

1. Details of Module and its structure

Subject Name	Physics
Course Name	Physics 03 (Physics Part 1 Class XII)
Module Name/Title	Unit-01, Module-04: Electric dipole Chapter-01: Electric charges and Fields
Module Id	Leph_10104_eContent
Pre-requisites	Electric force, Electric field, uniform and non-uniform field and Rules of vector addition
Objectives	<p>After going through this module, the learners will be able to</p> <ul style="list-style-type: none"> • Understand the concept of electric dipole and its physical significance. • Derive an expression for the electric field due to an electric dipole at points lying on its axial and equatorial line • Comprehend the motion of electric dipole in a uniform electric field. • Deduce the torque experienced by an electric dipole when displaced in a uniform field from its equilibrium position • Calculate the work done to rotate an electric dipole in a uniform electric field • Apply the characteristics of an electric dipole to solve simple problems.
Keywords	Electric dipole, Axial and equatorial field, Uniform and Non Uniform electric field, Torque and Electrostatic potential energy

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1. UNIT-SYLLABUS

Chapter-1: Electric Charges and Fields

Electric Charges; Conservation of charge, Coulomb's law-force between two point charges, forces between multiple charges; superposition principle and continuous charge distribution.

Electric field; electric field due to a point charge, electric field lines, electric dipole, electric field due to a dipole, torque on a dipole in uniform electric field.

Electric flux, statement of Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside).

Chapter-2: Electrostatic Potential and Capacitance

Electric potential, potential difference, electric potential due to a point charge, a dipole and system of charges; equipotential surfaces, electrical potential energy of a system of two point charges and of electric dipole in an electrostatic field.

Conductors and insulators, free charges and bound charges inside a conductor. Dielectrics and electric polarization, capacitors and capacitance, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor

2. MODULE WISE DISTRIBUTION OF UNIT SYLLABUS

The above unit is divided into 11 modules for better understanding.

11 Modules

Module 1	<ul style="list-style-type: none"> • Electric charge • Properties of charge • Coulomb's law • Characteristics of coulomb force • Effect of intervening medium on coulomb force numerical
Module 2	<ul style="list-style-type: none"> • Forces between multiple charges • Principle of superposition • Continuous distribution of charges numerical
Module 3	<ul style="list-style-type: none"> • Electric field E • Importance of field and ways of describing field • Point charges superposition of electric field numerical
Module 4	<ul style="list-style-type: none"> • Electric dipole • Electric field of a dipole • Charges in external field • Dipole in external field Uniform and non-uniform

Module 5	<ul style="list-style-type: none"> • Electric flux , • Flux density • Gauss theorem • Application of gauss theorem to find electric field for charge distribution <p>Numerical</p>
Module 6	<ul style="list-style-type: none"> • Application of gauss theorem: Field due to field infinitely long straight wire Uniformly charged infinite plane Uniformly charged thin spherical shell (field inside and outside)
Module 7	<ul style="list-style-type: none"> • Electric potential, • Potential difference, • Electric potential due to a point charge, a dipole and system of charges; • Equipotential surfaces, • Electrical potential energy of a system of two point charges and of electric dipole in an electrostatic field. <p>Numerical</p>
Module 8	<ul style="list-style-type: none"> • Conductors and insulators, • Free charges and bound charges inside a conductor. • Dielectrics and electric polarization
Module 9	<ul style="list-style-type: none"> • Capacitors and capacitance, • Combination of capacitors in series and in parallel • Redistribution of charges , common potential <p>Numerical</p>
Module 10	<ul style="list-style-type: none"> • Capacitance of a parallel plate capacitor with and without dielectric medium between the plates • Energy stored in a capacitor

3. Words you must know

Let us recollect the words we have been using in our study of this physics course.

