1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 03 (Physics Part-1, Class XII)
Module Name/Title	Unit-03, **Module-04: Cyclotron** Chapter-04: Moving Charges and Magnetism
Module Id	Leph_10404_eContent
Pre-requisites	Vector algebra, magnetic field around a current carrying conductor , circular motion , kinematics and dynamics of motion, basic mathematics
Objectives	**After going through this module, the learners will be able to:** • **Realize** why a force acts on a moving charge in a magnetic field • **Justify** conditions for maximum and minimum force on moving charges as they move in a uniform magnetic field • **Understand** right hand palm rule and its application • **Explain** the path of charged particles in uniform electric and uniform magnetic field • **Know** the principle of velocity selector • **Comprehend and appreciate** the principle, design and working of a cyclotron • **Recognize** the limitations of a cyclotron
Keywords	Magnetic field, Lorentz force, force on current carrying conductor, motion of charged particle in magnetic field and cyclotron, velocity selector

2. Development Team

Role	Name	Affiliation
National MOOC Coordinator (NMC)	Prof. Amarendra P. Behera	Central Institute of Educational Technology, NCERT, New Delhi
Programme Coordinator	Dr. Mohd Mamur Ali	Central Institute of Educational Technology, NCERT, New Delhi
Course Coordinator / PI	Anuradha Mathur	Central Institute of Educational Technology, NCERT, New Delhi
Subject Matter Expert (SME)	Alpana Malik	St. Thomas School, Mandir Marg, New Delhi-110001.
Review Team	Associate Prof. N.K. Sehgal (Retd.) Prof. V. B. Bhatia (Retd.) Prof. B. K. Sharma (Retd.)	Delhi University Delhi University DESM, NCERT, New Delhi

TABLE OF CONTENTS

1. UNIT SYLLABUS

UNIT –III: Magnetic Effects of Current and Magnetism, 10 Modules

Chapter-4: Moving Charges and Magnetism

Concept of magnetic field, Oersted's experiment.

Biot - Savart law and its application to current carrying circular loop.

Ampere's law and its applications to infinitely long straight wire, Straight and toroidal solenoids, Force on a moving charge in uniform magnetic and electric fields, Cyclotron.

Force on a current-carrying conductor in a uniform magnetic field. Force between two parallel current-carrying conductors-definition of ampere. Torque experienced by a current loop in uniform magnetic field; moving coil galvanometer-its current sensitivity and conversion to ammeter and voltmeter.

Chapter-5: Magnetism and Matter

Current loop as a magnetic dipole and its magnetic dipole moment. Magnetic dipole moment of a revolving electron. Magnetic field intensity due to a magnetic dipole (bar magnet) along its axis and perpendicular to its axis. Torque on a magnetic dipole (bar magnet) in a uniform magnetic field; bar magnet as an equivalent solenoid, magnetic field lines; Earth's magnetic field and magnetic elements.

Para-, dia- and ferro - magnetic substances, with examples. Electromagnets and factors affecting their strengths. Permanent magnets.

2. MODULE WISE DISTRIBUTION OF UNIT SYLLABUS 10 MODULES

The above unit is divided into 10 modules for better understanding

Module 1	• Introducing moving charges and magnetism • Direction of magnetic field produced by a moving charge • Concept of Magnetic field • Oersted's Experiment • Strength of the magnetic field at a a point due to current carrying conductor • Biot-Savart Law • Comparison of coulomb's law and Biot Savarts law
Module 2	• Applications of Biot- Savart Law to current carrying circular loop, straight wire • Magnetic field due to a straight conductor of finite size Examples
Module 3	• Ampere's Law and its proof • Application of amperes circuital law: straight wire and toroidal solenoids. • Force on a moving charge in a magnetic field • Unit of magnetic field Examples
Module 4	• Force on moving charges in uniform magnetic field and uniform electric field.

	• Lorentz force • Cyclotron
Module 5	• Force on a current carrying conductor in uniform magnetic field • Force between two parallel current carrying conductors • Definition of ampere
Module 6	• Torque experienced by a current rectangular loop in uniform magnetic field • Direction of torque acting on current carrying rectangular loop in uniform magnetic field • Orientation of a rectangular current carrying loop in a uniform magnetic field for maximum and minimum potential energy
Module 7	• Moving coil Galvanometer- • Need for radial pole pieces to create a uniform magnetic field • Establish a relation between deflection in the galvanometer and the current • its current sensitivity • Voltage sensitivity • conversion to ammeter and voltmeter Examples
Module 8	• Magnetic field intensity due to a magnetic dipole (bar magnet) along its axis and perpendicular to its axis. • Torque on a magnetic dipole in uniform magnetic field. • Explanation of magnetic property of materials
Module 9	• Dia, Para and ferromagnetic substances with examples. Electromagnets and factors affecting their strengths, permanent magnets.
Module 10	• Earth's magnetic field and magnetic elements.

Module 4

3. WORDS YOU MUST KNOW

- **Coulomb's law**: The mutual force of attraction or repulsion between two point charges is directly proportional to the product of two charges (q_1 and q_2) and inversely proportional to the square of the distance between them. It acts along the line joining them.

- **Electric field**: Electric force due to a charge is experienced in the space surrounding it at a location.at a location the electric field is expressed as the electrostatic force per unit charge E (r) placed at the location.. It is a vector.. Electric field follows the superposition principle.
- **Principle of superposition**: The combined electric or magnetic field due to several sources is the vector addition of electric and magnetic field due to each individual source.
- **Electric current**: The time rate of flow of charge in a conductor.
- **Magnetic field**: Magnets, Currents and moving charges produce magnetic fields. Denoted by B(r). It is a vector field like an electric field.
- **Electric and Magnetic field line**: It is a curve, the tangent to which a point gives the direction of the electric or magnetic field at that point.
- **Maxwell's cork screw rule or right hand screw rule**: It states that if the forward motion of an imaginary right handed screw is in the direction of the current through a linear conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor.
- **Biot-Savart law**: According to Biot-Savart law, the magnetic field *dB* at *P* due to the current element Idl is given by

$$\mathbf{dB = \mu_0 \frac{Idl\sin\theta}{r^2}}$$

- **Ampere's circuital law:** $\oint \mathbf{B.dl} = \mu_0 \mathbf{I}$

  An alternative way to determine the magnetic field at a location around a conductor carrying current .An amperian loop is imagined tm symmetrically encircle the conductor or conductors with steady currents . The line integral of B.dl along a closed loop is equal to the product of permeability and the net current enclosed by the amperian loop.

- **Solenoid:** A coil with a large number of turns. On passing current in the coil, it has a strong uniform magnetic field inside its core, $B = \mu_0 n I$.
- **Toroid:** A hollow circular ring on which a large number of turns of a wire are closely would. On passing current the magnetic field is constant and restricted within the coil, $B = \mu_0 n I$.

