Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 03 (Physics Part 1 Class XII)
Module Name/Title	Unit-03**, Module –02: Problems based on Biot Savart's law** Chapter-04 Moving Charges and Magnetism
Module Id	Leph_10402_eContent
Pre-requisites	Knowledge about Electric Current, Magnetic field, Biot Savart's Law, permittivity ,, permeability
Objectives	**After going through this module , the learners will be able to** : • **Apply Biot Savarts law to** ✓ Calculate the magnitude of magnetic field at a distance away from a current carrying conductor ✓ Predict the direction of magnetic field at a location using right hand rule ✓ Determine the magnetic field due to a straight conductor of finite size. ✓ Determine the magnetic field on the axis of a circular current loop. ✓ Solve problems using Biot Savart's law
Keywords	Magnetic field due to current carrying circular loop, Right hand thumb rule, Biot Savart law, applications of Biot Savart's law.

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Current loop as a magnetic dipole and its magnetic dipole moment. Magnetic dipole moment of a revolving electron. Magnetic field intensity due to a magnetic dipole (bar magnet) along its axis and perpendicular to its axis. Torque on a magnetic dipole (bar magnet) in a uniform magnetic field; bar magnet as an equivalent solenoid, magnetic field lines; Earth's magnetic field and magnetic elements.

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The above unit is divided into 10 modules for better understanding.

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MODULE 2

3. WORDS YOU MUST KNOW

- **Coulomb's law**: The force of mutual attraction or repulsion between two point charges is directly proportional to the product of two charges (q_1 and q_2) and inversely proportional to the square of the distance between them. It acts along the line joining them.
- **Electric current:** The time rate of flow of charge.
- **Magnetic field lines**: It is a curve, the tangent to which a point gives the direction of the magnetic field at that point.

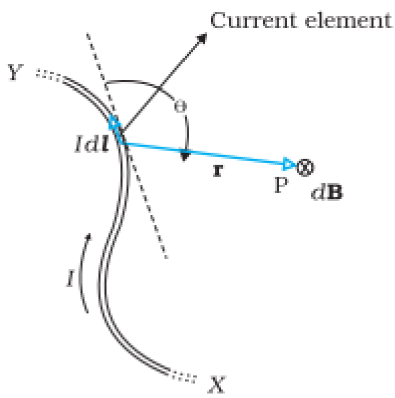
- **Biot-Savart law**: According to Biot-Savart law, the magnetic field *dB* at *P* due to the current element *Idl* is given by
  - $dB = \mu_0 \, Idl \sin\theta / 4\pi r^2$

- **Right hand thumb rule or curl rule:** If a current carrying conductor is imagined to be held in the right hand such that the thumb points in the direction of the current, then the tips of the fingers encircling the conductor will give the direction of the magnetic lines of force.
- **Maxwell's cork screw rule or right hand screw rule**: It states that if the forward motion of an imaginary right handed screw is in the direction of the current through a linear conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor.

4. INTRODUCTION

What is the Biot-Savart Law?

You will recall from our previous modules that electric fields and magnetic fields might seem different, but they're actually part of one larger force called the electromagnetic force. Charges that aren't moving produce electric fields. But when those charges do move, they instead create magnetic fields. Charges moving in an electric wire also produce magnetic fields. If we move a compass near to an electric wire, the compass needle changes direction or deflects.



The **Biot-Savart Law is a mathematical expression that describes the magnetic field created by a current-carrying wire, and allows you to calculate its strength at various points.**

To derive this law, we first take this equation for electric field. This is the full version, where we use $\mu_0 / 4\pi$ instead of the electrostatic constant *k*. Since we're looking at a wire, we replace the charge *q* with *I dl*, which is the current in the wire, multiplied by a length element in the wire. Basically it's treating this little chunk of the wire as our charge. And we also replace the electric field *E* with a magnetic field element *dB* because a moving charge produces a magnetic field, not an electric field.

