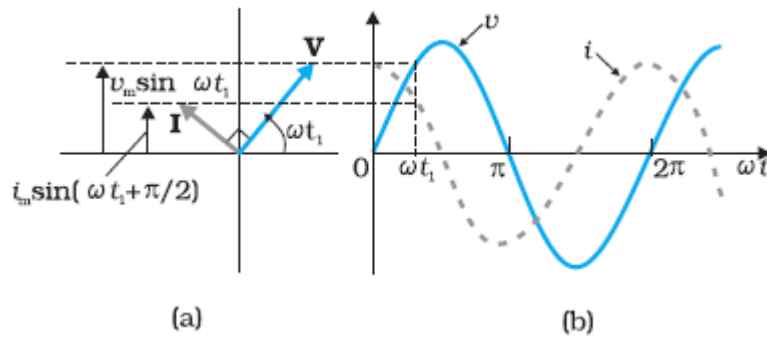
$$I = I_0 \sin(\omega t + \frac{\pi}{2})$$

the current is  $\pi/2$  ahead of voltage.

The graph shows the phasor diagram at an instant  $t_1$ . Here the current phasor  $I$  is  $\pi/2$  ahead of the voltage phasor  $V$  as they rotate counterclockwise.

The graph shows the variation of voltage and current with time.

We see that the current reaches its maximum value earlier than the voltage by one-fourth of a period.



Notice that: at  $t = 0$ , the voltage across the capacitor is zero while the current in the circuit is at a maximum. Thus, we say that the current leads the voltage by  $\pi/2$  in a capacitive circuit.

- Does a capacitor consume power like a resistance?**

The instantaneous power supplied to the capacitor is

$$\begin{aligned}
 p &= I(t)V(t) = I(t) = I_0 \sin\left(\theta - \frac{\pi}{2}\right) \times V_0 \sin \omega t \\
 &= -I_0 V_0 \cos(\omega t) \sin(\omega t) \\
 &= -\frac{I_0 V_0}{2} \sin(2\omega t)
 \end{aligned}$$

**So like in the case of an inductor the average power = 0**



As  $\langle \sin(2\omega t) \rangle = 0$  over a complete cycle

- **Does the capacitor store energy?**

As charge increases in the capacitor, it sets up an electric field inside it and energy is stored in the form of electrostatic energy

$$\text{Electrostatic energy} = \frac{1}{2} CV^2$$

Maximum value would correspond to maximum value of voltage

$$\text{Maximum electrical energy stored} = \frac{1}{2} CV_0^2$$

Thus, we see that

- **in the case of an inductor, the** current lags the voltage by  $\pi/2$
- **in the case of a capacitor, the** current leads the voltage by  $\pi/2$ .

### EXAMPLE

A lamp is connected in series with a capacitor. Predict your observations for dc and ac connections. What happens in each case if the capacitance of the capacitor is reduced?

### SOLUTION

When a dc source is connected to a capacitor, the capacitor gets charged and after charging no current flows in the circuit and the lamp will not glow. There will be no change even if  $C$  is reduced. With ac source, the capacitor offers capacitive reactance ( $1/\omega C$ ) and the current flows in the circuit. Consequently, the lamp will shine.

Reducing  $C$  will increase reactance and the lamp will shine less brightly than before.

### EXAMPLE

A  $15.0 \mu\text{F}$  capacitor is connected to a  $220 \text{ V}$ ,  $50 \text{ Hz}$  source.

Find the

- a) capacitive reactance and

b) the current (rms and peak) in the circuit.

If the frequency is doubled, what happens to the capacitive reactance and the current?

### SOLUTION

a) The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 50 \times 15 \times 10^{-6}} = 212 \text{ ohms}$$

b) the rms current is

$$I = \frac{V}{X_C} = \frac{220}{212} = 1.04 \text{ A}$$

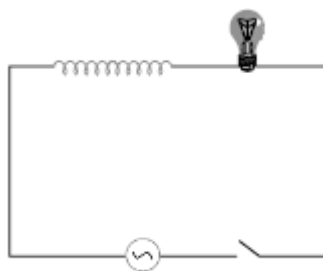
$$\text{peak current will be } I_0 = \sqrt{2} I = 1.41 \times 1.04 = 1.47 \text{ A}$$

This current oscillates between +1.47A and -1.47 A, and is ahead of the voltage by  $\pi/2$ .

If the frequency is doubled, the capacitive reactance is halved and consequently, the current is doubled

### EXAMPLE

A light bulb and an open coil inductor are connected to an ac source through a key as shown in Fig



The switch is closed and after sometime, an iron rod is inserted into the interior of the inductor. The glow of the light bulb

- (a) increases;
- (b) decreases;
- (c) is unchanged,

**As the iron rod is inserted. Give your answer with reasons.**

### **SOLUTION**

As the iron rod is inserted, the magnetic field inside the coil magnetizes the iron increasing the magnetic field inside it. Hence, the inductance of the coil increases. Consequently, the inductive reactance of the coil increases. As a result, a larger fraction of the applied ac voltage appears across the inductor, leaving less voltage across the bulb. Therefore, the glow of the light bulb decreases.