- **Electric Charge:** Electric charge is an intrinsic characteristic, of many of the fundamental particles of matter that gives rise to all electric and magnetic forces and interactions.
- **Conductors:** Some substances readily allow passage of electricity through them, others do not. Those which allow electricity to pass through them easily are called *conductors*. They have electric charges (electrons) that are comparatively free to move inside the material. Metals, human and animal bodies and earth are all conductors of electricity.
- **Insulators:** Most of the non-metals, like glass, porcelain, plastic, nylon, wood, offer high opposition to the passage of electricity through them. They are called *insulators*.
- **Point Charge:** When the linear size of charged bodies is much smaller than the distance separating them, the size may be ignored and the charge bodies can then be treated as *point charges*.
- **Conduction:** Transfer of electrons from one body to another, it also refers to flow of charges or electrons in metals and ions in electrolytes and gases
- **Induction:** The temporary separation of charges in a body due to a charged body in the vicinity. The effect lasts as long as the charged body is held close to the body in which induction is taking place
- **Quantization of charges:** Charge exists as an integral multiple of basic electronic charge. Charge on an electron is $1.6 \times 10^{-19} \text{ C}$
- **Electroscope:** A device to detect charge.
- **Coulomb:** S.I unit of charge defined in terms of 1 ampere current flowing in a wire to be due to 1 coulomb of charge flowing in 1 s

1 coulomb = collective charge of 6×10^{18} electrons

- **Conservation of charge:** Charge can neither be created or destroyed in an isolated system it(electrons) only transfers from one body to another
- **Coulomb's law:** the mutual force of attraction or repulsion between two stationary point charges is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

For two charges located in free space or vacuum

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$$

- **Coulomb's Force:** It is the electrostatic force of interaction between the two point charges.
- **Vector form of coulombs law:** A mathematical expression based on coulombs law to show the magnitude as well as direction of mutual electrostatic force between two or more charges.

$$F_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

- **Laws of vector addition:**

Triangle law of vector addition: If two vectors are represented by two sides of a triangle in order, then the third side represents the resultant of the two vectors

Parallelogram law of vector addition: If two vectors are represented in magnitude and direction by adjacent sides of a parallelogram then the resultant of the vectors is given by the diagonal passing through their common point

Also resultant of vectors P and Q acting at angle of θ is given by

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

Polygon law of vector addition: Multiples vectors may be added by placing them in order of a multisided polygon, the resultant is given by the closing side taken in opposite order.

Resolution of vectors into components and then adding along x, y and z directions

- **Linear charge density:** The *linear charge density*, λ is defined as the charge per unit length.
- **Surface charge density:** The *surface charge density* σ is defined as the charge per unit surface area.
- **Volume charge density:** The *volume charge density* ρ is defined as the charge per unit volume.
- **Superposition Principle:** For an assembly of charges q_1, q_2, q_3, \dots , the force on any charge, say q_1 , is the vector sum of the force on q_1 due to q_2 , the force on q_1 due to q_3 , and so on. For each pair, the force is given by the Coulomb's law for two point charges.
- **Torque:** Torque is the tendency of a force to rotate an object about an axis.
- **Electric field lines:** An electric field line is a curve drawn in such a way that the tangent at each point on the curve gives the direction of electric field at that point.

- **Source and test charge:** The charge, which is producing the electric field, is called a *source charge* and the charge, which tests the effect of source charge, is called a *test charge*.
- **Uniform Field:** A *uniform electric field* is one whose magnitude and direction is same at all points in space and it will exert same force of a charge regardless of the position of charge.
- **Non uniform field:** We know that electric field of point charge depends upon location of the charge. Hence has different magnitude and direction at different points. We refer to this field as non-uniform electric field

4.INTRODUCTION

In the molecule of HCl , the red represents chlorine and the purple hydrogen ion (figure 1).

In water molecule the negative represents oxygen and the positive hydrogen (figure 2).

You must have done the molecular structure in your chemistry courses.