Last of all, we have to realize that a current has a direction (unlike a charge). So we need to make sure the direction of the current affects our result. We do that by adding sine of the angle between the current and the radius. That way, if the wire is curvy, we'll take that into account. And that's it - that's the Biot-Savart law.

The magnetic field dB due to this element is to be determined at a point P which is at a distance r from it. Let θ be the angle between dl and the displacement vector r.

According to Biot-Savart's law, the magnitude of the magnetic field dB is proportional to the current I, the element length |dl|, and inversely proportional to the square of the distance r.

Its direction is perpendicular to the plane containing dl and r .

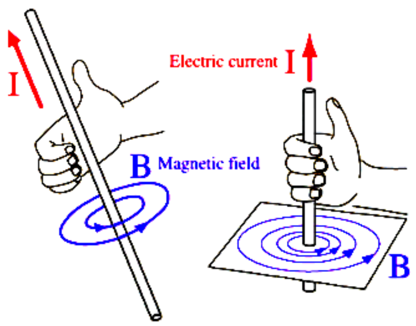
Thus, in vector notation:

$$\mathrm{dB} \propto \frac{\mathrm{Idl} \times \mathrm{r}}{\mathrm{r}^3}$$

$$\mathrm{dB} = \frac{\mu_0}{4\pi}\frac{\mathrm{Idl} \times \mathrm{r}}{\mathrm{r}^3}$$

Direction of the field is given by **Right hand grip rule**

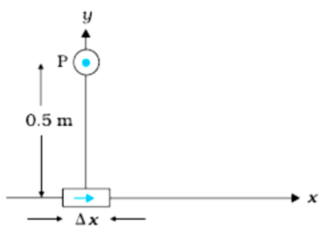
I want you to give the screen a thumb up, right now. I'm serious - give the screen a thumb up with your right hand. It has to be with your right hand. If you point your thumb in the direction of the current for this wire, your fingers will curl in the direction of the magnetic field. They'll follow the arrows of the concentric circles. And that's how you figure out the direction.



5. APPLICATION OF BIOT-SAVART LAW

(A) MAGNETIC FIELD DUE TO A STRAIGHT CURRENT CARRYING CONDUCTOR A FINITE SIZE

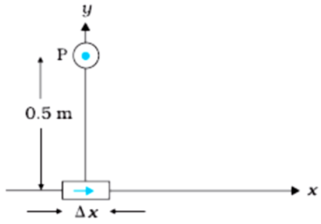
An element _l = *x* ˆ**i** is placed at the origin and carries a large current I = *10 A*. What is the magnetic field on the *y*-axis at a distance of 0.5 m. Δ*x* = 1 cm.



$$|dB| = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

dl = Δ*x* = 10^{-2} m , I = 10 A, r = 0.5 m = y, $\frac{\mu_0}{4\pi} = 10^{-7} Tm/A$

$\theta = 90^0, sin\theta = 1$



$$|dB| = \frac{10^{-7} \times 10 \times 10^{-2}}{25 \times 10^{-2}} = 4 \times 10^{-8} T$$

The direction of the field is in the + z direction:

$$dl \times r = \Delta x\hat{\imath} \times y\hat{\jmath} = y\Delta x(\hat{\imath} \times \hat{\jmath}) = y\Delta x\hat{k}$$

This is because of the cyclic property of cross products:

$\hat{\imath} \times \hat{\jmath} = \hat{k};\ \hat{\jmath} \times \hat{k} = \hat{\imath};\ \hat{k} \times \hat{\imath} = \hat{\jmath}$