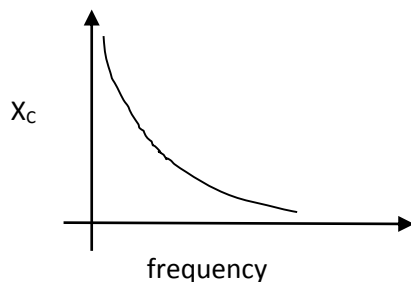
### **EXAMPLE**

**Plot a  $X_C - f$  graph**

### **SOLUTION**

$$X_C = \frac{1}{2\pi fC}$$

$X_C$  inversely proportional to frequency if capacitance is constant



### **PROBLEMS FOR PRACTICE**

- i). Consider a purely capacitive circuit (a capacitor connected to an AC source).
  - (a) How does the capacitive reactance change if the driving frequency is doubled?  
Halved?
  - (b) Are there any time instants when the capacitor is supplying power to the AC source?

ii). a) A capacitor of  $0.5 \mu\text{F}$  capacitance, is connected, as shown in Figure (x) to an AC generator with  $V = 300 \text{ V}$ . What is the amplitude  $I_0$  of the resulting alternating current if the angular frequency  $\omega$  is (i)  $100 \text{ rad/s}$ , and (ii)  $1000 \text{ rad/s}$ ?

(b) A  $45\text{-mH}$  inductor is connected, as shown in Figure (y), to an AC generator with  $V = 300 \text{ V}$ . The inductor has a reactance  $X_L = 1300\Omega$ .

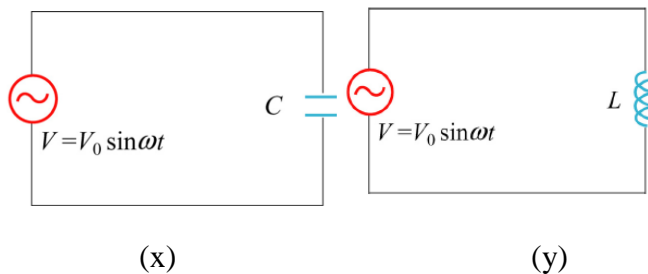
What must be?

(i) The applied angular frequency  $\omega$  and

(ii) The applied frequency  $f$

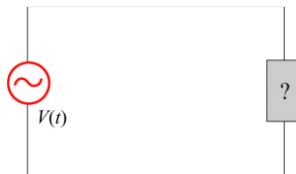
for this to be true?

What is the amplitude  $I_0$  of the resulting alternating current?



**Figure (x) a purely capacitive circuit, and (y) a purely inductive circuit.**

iii) An AC voltage source is connected to a “black box” which contains a circuit, as shown in Figure x. Figure x A “black box” connected to an AC voltage source. The elements in the circuit and their arrangement, however, are unknown. Measurements outside the black box provide the following information:



$$V(t) = (80 \text{ V}) \sin \omega t$$

$$I(t) = (1.6 \text{ A}) \sin(\omega t + 45^\circ)$$

- (a) Does the current lead or lag the voltage?
- (b) Is the circuit in the black box largely capacitive or inductive?
- (c) Is the circuit in the black box at resonance?
- (d) What is the power factor?
- (e) Does the box contain a resistor? A capacitor? An inductor?
- (f) Compute the average power delivered to the black box by the AC Source.

iv) A resistor of  $200\ \Omega$  and a capacitor of  $15.0\ \mu\text{F}$  are connected in series to a  $220\ \text{V}$ ,  $50\ \text{Hz}$  ac source. Calculate

- (a) The current in the circuit;
- (b) The voltage (rms) across the resistor and the capacitor.

Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

a)  $0.755\ \text{A}$ ; b)  $220\ \text{V}$

v) **At an airport, a person is made to walk through the doorway of a metal detector, for security reasons. If she/he is carrying anything made of metal, the metal detector emits a sound. On what principle does this detector work?**

### **SOLUTION:**

The metal detector works on the principle of resonance in ac circuits. When you walk through a metal detector, you are, in fact, walking through a coil of many turns. The coil is connected to capacitor tuned so that the circuit is in resonance. When you walk through with metal, in your pocket, the impedance of the circuit changes; resulting in significant change in current in the circuit. This change in current is detected and the electronic circuitry causes a sound to be emitted as an alarm.

**Suppose the frequency of the source in the previous example can be varied.**

- (a) What is the frequency of the source at which resonance occurs?

(b) Calculate the impedance, the current, and the power dissipated at the resonant condition.

a) 35.4Hz; b)  $3\Omega$ , 66.7A, 13.35 kW

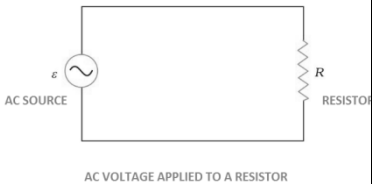
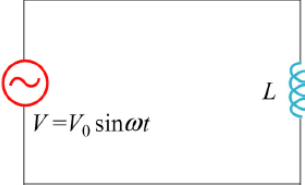
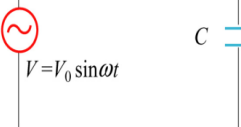
vi). When an electric device x is connected to 220V AC supply, the current is 0.5 A and is in the same phase as potential difference, when another device y is connected to the same supply the current is again 0.5 A but it leads the potential difference by  $\pi/2$  radians. Name the device x and y. (x resistance and y is a capacitor)

vii). What is the meaning of wattles current?

## 10. SUMMARY

In this module we have learnt:

- ac circuits
- components in ac circuits
- comparison of circuit component in ac circuit with that if used in dc circuit
- reactance mathematically
- ac circuit with pure R
- ac circuit with pure L
- ac circuit with Pure C
- Phasor, graphs for each

	Pure resistance circuit	Pure inductive circuit	Pure capacitive circuit
Circuit diagram			
Input voltage	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$
Current	$I = \frac{V_0}{R} \sin \omega t$	$I = I_0 \sin(\omega t - \frac{\pi}{2})$	$I = I_0 \sin(\omega t + \frac{\pi}{2})$

	$I = I_0 \sin \omega t$		
reactance	<b>R</b>	$X_L = 2\pi fL$	$X_C = 1/2\pi fC$
Current voltage graph			
Current voltage phasor			