4. INTRODUCTION

We have been discussing the relation between electric charge and electric fields. In our unit 1 of this course we have studied in detail about the influence of stationary charges in the surrounding space, we called this region of influence electric field.**. We also closely looked at moving charges, currents in wires. In the previous modules of this unit, our concentration was on study of magnetic field due to current in conductors. We made use of Biot Savart's law and Ampere's circuital law to determine the value of magnetic field due to a steady current in a wire.**

The fact that currents produce magnetic fields, moving charges are also associated with current like behaviour. It is for this reason that the magnetic field due to a moving charge, or a current carrying conductor will interact with an external magnetic field.

We will now study the mechanical force (Lorentz force) on a charge moving in an external magnetic field. We will also learn methods to calculate and find the direction of this force

5. LORENTZ FORCE

CHARGED PARTICLE MOVING IN AN UNIFORM ELECTIC FIELD

Let us suppose that there is a point charge *q* (moving with a velocity v in presence of electric field E

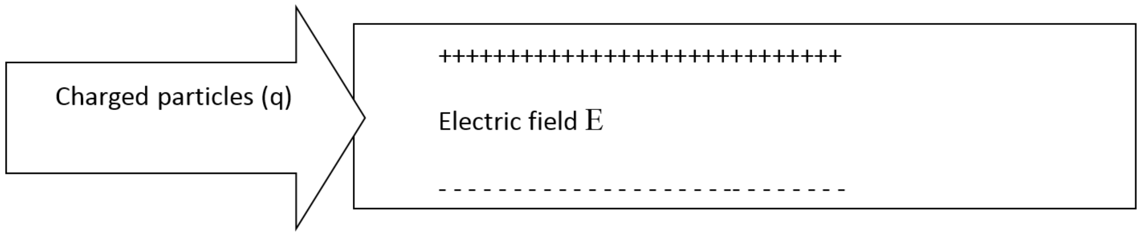


Figure shows a stream of charged particles moving into a uniform electric field directed from positive to negative

The force on an electric charge *q* due to electric field E will be

$$F_{electric} = q\,E \text{ or } F = q\,E$$

Direction of electric force will depend upon

- **Sign on charge +ve or – ve**
- **Magnitude of charge**
- **Magnitude of electric field E**
- **Direction of field E**

EXAMPLE

A and B represent two metallic discs. They are alternately charged + ve and – ve . A small charged balloon (balloon rubbed with woolen cloth) is floated in the space between A and B.

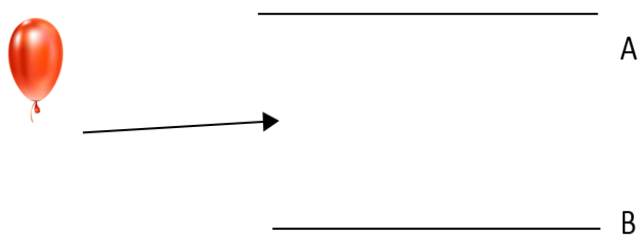


Figure shows a charged balloon moving into a fluctuating electric field

Describe the path of the balloon in the space between the discs

SOLUTION

The balloon will be negatively charged. It would be alternately attracted by the positive plate, it would go up and down. This will continue till it loses its charge to the atmosphere or is out of the influence of the electric field between the metallic discs.

CHARGED PARTICLE MOVING IN A UNIFORM MAGNETIC FIELD

If we have the charged particles *q* (moving with a velocity v and, located at r at a given time *t*) in presence of uniform magnetic field B

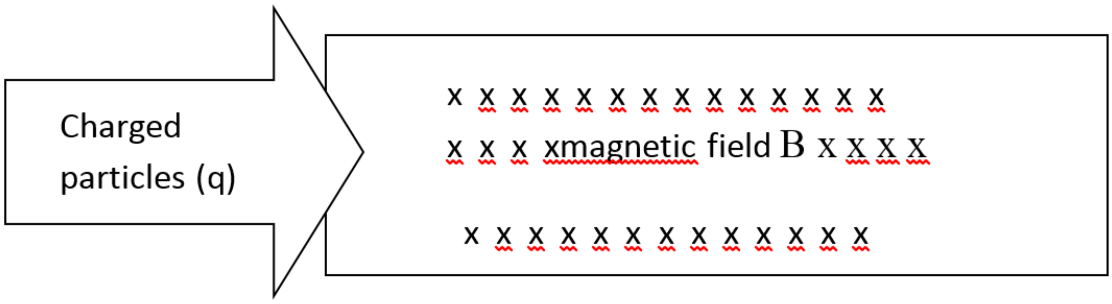


Figure shows a stream of charged particles moving into a uniform magnetic field directed into the plane of the screen

From the previous module

$$\mathbf{F}_{\mathbf{magnetic}} = \mathbf{q}\,\mathbf{v} \times \mathbf{B}(\mathbf{r})$$

Here direction of magnetic force will depend upon

- **Sign on charge +ve or –ve**
- **Magnitude of charge**
- **Velocity of charged particle v (both magnitude of velocity and direction of velocity)**
- **Magnetic field B (r) (both magnitude and direction)**
- **The angle between vectors v and B (r)**

In case both electric and magnetic fields act on the moving charged particle simultaneously

The net force will be vector addition of F electric and F magnetic

$$\mathbf{F}_{net} = \mathbf{F}_{electric} + \mathbf{F}_{magnetic} = q\,[\mathbf{E}(\mathbf{r}) + \mathbf{v} \times \mathbf{B}(\mathbf{r})]$$

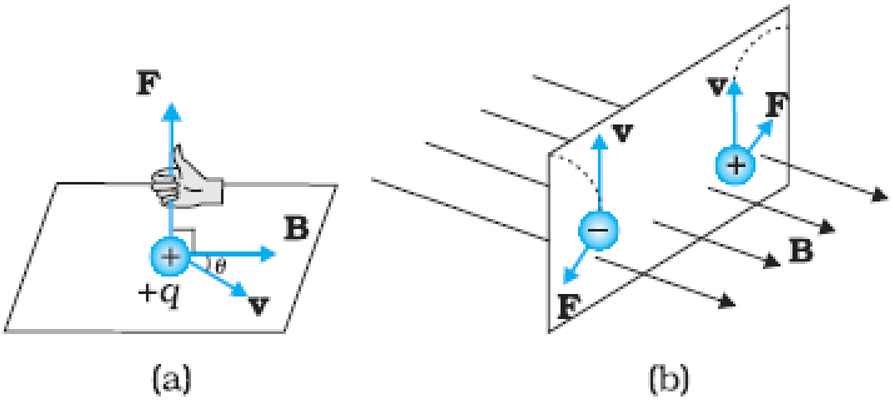
This force was given first by H.A. Lorentz based on the extensive experiments of Ampere and others. It is called the Lorentz force.

You have already studied in detail the force on a charged particle due to an electric field. If we look at the interaction with the magnetic field, we find the following features.

(i) **It depends on *q*, v and B (charge of the particle, the velocity and the magnetic field).** ***Force on a negative charge is opposite to that on a positive charge.***

(ii) The magnetic force *q* [v × B] includes a vector product of velocity and magnetic field. The vector product makes the force due to magnetic field vanish (become zero) if velocity and magnetic field are parallel or anti-parallel.