In effect for a straight wire we can say

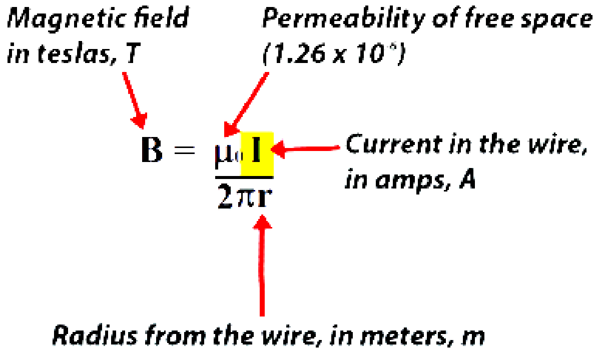
$B = \mu_0 I / 2\pi r$

Or

In other words, the magnetic field, *B*, measured in tesla is equal to the permeability of free space μ_0, multiplied by the current going through the wire *I*, measured in amps, divided by 2π times the distance away from the wire *r*, measured in meters.

So this equation helps us figure out the magnetic field in a radius *r* from a straight wire carrying a current *I*.

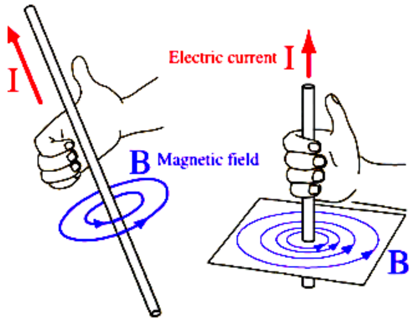
Notice the circle of radius r is perpendicular to the wire.



The equation gives us the magnitude of the magnetic field and direction by right hand grip rule.

This rule is used to know the direction of magnetic field due to a current-carrying conductor

According to this rule, "if we grasp a section of the wire conductor in our right hand such that the thumb points in the direction of current, then the fingers will encircle the conductor in the direction of the magnetic field."



EXAMPLE

A wire carrying a current of 0.1 amps, calculate the magnetic field at a distance of 0.5 meters from the wire.

SOLUTION

I is equal to 0.1 amps,

r is equal to 0.5 meters.

$\Theta = 90^0$

so B = ?.

$B = \mu_0 I / 2\pi r$

$B = 10^{-7} \times .1 / .5 =$ **4×10^{-8} Tesla**

THINK ABOUT THESE

- **Would the magnitude of B depend upon the length of the conductor?**
- **Would the magnitude of B depend upon the orientation of the conductor?**
- **Would the magnitude of B depend upon the current through the conductor?**
- **Would the magnitude of B depend upon environment around the conductor?**
- **Would the magnitude of B depend upon variation in the magnitude of current in the conductor?**

(B) MAGNETIC FIELD ON THE AXIS OF A CIRCULAR CURRENT LOOP

In this section, we shall evaluate **the magnetic field due to a circular coil along its axis**. The evaluation entails summing up the effect of infinitesimal current elements (I dl) mentioned in the previous section.

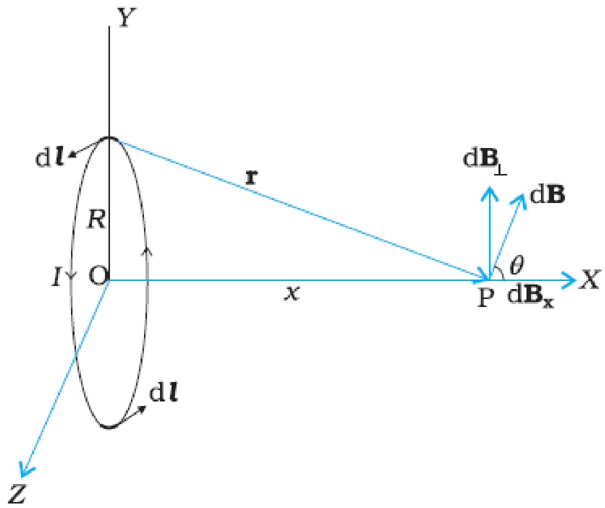
We assume that the current I is steady and that the evaluation is carried out in free space (i.e., vacuum).

Figure 4.11 depicts a circular loop carrying a steady current I. The loop is placed in the y-z plane with its centre at the origin O and has a radius R.

The x-axis is the axis of the loop.