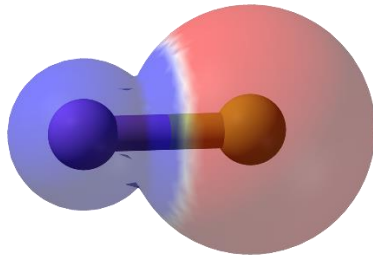


Figure 1

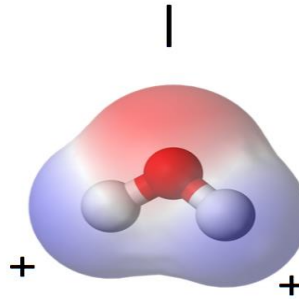


Figure 2

The figures show molecules in stable condition, the negative electrons forming a negative charge cloud separated from the positive charge.

This arrangement has positive and negative charges very close to each other, but their centers of distribution are not coincident. This is called an electric dipole.

Many molecules of materials around us are electric dipoles.

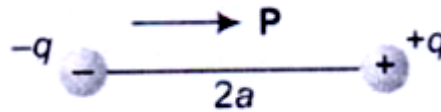
In this module

- you will learn about the electric dipole.
- You will learn to apply Coulomb's Law to find the electric field of an electric dipole.
- You will investigate the properties of the electric field associated with an electric dipole.
- You will also explore what happens to an electric dipole when it is placed in a uniform external electric field.

5.ELECTRIC DIPOLE

Let us first understand what is an electric dipole? And why is it called so?

An electric dipole is a pair of equal and opposite point charges q and $-q$, separated by a distance $2a$. The line connecting the two charges defines a direction in space is called **dipole axis**. By convention, the direction from $-q$ to q is said to be the **direction of the dipole**. The mid-point of locations of $-q$ and q is called the center of the dipole.



Every electric dipole is characterized by its electric **dipole moment** which is a **vector p** directed from the negative to the positive charge.

- The magnitude of dipole moment is, $p = (2a)q$
- Dipole moment has a direction, from negative to positive
- Its SI unit is coulomb meter or Cm
- Here, $2a$ is the distance between the two charges as shown in the figure.

It is a useful concept in atoms and molecules where the effects of charge separation are measurable, but the distances between the charges are too small to be easily measurable. It is also a useful concept in dielectrics and other applications in solid and liquid materials.

Total charge on a dipole is zero but it has an electric field

We can realize that the total charge of the electric dipole is zero. This does not mean that the field of the electric dipole is zero. Since the charge q and $-q$ is separated by some distance, the electric fields due to them, when added, do not exactly cancel out. However, at distances much larger than the separation of the two charges forming a dipole ($r \gg 2a$), the fields due to q and $-q$ nearly cancel out. The electric field due to a dipole therefore falls off at large distance.

EXAMPLE

From definition of electric dipole moment how can we say it is a vector? what is its unit? what does the magnitude of dipole moment depend upon?

SOLUTION

Since

$$p = (2a)q$$

P is a vector and its response to an external electric field will depend upon the direction of external electric field.

Dipole aligns in the direction of external electric field or the dipole moment must be associated with direction or should be a vector. direction of dipole moment is from negative to positive.

Unit Cm

EXAMPLE

A system has two charges $q_A = 2.5 \times 10^{-7} \text{ C}$ and $q_B = -2.5 \times 10^{-7} \text{ C}$ located at points A: (0, 0, -15 cm) and B: (0,0, +15 cm), respectively.

What are the total charge and electric dipole moment of the system?

SOLUTION

Total charge = 0

Dipole moment = $2 q a$

$$p = 2 \times 2.5 \times 10^{-7} \text{ C} \times 30 \times 10^{-2} \text{ m} = 15 \times 10^{-8} \text{ Cm},$$

6.ELECTRIC FIELD OF AN ELECTRIC DIPOLE

The electric field of the pair of charges ($-q$ and q) at any point in space can be found out from Coulomb's law and the superposition principle.

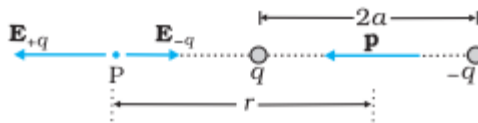
The results are simple for the first two of the following cases:

- (i) When the point under consideration is on the **dipole axis**,
- (ii) When it is in the **equatorial plane** of the dipole, i.e., on a plane perpendicular to the dipole axis through its center.