(iii) The force acts in a (sideways) direction perpendicular to both the velocity and the magnetic field. Its direction is given by the screw rule or right hand rule for vector (or cross) product as illustrated in Fig.



(iv) The magnetic force is zero if charge is not moving (as then |v|= 0).

(v) **Only a moving charge experiences the magnetic force.**

(vi) **The expression for the magnetic force helps us to define the unit of the magnetic field, if one takes *q*, F and v, all to be unity in the force equation**

F = *q* [v × B] =*q v B* sin θ ˆn , where θ is the angle between v and B Fig (a)

We can say, the magnitude of magnetic field *B* is 1 SI unit when the force acting on a unit charge (1 C), moving perpendicular to B with a speed 1m/s.

Dimensionally, we have [*B*] = [*F/qv*] and the unit of B,are Newton second / (coulomb meter).
This unit is called *tesla* (T) named after Nikola Tesla (1856 – 1943).
Tesla is a rather large unit.
A smaller unit (non-SI) called *gauss* (=10^{-4} tesla) is also often used.
The earth's magnetic field is about 3.6 × 10^{-5} T.

THINK ABOUT THIS

Lorentz force is the combination of electric and magnetic force on a point charge due to electromagnetic fields.

So what are your predictions for?

a) **Stationary charge in only an electric field**
   - **field is uniform /**
   - **field is non uniform**
b) **Stationary charge in only a magnetic field**
   - **(field is uniform /**
   - **field is non uniform**
c) **Moving charge in a uniform electric field,**
   - **Moving in the direction of the field,**
   - **opposite to the direction of the field,**

- **perpendicular to the direction of the field or**
- **at an angle to the field direction**

d) Moving charge in a uniform magnetic field

- **Moving in the direction of the field,**
- **opposite to the direction of the field,**
- **perpendicular to the direction of the field or**
- **at an angle to the field direction**

And also …

If we clean our spectacles with silk the glass in the spectacle acquires a +ve charge. Now if we move the spectacles in a magnetic field (produced by a magnet or a steady current in a wire) , hold it stationary in magnetic field , make the spectacle move in a circle in the magnetic field ,. Would the Lorentz force act? Would its value (magnitude and direction) change? Will we feel the effect of the Lorentz force?

6. MAGNETIC FORCE ON A CURRENT CARRYING CONDUCTOR PLACED IN AN EXTERNAL MAGNETIC FIELD

We can extend the analysis for force due to magnetic field on a single moving charge to a straight rod carrying current.

Consider a rod of a uniform cross-sectional area *A* and length *L*.

We shall assume one kind of mobile carriers as in a solid conductor (electrons). Let the number density (number of electrons per unit volume) of these mobile charge carriers in it be *n*.

Then the total number of mobile charge carriers in it **is n L A.**

For a steady current *I* in this conducting rod, we may assume that each mobile carrier has an average drift velocity v_d.

In the presence of an external magnetic field B, the force on these carriers is:

$$\mathbf{F} = (nLA)q\ \mathbf{v_d} \times \mathbf{B}$$

Where, *q* is the value of the electron charge

Now **n q v_d = current density j**

Hence ,

| (nq v_d) | A = current I

Thus,

F = [(nqv_d) L A] × B = [j A L] × B

= I L × B

Where *L* is a vector of magnitude | *L*|, the length of the rod, and with a direction identical to the current *I*.

Note that the current *I* is not a vector.

Equation F = **I L × B** holds for a straight rod.

Quick to remember would be F = B I L

In this equation, **B is the external magnetic field**. **It is not the field produced by the current-carrying rod.**

If the rod or wire has an arbitrary shape we can calculate the Lorentz force on it by considering it as a collection of linear strips d*l and* summing up or using the method of integration

F = Σ I dL × B

7. PREDICTING DIRECTION AND MAGNITUDE OF LORENTZ FORCE DUE TO MAGNETIC FIELDS, CURRENT OR VELOCITY OF CHARGED PARTICLE

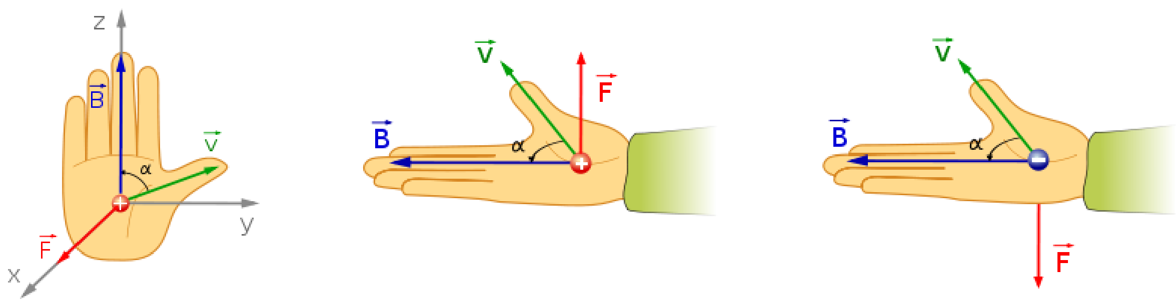
F = q v x B

F= q |v x B|= q v B sin θ, where θ is the angle between velocity vector and vector B

F= |BIL|

Prediction for directions can be done using right hand palm rule.

If we stretch the palm of the right hand and hold the thumb perpendicular to the fingers If we take the fingers to be pointing in the direction of the magnetic field B, the thumb to show the current or path of +ve charge, movement of the palm will be the direction of Lorentz force



- **What will be the path of a charged particle moving along the direction of uniform magnetic field?**

  F = q v B sin θ

  θ = 0 so F = 0

  charged particle is not deflected from its original path.

- **Under what condition an electron moving through a magnetic field experience maximum force?**

  F = q v B sin θ

  If θ = 90   so, F = q v B

  Electron should move perpendicular to the direction of magnetic field

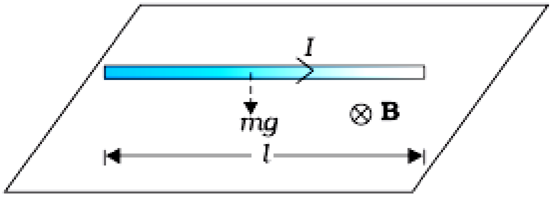
- **Under what condition will a proton experience minimum force in a magnetic field directed east to west?**

  Proton should move east to west or west to east for F to be zero or f to be minimum.

EXAMPLE

A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B. What is the magnitude of the magnetic field?

SOLUTION



The Lorentz force on the rod is upward force F

F= BIL

For mid-air suspension, this must be balanced by mg the weight of the rod

BIL = mg

$B = \frac{mg}{IL}$

$= \frac{0.2\ kg \times 9.8 ms^{-2}}{2A \times 1.5\ m}$

= **0.65 T**

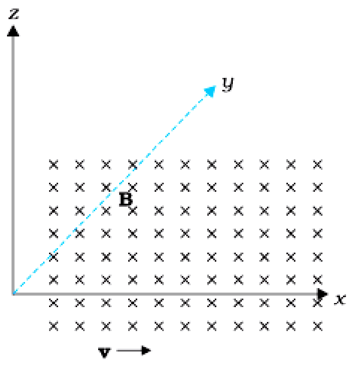
EXAMPLE

If the magnetic field is parallel to the positive *y*-axis and the charged particle is moving along the positive *x*-axis which way would the Lorentz force be if the charged particle is :

(a) an electron (negative charge),

(b) a proton (positive charge).