We wish to calculate the magnetic field at the point P on this axis.

Let x be the distance of P from the centre O of the loop.



Consider a conducting element dl of the loop. This is shown in Figure. The magnitude dB of the magnetic field due to dl is given by the Biot-Savart law:

$$\mathbf{dB} = \frac{\mu_0}{4\pi}\frac{\mathbf{I|dl \times r|}}{\mathbf{r^3}}$$

Notice

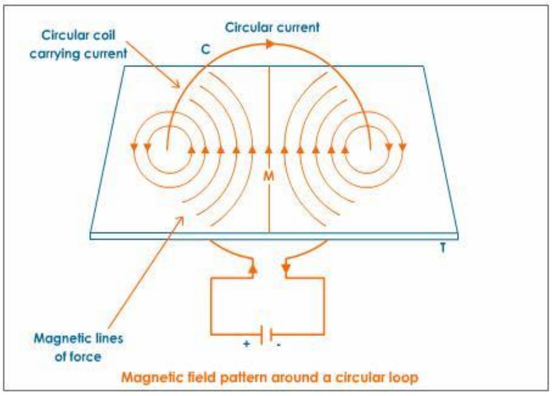
- ✓ $\mathbf{r^2 = x^2 + R^2}$

- ✓ any element of the loop will be perpendicular to the displacement vector 'r' from
  - **The element dl to the axial point.**
  - **The element d*l* in Figure is in the *y-z* plane whereas the displacement vector r from d*l* to the axial point P is in the *x-y* plane.**

Hence |dl × r| = r dl. Thus,

$$\mathbf{dB} = \frac{\mu_0}{4\pi}\frac{\mathbf{Idl}}{\mathbf{(x^2 + R^2)}}$$

- ✓ The direction of dB is shown in Figure. It is perpendicular to the plane formed by d*l* and r.
- ✓ It has an *x*-component dB *x* and a component perpendicular to *x*-axis, dB⊥. (in the y-z plane)
- ✓ When the components perpendicular to the *x*-axis are summed over, they cancel out and the net value =0.



For example, the dB⊥ component due to d*l* is cancelled by the contribution due to the diametrically opposite d*l* element, shown in Figure

- ✓ The net contribution is along the x direction

Its magnitude is obtained by integrating d*B x* = d*B* cos θ over the loop.

$$cos\theta = \frac{R}{(x^2 + R^2)^{1/2}}$$

Hence

$$\mathbf{dB = \frac{\mu_0 Idl}{4\pi}\frac{R}{(x^2 + R^2)^{3/2}}}$$

The summation of elements d*l* over the loop yields 2π*R*, the circumference of the loop. $\int \mathbf{dl = 2\pi R}$

Thus, the magnetic field at P due to entire circular loop is:

$$\mathbf{B = B_x \hat{i} = \frac{\mu_0 I}{2}\frac{R^2}{(x^2 + R^2)^{3/2}}}$$

SPECIAL CASES:

- ✓ **When point P is at the centre of the coil, then**

$$\mathbf{B_0 = \frac{\mu_0 I}{2R}}$$

the field is given by right hand rule at the centre of the coil

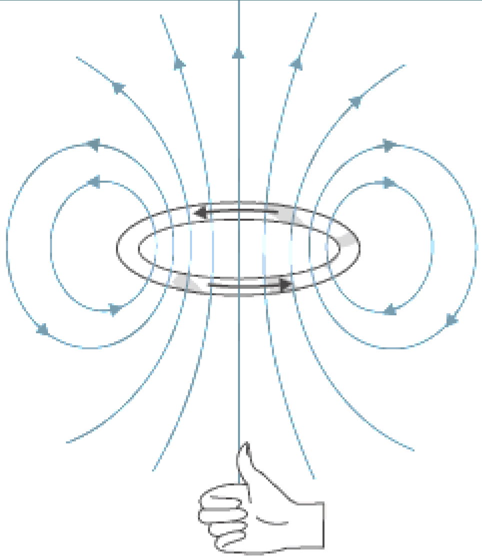


Figure shoes The magnetic field lines for a current loop. The direction of the field is given by the right-hand thumb rule described in the text. The upper side of the loop may be thought of as the north pole and the lower side as the south pole of a magnet