- (iii) The electric field at **any general point P** is obtained by adding the electric fields E_{-q} due to the charge $-q$ and E_{+q} due to the charge q , by the parallelogram law of vectors.

i) FOR POINTS ON THE DIPOLE AXIS:

Let the point P be at distance r from the center of the dipole on the side of the charge q , as shown in Figure.



Electric field at a point P on the dipole axis

Let the point P be at distance r from the centre of the dipole on the side of the charge $+q$, as shown in Fig.

$$\mathbf{E}_{-q} = -\frac{q}{4\pi\epsilon_0(r+a)^2}\hat{\mathbf{p}}$$

Where, $\hat{\mathbf{p}}$ is the unit vector along the dipole axis (from $-q$ to q). Also,

$$\mathbf{E}_{+q} = \frac{q}{4\pi\epsilon_0(r-a)^2}\hat{\mathbf{p}}$$

The total field at P is:

$$\mathbf{E} = \mathbf{E}_{+q} + \mathbf{E}_{-q} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right] \hat{\mathbf{p}} = \frac{q}{4\pi\epsilon_0} \frac{4ar}{(r^2-a^2)^2} \hat{\mathbf{p}}$$

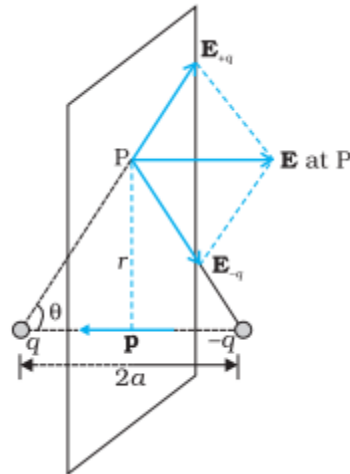
Or

For $r \gg a$

$$\mathbf{E} = \frac{4qa}{4\pi\epsilon_0 r^3} \hat{\mathbf{p}}$$

Notice: for a dipole E is proportional to $1/r^3$ and not $1/r^2$ as in the case of an electric field due to a point charge.

ii) FOR POINTS ON THE EQUATORIAL PLANE



The magnitudes of the electric fields due to the two charges $+q$ and $-q$ are given by:

$$E_{+q} = \frac{q}{4\pi\epsilon_0} \frac{1}{(r^2 + a^2)}$$

and

$$E_{-q} = \frac{q}{4\pi\epsilon_0} \frac{1}{(r^2 + a^2)}$$

are equal.

The directions of E_{+q} and E_{-q} are shown in Fig. Clearly, the components normal to the dipole axis cancel away. The components along the dipole axis add up.

The total electric field is opposite to \hat{p} . We have:

$$\begin{aligned} \mathbf{E} &= -(\mathbf{E}_{+q} + \mathbf{E}_{-q}) \cos \theta \hat{p} \\ &= -\frac{2qar}{4\pi\epsilon_0(r^2 + a^2)^2} \hat{p} \end{aligned}$$

At large distances ($r \gg a$), this reduces to:

$$\mathbf{E} = -\frac{2qa}{4\pi\epsilon_0 r^3} \hat{\mathbf{p}}$$

It is clear that the dipole field at large distances does not involve q and a separately; it depends on the product qa .

This suggests the **definition of dipole moment**.

The **dipole moment vector** \mathbf{p} of an electric dipole is defined by:

$$\mathbf{p} = q \times 2a \hat{\mathbf{p}}$$

That is, **it is a vector whose magnitude is charge q times the separation $2a$ (between the pair of charges $+q, -q$) and the direction is along the line from $-q$ to q .**

In terms of \mathbf{p} , the electric field of dipole at large distances takes simple forms:

At a point on the dipole axis:

$$\mathbf{E} = \frac{2\mathbf{p}}{4\pi\epsilon_0 r^3} \quad \text{for } r \gg a$$

At a point on the equatorial plane:

$$\mathbf{E} = -\frac{\mathbf{p}}{4\pi\epsilon_0 r^3} \quad \text{for } r \gg a$$

Notice

- the dipole field at large distances falls off not as $1/r^2$ but as $1/r^3$.
- the magnitude and the direction of the dipole field depend not only on the distance r but also on the *angle* between the position vector \mathbf{r} and the dipole moment \mathbf{p} .