SOLUTION



Let us understand this.

The velocity v of particle is along the + ve *x* -axis,

While B, the magnetic field is along the +ve *y* -axis, i.e into the screen,

So v × B is along the *z*-axis (right-hand thumb rule or right hand palm rule).

So, (a) for electron it will be along –*z* axis. (b) for a positive charge (proton) the force is along +*z* axis.

Why is the direction of Lorentz force different n the two cases? The reason for this is that the current direction is taken according to conventional direction of current.

Watch the video for more.
https://www.youtube.com/watch?v=fwiKRis145E

EXAMPLE

In a chamber, a uniform magnetic field of 6.5 G (1 G = 10 $^{-4}$ T) is maintained. An electron is shot into the field with a speed of 4.8 × 10^6 m s^{-1} normal to the field. Calculate the force on the electron also give the direction of force
(*e* = 1.6 × 10 $^{-19}$ C)

SOLUTION

F= q v B

$= 1.6 \times 10^{-19} C \times 4.8 \times 10^{6} ms^{-1} \times 6.5 \times 10^{-4} T$

=49.92 x 10^{-17} N

Direction will depend upon direction of uniform magnetic field, direction in which electron enters the field and using right hand palm rule.

EXAMPLE

A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire?

SOLUTION

F= B I L

$=0.27\,T \;\times 10A \times 3 \;\times 10^{-2}m$

=8.1 x 10^{-2} N, direction given by right hand palm rule .

EXAMPLE

What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30º with the direction of a uniform magnetic field of 0.15 T?

SOLUTION

F = BIL sin θ

$= 0.15T \times 8 \;\times \sin 30$

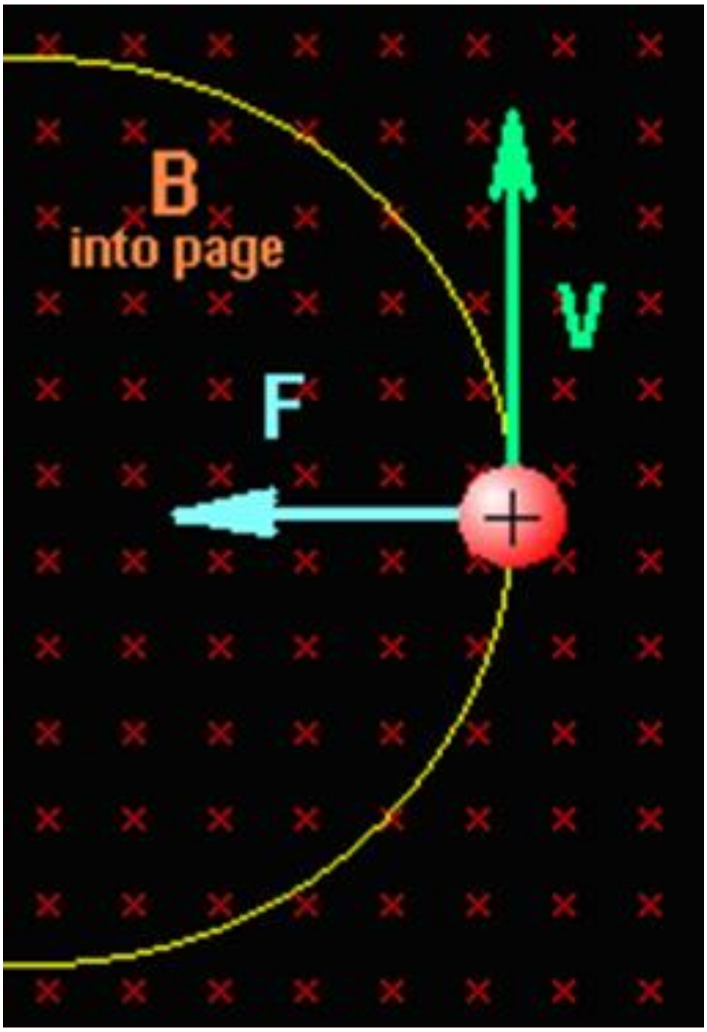
=0.6 N m^{-1}

8. MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

We will now consider, **the motion of a charge moving in a magnetic field**. We have learnt in Mechanics that a force on a particle does work if the force has a component along (or opposite to) the direction of motion of the particle.

Note since in case of motion of a charge in a magnetic field,, the magnetic force is perpendicular to the velocity of the charged particle. **So no work is done and no change in the magnitude of the velocity is produced** (though the direction of momentum may change).

http://www.phys.hawaii.edu/~teb/optics/java/partmagn/index.html



The graphic shows magnetic field directed into the screen, the force on the charged particle is perpendicular to the velocity direction. This should make the charged particle move in a circular track

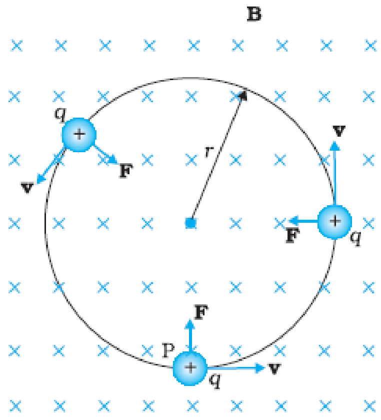
Remember that this is unlike the force due to an electric field, *q*E, which *can* have a component parallel (or antiparallel) to motion and thus can transfer energy in addition to momentum.

We shall consider motion of a charged particle in a ***uniform* magnetic field**. First consider the case of v perpendicular to B.

The perpendicular force, *q* v × B, acts as a **centripetal force** and produces **a circular motion** perpendicular to the magnetic field.

The particle will describe a circle if v and B are perpendicular to each other

If velocity has a component along B, this component remains unchanged as the motion along the magnetic field will not be affected by the magnetic field. The motion in a plane perpendicular to B is as before a circular one, thereby producing **a *helical* motion**



You have already learnt in earlier classes (See Class XI, Chapter 4) that if *r* is the radius of the circular path of a particle, then a force of

$$\mathbf{F} = \frac{\mathbf{mv^2}}{\mathbf{r}}$$

acts perpendicular to the path towards the centre of the circle, and is called the **centripetal force.** If the velocity v is perpendicular to the magnetic field B, the magnetic force is perpendicular to both v and B and acts like a centripetal force.