- ✓ When point P is at the centre of the coil of N turns then

$$B_0 = \frac{\mu_0 N\, I}{2R}$$

the field is given by right hand rule at the centre of the coil

- ✓ When P is very far away from the centre of the coil, then x >>> R, so:

$$B = \frac{\mu_0 I R^2}{2x^3}$$

Here, the magnetic field is along the axis of the coil.

- ✓ When P is at a distance equal to the radius of the coil , from the centre of the coil, hen x = R, so:

$$B = B_x \hat{i} = \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + R^2)^{3/2}} = \frac{\mu_0 I}{2^{\frac{3}{2}} \ R}$$

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there are two rules for prediction of direction of magnetic field due to a circular current loop (a conductor loop in which current is flowing)

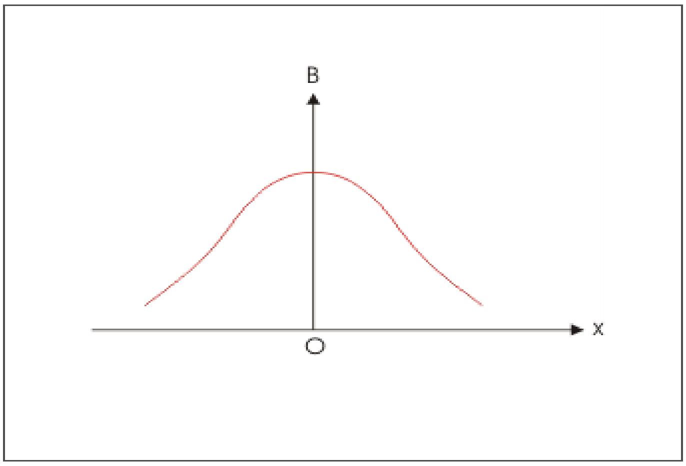
i) **Right hand thumb rule, if we curl the palm of our right hand around the circular wire with the fingers pointing in the direction of the current then the extended thumb gives the direction of the magnetic field.**

ii) **The rule gives the polarity of any face of the circular current loop. If the current round any face of the coil is in anticlockwise direction, it behaves like a north pole with magnetic field line emerging from it**

VARIATION OF THE MAGNETIC FIELD ALONG THE AXIS OF A CIRCULAR CURRENT LOOP.

The figure shows the variation of the magnetic field along the axis of a circular loop with distance from the centre.

The value of B is maximum at the centre and it decreases as we go away from the centre on either side of the loop.



Variation of B along the axis of a circular current loop

6. EXAMPLES BASED ON THE APPLICATION OF BIOT SAVART LAW

EXAMPLE:

Consider a tightly wound 100 turns coil of radius 10 cm, carrying a current of 1A. What is the magnitude of the magnetic field at the centre of the coil?

SOLUTION:

Since the coil is tightly wound, we may take each circular element to have the same radius R = 10 cm = 0.1 m. The number of turns N = 100. The magnitude of the magnetic field at the centre is:

$B = \mu_0 NI / 2R = 4\pi \times 10^{-7} \times 10^{2} \times 1/ 2 \times 10^{-1} = 2\pi \times 10^{-4} = \mathbf{6.28 \times 10^{-4}\ T}$