It has a magnitude *q v B.*

Equating the two expressions for centripetal force,

$$\frac{\mathbf{mv^2}}{\mathbf{r}} = qvB$$

This gives us the radius of the circular path

$$\mathbf{r} = \frac{\mathbf{m\,v}}{\mathbf{qB}}$$

This will be the **radius of the circle described by the charged particle.**

So the radius would depend upon

- **Mass of the charged particle**
- **Velocity of the charged particle**

- **Charge on the charged particle**
- **Strength of the magnetic field**

Notice we have assumed that if all the above remain the same the charged particle will continue in a circular motion with no loss of energy

THINK ABOUT THESE

- **What if the mass of the particle changed?**
- **What if the speed or direction of particle is changed? Speed increases or decreases?**
- **What if some charge leaks from the particle? Or it attracts more charge?**
- **What if the field space ha non uniform field?**
- **What is the magnetic field strength increases?**
- **What if the field strength increases?**

For a uniform magnetic field

If ω is the angular frequency, then $v = \omega r$. So,

$$\omega = 2\pi f = \frac{qB}{m}$$

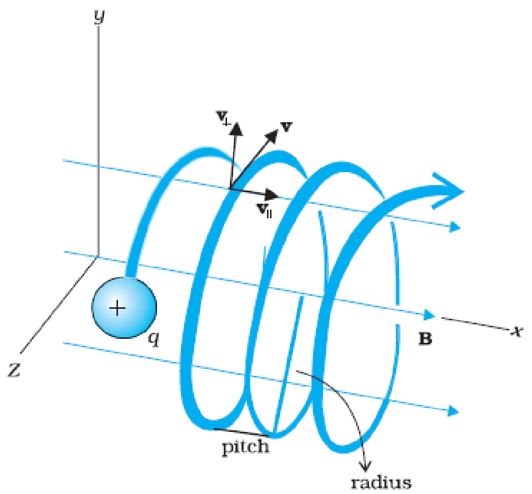
which is independent of the velocity or energy. **Here f is the frequency of rotation**.

The independence of 'f' from energy has important application in the design of a cyclotron.

The time taken for one revolution is

$$T = 2\pi/\omega \equiv 1/f.$$

If there is a component of the velocity parallel to the magnetic field (denoted by $v_{||}$), it will make the particle move along the field and the path of the particle would be a helical one



The distance moved along the magnetic field in one rotation is called pitch *p*.

$$p = v_{||}T = 2\pi m\, v_{||} / q B$$

The radius of the circular component of motion is called the *radius* of the *helix*.

HELICAL MOTION OF CHARGED PARTICLES AND AURORA BOREALIS

Beauty of magnetic effect on charged particles , just for your interest

In Polar Regions like Alaska and Northern Canada, a splendid display of colors is seen in the sky.

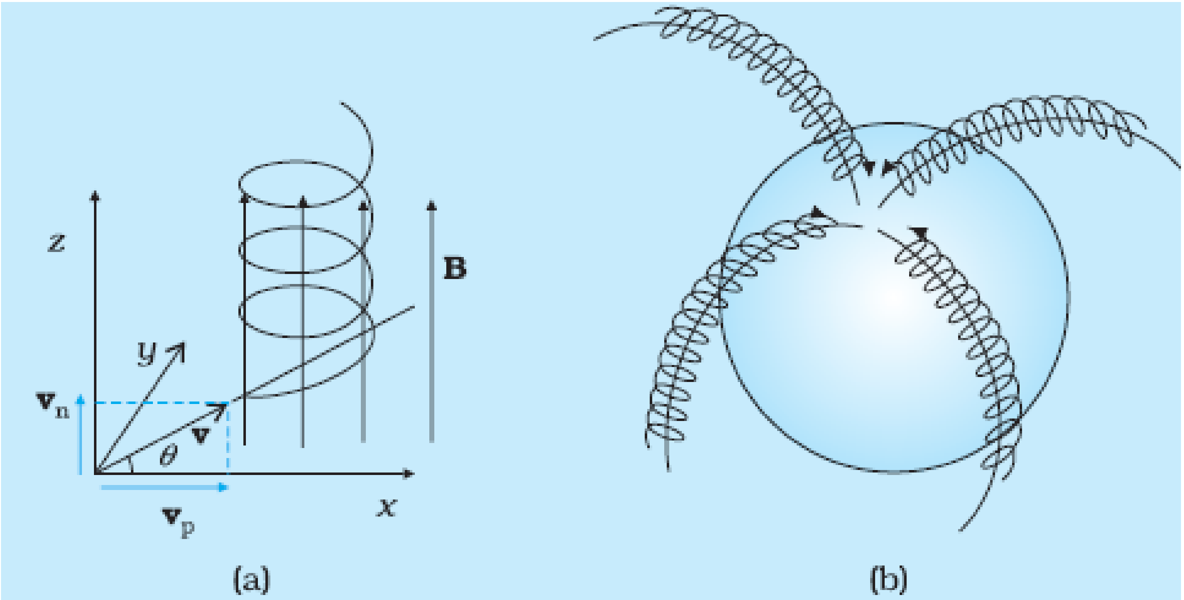
The appearance of dancing green pink lights is fascinating, and equally puzzling.

An explanation of this natural phenomenon is now found in physics, in terms of what we have studied here.

Consider a charged particle of mass *m* and charge *q*, entering a region of magnetic field B with an initial velocity v. Let this velocity have a component v_p parallel to the magnetic field and a component v_n normal to it. There is no force on a charged particle in the direction of the field. Hence the particle continues to travel with the velocity v_p parallel to the field. The normal component v_n of the particle results in a Lorentz force ($v_n \times B$) which is perpendicular to both v_n and B. As seen the particle thus has a tendency to perform a circular motion in a plane perpendicular to the magnetic field.

When this is coupled with the velocity parallel to the field, the resulting trajectory will be a helix along the magnetic field line, as shown in Figure (a) here.

Even if the field line bends, the helically moving particle is trapped and guided to move around the field line. Since the Lorentz force is normal to the velocity of each point, the field does not work on the particle and the magnitude of velocity remains the same.



During a solar flare, a large number of electrons and protons are ejected from the sun.

Some of them get trapped in the earth's magnetic field and move in helical paths along the field lines. The field lines come closer to each other near the magnetic poles; see figure (b).

Hence the density of charges increases near the poles. These particles collide with atoms and molecules of the atmosphere. Excited oxygen atoms emit green light and excited nitrogen atoms emits pink light. This phenomenon is called *Aurora Borealis* in physics

EXAMPLE

What is the radius of the path of an electron (mass 9×10^{-31} kg and charge 1.6×10^{-19} C) moving at a speed of 3×10^{7} m/s in a magnetic field of 6×10^{-4} T perpendicular to it? What is its frequency?

SOLUTION

$$r = \frac{m\,v}{qB}$$

$$= \frac{9\times10^{-31}\text{kg}\times3\times10^{7}\text{ms}^{-1}}{1.6\times10^{-19}\text{C}\times6\times10^{-4}\text{T}}$$

$=26 \text{ x } 10^{-2}\text{m} = 26 \text{ cm}$

Frequency = speed / circumference

$=2 \times 10^{6}\text{s}^{-1}$

= 2 MHz

In the above example how would the result change, if the electron travelled at only a very small angle to the magnetic field?

EXAMPLE

An alpha particle and a proton are moving in the plane of the screen in a region under the influence of a uniform magnetic field. If the particles have equal linear momentum, what would be the ratio of the radii of their trajectories?

SOLUTION

$r = \frac{m\,v}{qB}$

Or $r = \frac{p}{qB}$

$r \propto \frac{1}{q}$ as p and B are the same

$\frac{r_\alpha}{r_p} = \frac{q_\alpha}{q_p} = \frac{+e}{+2e} = \frac{1}{2}$

The radius of the circular track of alpha particle will be half of that for the proton.

How would the ratio change if the particles were alphas particle and an electron?

EXAMPLE

An electron and a proton are introduced into a uniform magnetic field. The speed of both particles is the same, and the initial direction is perpendicular to the magnetic field. What is the ratio of the radius of the circular paths of the electron and proton?

SOLUTION

$r = \frac{m\,v}{qB}$

$\boldsymbol{r \propto m}$ **Since q, B and v are the same**

$\frac{r_e}{r_p} = \frac{m_e}{m_p} = \frac{1}{1840}$

The radius of circular path of the proton is 1840 times the radius of circular path of the electron

9. MOTION IN COMBINED ELECTRIC AND MAGNETIC FIELDS – VELOCITY SELECTOR

We know that a charge q moving with velocity v in presence of both electric and magnetic fields experience a force given by equation

$$\mathbf{F} = \mathbf{q}[\mathbf{E}(\mathbf{r}) + \mathbf{v} \times \mathbf{B}(\mathbf{r})] = \mathbf{F}_{\mathbf{electric}} + \mathbf{F}_{\mathbf{magnetic}}$$

Or $$F = q[E + v\,x\,B] = F_E + F_B$$

We shall consider the simple case in which electric and magnetic fields are perpendicular to each other and also perpendicular to the velocity of the particle, as show in figure 4.

We have,

$$E = E\hat{j}$$

$$B = B\hat{k}$$

$$v = v\hat{\imath}$$

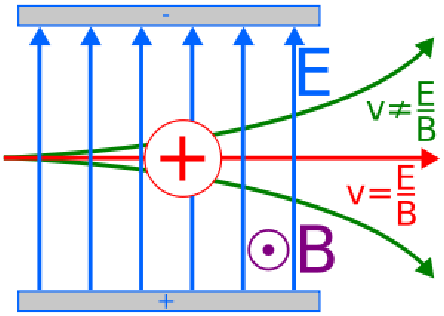
$$F_E = qE = qE\hat{j}$$

$$F_B = qV \times B = q\left[v\hat{\imath} \times B\hat{k}\right] = -qB\hat{j}$$

Therefore, $$\mathbf{F} = \mathbf{q}\,[\mathbf{E} - \mathbf{VB}]\hat{\mathbf{j}}$$

Thus, electric and magnetic forces are in opposite directions as shown in the figure, suppose we adjust the value of E and B such that the magnitude of the two forces is equal. **Then total force on the charge is zero and the charge will move in the fields un-deflected. This happens when,**

$$\mathbf{qE} = \mathbf{qvB} \quad \mathbf{or} \quad \mathbf{v} = \frac{\boldsymbol{E}}{\boldsymbol{B}}$$



The above discussion in this section allows us to use it for a device called **velocity selector. The purpose of such an arrangement would be to use velocity of a charged particle of known and desired values.**

This condition can be used to select charged particles of a particular velocity out of a beam containing charges moving with different speeds (irrespective of their charge and mass). The crossed *E* and *B* fields, therefore, serve as a *velocity selector*. Only particles with speed *E*/*B* pass un-deflected through the region of crossed fields.

This method was employed by J. J. Thomson in 1897 to measure the c**harge to mass ratio (e/m) of an electron.**

The principle is also employed in Mass Spectrometer – a device that separates charged particles, usually ions, according to their charge to mass ratio.

10. CYCLOTRON

The **Cyclotron was** invented by E.O. Lawrence and M.S. Livingston in 1934, for accelerating positively charged particles, such as protons & deuterons to very high energy so that they could be used in disintegration experiments to study physics of the nucleus.

PRINCIPLE: Cyclotron works on two principles:

a) The cyclotron uses both electric and magnetic fields in combination to increase the energy of charged particles. As the fields are perpendicular to each other they are called *crossed fields*.

b) Cyclotron uses the fact that the frequency of revolution of the charged particle in a magnetic field is independent of its energy.

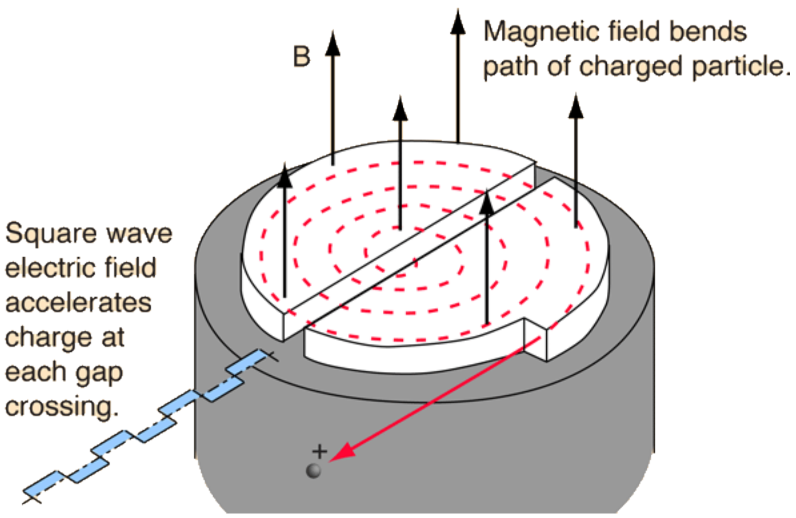
So the device uses an electric field to increase the magnitude of speed and the magnetic field brings it back to be accelerated again. The repeated entry of the charged particle into uniform electric and uniform magnetic field results in increase in energy of the charged particle

DESCRIPTION AND DIAGRAM:

It consist of two horizontal D shaped hollow aluminum or copper metal segments D_1 and D_2 (called the dees) with a small gap between them **see Figure**

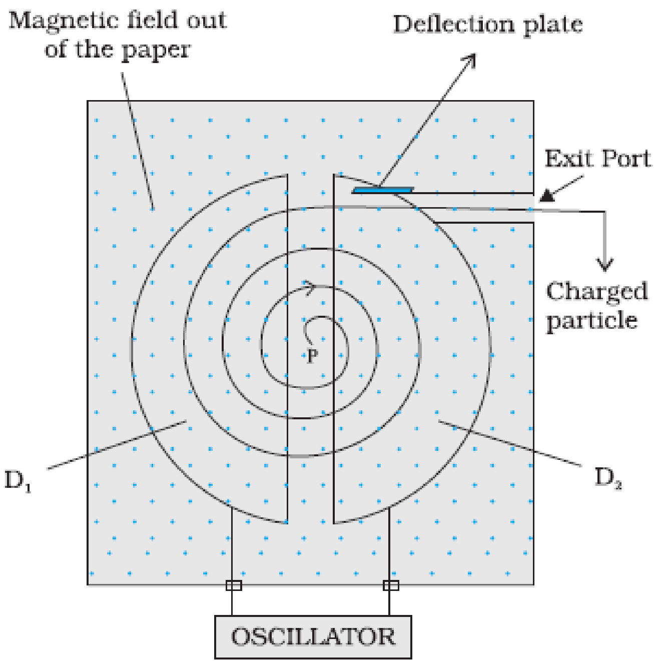
An alternating potential difference of the order of 10^5 volt, at a frequency of 10-15 MHz, is applied across the dees. An intense magnetic field $\vec{B}$ of about 1.6 tesla is set up perpendicular to the plane of dees by a large electromagnet.