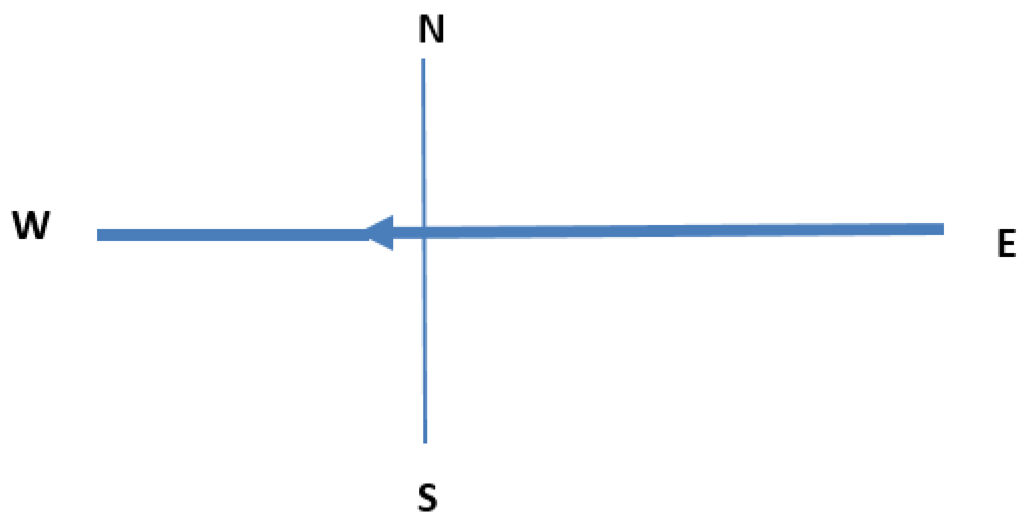
EXAMPLE:

A current of 10 A is flowing east to west in along wire kept horizontally in the east west direction. Find magnetic field

a) in a horizontal plane at a distance of 10 cm north

b) in a horizontal plane at a distance of 20 cm south of the wire

SOLUTION



a) magnetic field in the horizontal plane at a distance of 10 cm north of the wire is

$$B_N = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.10} = \mathbf{2 \times 10^{-5}} \boldsymbol{T}$$

according to right hand rule the direction will be in the horizontal plane into the screen

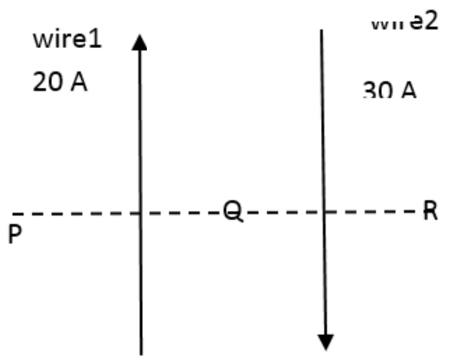
b) magnetic field in the horizontal plane at a distance of 20 cm South of the wire is

$$B_S = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.20} = \mathbf{1 \times 10^{-5}} \boldsymbol{T}$$

according to right hand rule the direction will be in the horizontal plane out of the screen

EXAMPLE

In the figure are shown two current carrying wires 1 & 2. Find the magnitudes and directions of the magnetic field at the points at P, Q and R. The distance between P and wire 1 is 10 cm, Q is at the midpoint between the wires 1 and 2.The distance between R and wire 2 is also 10 cm . the current in wire 1 is 20 A and in wire 2 is 30 A



SOLUTION:

According to right hand rule the field at P due to the current in wire 1 will be perpendicular to the screen pointing outward and that at Q & R pointing inward into the screen. Similarly, the field due to the current in wire 2 will be inward into the screen at P and Q, and out ward at R. **Thus at P and R the direction of magnetic fields due to the two wires are opposite but at Q they are in the same direction.**

Therefore **resultant field at P** is $B_P = B_1 - B_2$

$$B_P = B_1 - B_2 = \frac{\mu_0 I_1}{2\pi(P1)} - \frac{\mu_0 I_2}{2\pi(2P)} = \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{20}{0.10} - \frac{30}{0.3}\right]$$

= 2×10^{-5} T

It will be perpendicular to the plane of the screen pointing outwardward.