The whole space inside the dees is evacuated to a pressure of about 10^{-6} mm of mercury.



Schematic sketch of the cyclotron

Top view

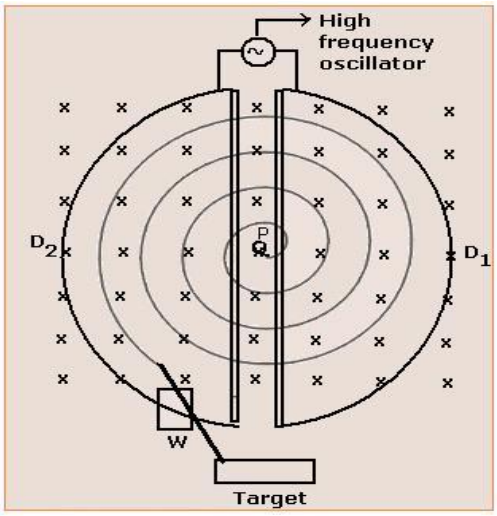


An ion source is located at the centre *P* in the gap between the dees. It consists of a small chamber, containing a heated filament and a gas such as hydrogen (for protons) or deuterium (for deuterons). The thermions given out of filament produce positive ions by ionization of the gas. The ions come out through a small hole in the ion source and are available to be accelerated.

The charged particle moves in circular tracks in the dees, D1 and D2, on account of a uniform perpendicular magnetic field B. An alternating voltage source accelerates these ions to high speeds. The ions are eventually 'extracted' at the exit port.

WORKING:

A positive ion is obtained from the source *P*. Electric field is created in the gap with help of H.T. Oscillator due to this strong electric field in the gap of charge gets accelerated and its velocity increases with increased velocity it enters one Dee say D_2. Inside the dee there is no electric field as **the charge resides only n the surface of conductors** but magnetic field does. Now due to magnetic field the charge particle turns in the circular path. After making half the circle, it comes out from D_2 enter the gap from opposite direction. During this time period the oscillator has reversed the polarity and the direction of electric field. Now electric field is from D_2 to D_1 which is along the velocity of charge particle, therefore, velocity measures and with this increased velocity the particle enters D_1. Once again electric field vanished inside the dee and the magnetic field turn the charge in circular path. After taking half the circle again it comes to the gap from D_1. By this time electric field is again reversed and once again it is along the direction of velocity i.e. from D_1 to D_2, hence acceleration takes place, the velocity further increases till it enters D_2.



The same process is repeated. Every time the particle enters the dee, its direction is turned around and every time when it enters the gap its velocity increases.

Mathematically

According to relation, $r = \frac{mv}{Bq}$, $r \propto v$

(The radius of circular path of charged particle keeps on increasing in every cycle; hence it makes a spiral with increasing radius.)

The time *t* required by ion to complete a semi-circle is given by

$$\mathbf{t} = \frac{\pi r}{v} = \frac{\pi m}{qB}$$

This shows that the time of passage of the ion through the dee is independent of the speed of the ion and of the radius of the circle.

It depends only on

- **The magnetic field *B* and**
- **The charge to mass ratio (*q/m*) of the ion.**

The frequency *of* the applied voltage is adjusted so that the polarity of the dees is reversed in the same time that it takes the ions to complete one half of the revolution. The requirement is called the *resonance condition*. The phase of the supply is adjusted so that when the positive ions arrive at the edge of D1, D2 is at a lower potential and the ions are accelerated across the gap. Inside the dees the particles travel in a region free of the electric field. The increase in their kinetic energy is qV each time they cross from one dee to another (*V* refers to the voltage across the dees at that time).

From

$$r = \frac{mv}{Bq}$$

It is clear that the radius of their path goes on increasing each time their kinetic energy increases. The ions are repeatedly accelerated across the dees until they have the required energy to have a radius approximately that of the dees. They are then deflected by a magnetic field and leave the system via an exit slit.

We have,

$v = \frac{qrB}{m}$ or $\frac{qRB}{m}$ (R is the radius of the dee)

or $v_{max} = \frac{qRB}{m}$ This is the maximum velocity an oscillator can create.

Finally the particle is allowed to come out from a window with help of a negatively charged plate or tube to hit the target.

Achievement of Resonant Condition: We have seen that the cyclotron operates under the condition that the frequency ν_o of the applied potential difference must be equal to the frequency ν of the circular revolution of the ion. That is

$$\nu = \nu_o$$

But $$\nu = \frac{1}{2t} = \frac{qB}{2\pi m}$$

$$\therefore \ \boldsymbol{\nu_o = \frac{qB}{2\pi m}}$$

In practice, the frequency ν_o of the electric oscillator is kept fixed, and the magnetic field B is varied until the above condition is satisfied.

MAXIMUM KINETIC ENERGY OF THE ACEELERATED CHARGED PARTICLE

The charged particle will attain maximum velocity near the periphery of the dee

We have already mentioned when $r = R$,

$$v_{max} = \frac{qBR}{m}$$

The corresponding kinetic energy of the particle is

$$K = \frac{1}{2}\mathbf{mv^2}_{\mathbf{max}} = \frac{\mathbf{q^2B^2R^2}}{\mathbf{2m}}$$

ENERGY GAINED PER REVOLUTION

If the electric field between the Dees is V.

Energy gained for one revolution will be qV + q V as the charged particle crosses the electric field **twice in one complete revolution.**

So **for n revolutions** the total energy will be 2 n q V.

This should be equal to the maximum kinetic energy.

$2\,n\,q\,V = \frac{q^2B^2R^2}{2m}$

LIMITATIONS :

- **The cyclotron cannot accelerate the particles to velocities as high as comparable to the velocity of light.** The reason is that at these velocities the mass m of a particle increase with increasing velocity according to

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where m_o is the rest mass of the particle and C is the velocity of light.

Therefore, the $t = \frac{\pi m}{Bq}$ taken by the particle to complete the successive semi-circular paths goes on increasing. Thus the particle becomes more and more out of step with the applied potential difference until it can no longer be accelerated further.

- **Electrons cannot be accelerated in a cyclotron**
  For a given energy, the velocity of an electron is much greater than that of a more massive particle like protons or deuterons and so the relativistic.....? Increase of mass is correspondingly much greater. Therefore, electrons very quickly get out of step with the potential difference. Hence the Cyclotron is not successful in accelerating electrons.