Resultant field at Q is $B = B_1 + B_2$

$$B_Q = B_1 + B_2 = \frac{\mu_0 I_1}{2\pi(1Q)} + \frac{\mu_0 I_2}{2\pi(2Q)} = \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{20}{0.10} + \frac{30}{0.10}\right]$$

$$\boldsymbol{B_Q = 1 \times 10^{-4} T}$$

It will be perpendicular to the plane of the screen pointing inward.

Resultant field at R is $B_R = B_2 - B_1$

$$B_R = B_2 - B_1 = \frac{\mu_0 I_2}{2\pi(2R)} - \frac{\mu_0 I_1}{2\pi(1R)} = \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{30}{0.10} - \frac{20}{0.30}\right]$$

$$\boldsymbol{B_R = 4.5 \times 10^{-5} T}$$

It will be perpendicular to the plane of the screen pointing out ward.

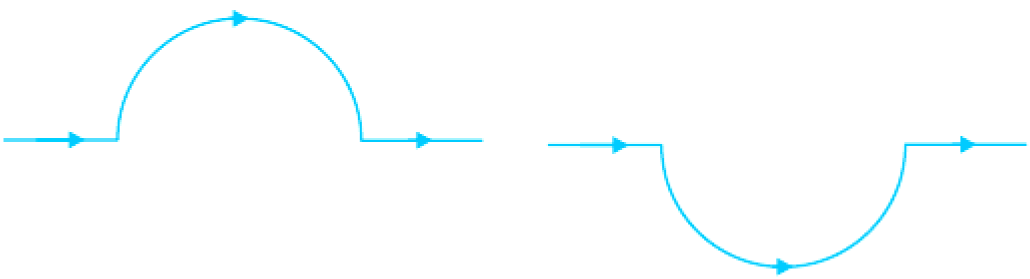
THINK ABOUT THESE

- **What if the current in the two wires was equal?**
- **What if it was in the same direction?**

EXAMPLE

A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown in Figure (a) Consider the magnetic field B at the centre of the arc.

(a) What is the magnetic field due to the straight segments?
(b) In what way the contribution to B from the semicircle differs from that of a circular loop and in what way does it resemble?
(c) Would your answer be different if the wire were bent into a semi-circular arc of the same radius but in the opposite way as shown in Figure (b)?



(a) (b)

SOLUTION

a) dl and r for each element of the straight segments are parallel. Therefore, dl × r = 0. Straight segments do not contribute to | B|.

b) For all segments of the semi circular arc, dl × r are all parallel to each other (into the plane of the paper). All such contributions add up in magnitude. Hence direction of B for a semi circular arc is given by the right-hand rule and magnitude is half that of a circular loop.

Thus B is 1.9×10^{-4} T normal to the plane of the paper going into it.

(c) Same magnitude of B but opposite in direction to that in (b).

7. PROBLEM FOR PRACTICE

i) In an orbital model of a hydrogen atom with one electron, revolving in circular orbit of radius 5.11×0^{-11} m at a frequency of 6.8×10^{15} Hz, what is the magnetic field at the centre of the orbit? Would the nucleus be subjected to this magnetic field?

(Hint: current equivalent of the revolving electron with frequency n = I = ne)

ii) The magnetic field due to a current carrying circular loop of radius 12 cm at its center is $0.50.\times 10^{-4}$ T . Find the magnetic field due to this loop at a point on the axis at a distance of 5.0 cm from the center.

(Hint: find the ratio of field at the centre of the loop and field at a point on the axis of the loop).

iii) Two coaxial circular loops of radii 2 cm and 4 am are laced such that their centres are 4 cm and 3 cm from a point O on the common axial line. The current in loop 1 is 1 A and is anticlockwise. What should be the magnitude and direction of current in loop 2 such that the net magnitude of B at O is zero?

8.SUMMARY

We have learnt:

- Meaning of Biot-Savart' law.
- Applications i.e., magnetic field due to straight conductor and special case to finite length.
- The magnetic field on the axis of circular coil carrying current followed by its special case to magnetic field at its centre.
- Problem solving using Biot Savart's Law.