- **In some machines the frequency ν_o potential difference is decreased on the particle accelerates in such a way that the product νm remains constant and the potential difference is always in step with the rotating particle. Such machines are called 'Syncho-Cyclotrons'**.
- **Neutrons cannot be accelerated by a cyclotron** as they are not charged

USES OF CYCLOTRON:

- The high energy particles are used to bombard nuclei and study nuclear reactions

- The high energy particles are used to produce other high energy particles such as neutrons by collisions. The fast neutrons are used in nuclear reactions.
- It is used to implant ions in solids and modify their properties or even synthesize new materials
- It is used in nuclear medicine for diagnostics, treatment and research

EXAMPLE

A chamber is maintained at a uniform magnetic field of $5x10^{-3}$T. An electron with a speed of $5x10^{7}$m/s enters the chamber in a direction normal to the field. Calculate the radius of the path and the frequency of revolution of the electron. (The mass of electron is 9.1 x 10^{-31} kg and its charge is 1.6 x 10^{-19}C)

SOLUTION: The electron entering a magnetic field B perpendicularly with speed v experiences a magnetic force evB which provides the required centripetal force $\frac{mv^2}{2}$ for its circular path in the field. Thus

$$evB = \frac{mv^2}{2}$$

$$r = \frac{mv}{eB} = \frac{9.1 \text{ x } 10^{-31} \text{ x } 5 \text{ x } 10^{7}}{1.6 \text{ x } 10^{-19} \text{ x } 5 \text{ x } 10^{-3}} \text{ m}$$

$$r = 5.7 \text{ x } 10^{-2} \text{ m} = 5.7 \text{ cm}$$

The period of revolution along the circular path is

$$T = \frac{2\pi r}{v}$$

and so the frequency is $n = \frac{v}{2\pi r} = \frac{v}{2\pi} \text{ x } \frac{eB}{mv}$

$$\nu = n = \frac{eB}{2\pi m}$$

Putting the values

$$n = \frac{1.6 \text{ x } 10^{-19} \text{ x } 5 \text{ x } 10^{-3}}{2 \text{ x } 3.14 \text{ x } 9.1 \text{ x } 10^{-31}} = 1.4 \text{ x } 10^{8} \text{ Hz}$$

EXAMPLE

An electron after being accelerated through a potential difference of 10^4 V enters a uniform magnetic field of 0.04T perpendicular to its direction of motion. Calculate the radius of curvature of its trajectory.

SOLUTION:

An electron (mass m_1 charge e) accelerated through a potential difference V acquires a speed v given by

$$\frac{1}{2}mv^2 = eV \qquad \rightarrow (1)$$

an entering perpendicularly a magnetic field B, the electron will adopt a circular path of radius r, given by

$$\frac{mv^2}{r} = evB \qquad \rightarrow (2)$$

Eliminating v in equation (1) and (2) we get ;

$$r = \frac{1}{B}\sqrt{\frac{2mv}{e}} = \frac{1}{0.04\text{ T}}\sqrt{\frac{2 \text{ x } 9.1 \text{ x } 10^{-31} \text{ x } 10^4}{1.6 \text{ x } 10^{-19}}}$$

$$r = 84.3 \text{ x } 10^{-4} \text{ m} = 8.43 \text{ mm}$$

EXAMPLE

A proton with kinetic energy 10 eV, moves on a circular path in a uniform magnetic field. What shall be the kinetic energies of (i) an α particle; and (ii) a deuteron moveing on the same circular path in the same field?

SOLUTION:

Let m be the mass and e the charge of a proton. Suppose it is moving with velocity ϑ on a circular path of radius r in a magnetic field. The magnetic force on the proton provides the necessary centripetal force. That is

$$evB = \frac{mv^2}{r}$$

Or $$v = \frac{eBr}{m}$$

And $$K.E_p = \frac{1}{2}mv^2 = \frac{1}{2}\frac{e^2B^2r^2}{m} = K_p$$

(i) Charge on a particle is 2e and its mass is 4m. Hence its kinetic energy will be :

$$K.E_{\alpha} = \frac{1}{2}\frac{(2e)^2B^2r^2}{4m} = K.E_p = K_p = 10 \text{ eV}$$

(ii) The charge on deuteron is e and its mass is 2m. Hence its kinetic energy will be :

$$K.E_d = \frac{1}{2}\frac{(e)^2B^2r^2}{2m} = \frac{K_p}{2} = 5 \text{ eV}$$

EXAMPLE

In a Cyclotron a magnetic field induction of 1.4T is used to accelerate protons. How rapidly should the electric field between the Dees be reversed? The mass and charge of proton are 1.67x10^{-27} kg and 1.6x10^{-19} respectively.

SOLUTION:

The time *t* required by a charge particle of mass *m* and charge *q* to complete a semicircle in a Dee is given by

$$t = \frac{\pi m}{qB} \quad \text{where B is the magnetic field.}$$

Thus,

$$t = \frac{3.14 \text{ x } 1.67 \text{ x } 10^{-27}}{1.6 \text{ x } 10^{-19} \text{ x } 1.4} = 2.34 \text{ x } 10^{-8} \text{ s}$$

For the working of Cyclotron, this should also be the time taken for one, half cycle of the electric field between the Dees. That is, the field should reverse after energy 2.34x10^{-8} s. In other words, the frequency of the applied field should be $\frac{1}{2 \text{ x } 2.34\text{x}10{-8}} =$ 2.14 x 10^{7} s .

11. SUMMARY:

In this module we have learnt

- Moving charges produce magnetic field

- A mechanical force acts on a moving charge or a current carrying conductor placed in a magnetic field as the two fields interact.
- Lorentz force on a charged particle could be due to electric field , magnetic field or both

  The total force on a charge q moving with velocity v in the presence of magnetic and electric fields B and E, respectively is called the *Lorentz force*. It is given by the expression:

  $F = q\,(v \times B + E)$

- Force on a charged particle in a uniform field is given by F = q v x B

  on a stationary charge – no force.

  on a moving charge:

  i) magnitude of force = | F| = q v B sin θ

  ii) direction is given by right hand palm rule

- A straight conductor of length l and carrying a steady current I experiences a force F in a uniform external magnetic field B,

  $F = I\,l \times B$

  Where |l| = l and the direction of l is given by the direction of the current.

- Path of a charged particle in uniform magnetic field is circular

  $$\frac{mv^2}{r} = qvB$$

  Radius of the path depends upon charge, mass, velocity, magnetic field.

  The magnetic force q (v × B) is normal to v and work done by it is zero.

- Velocity selector is a device which uses Electric and magnetic field to cancel the effect of each other on a charged particle moving with a certain velocity

- Cyclotron a device to accelerate charged particles and ions

- Principle, description, working, theory, calculations, limitations and uses of a cyclotron

  The fact that the frequency of revolution is independent of the particle's speed and radius. This fact is exploited in, the cyclotron, which is used to accelerate charged particles.

.