We can think of the limit when the dipole size $2a$ approaches zero, the charge q approaches infinity in such a way that the product $p = q \times 2a$ is finite.

Such a dipole is referred to as a **point dipole**. For a point dipole, the above equations are exact and true for any r .

THE SALIENT FEATURES DISTINGUISHING

Electric field due to a point charge and Electric field due to an electric dipole-

i). **Electric field intensity of a point charge falls as $1/r^2$ where as that of electric dipole falls as $1/r^3$.**

ii). **We can think of the limit when the dipole size $2a$ approaches zero, the charge q approaches infinity in such a way that the product $p = q \times 2a$ is finite. Such a dipole is referred to as a *point dipole or ideal dipole*.**

iii). **For an ideal dipole ($r \gg 2a$), the electric**

field intensity on the axial line is twice the value of electric field intensity on the equatorial line/plane for same distance r

iv). **Electric field intensity of an electric dipole on its axial line is parallel to the electric dipole moment.**

v). **Electric field intensity of an electric dipole on its equatorial plane is anti -parallel to the electric dipole moment**

vi). **Further, the magnitude and the direction of the dipole field at any point r can be obtained by resolving the dipole moment p along position vector r and perpendicular to r . It depends not only on the distance r but also on the *angle* between the position vector r and the dipole moment p .**

7. PHYSICAL SIGNIFICANCE OF ELECTRIC DIPOLE

In most molecules, the centers of positive charges and of negative charges lie at the same position and coincident.

Therefore, their dipole moment is zero.

CO₂ and CH₄ are examples of this type of molecules.

However, **they develop a dipole moment when an electric field is applied.**

But in some molecules, the centers of negative charges and of positive charges do not coincide.

Therefore they have a permanent electric dipole moment, even in the absence of an electric field. Such molecules are called **polar molecules.**

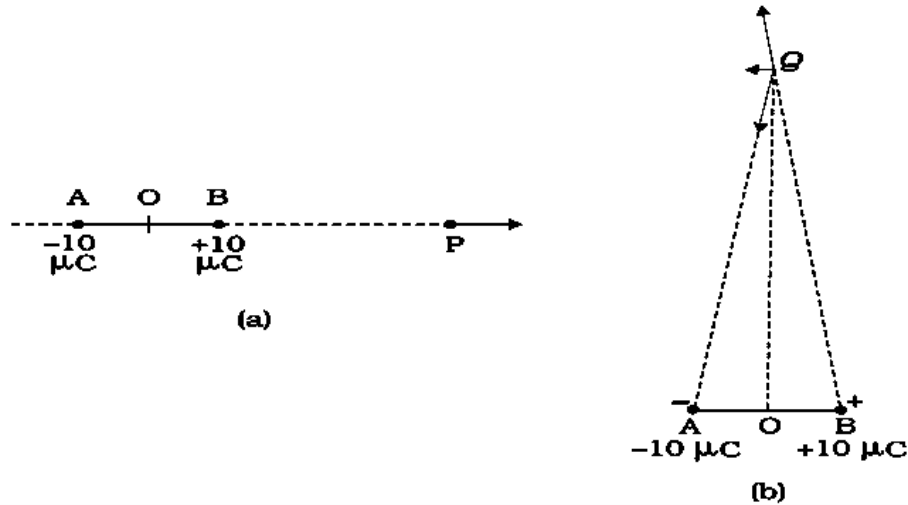
Water molecules, H₂O, are an example of this type. Various materials give rise to interesting properties and important applications in the presence or absence of electric field.

EXAMPLE

Two charges $\pm 10 \mu\text{C}$ are placed 5.0 mm apart. Determine the electric field at:

(a) a point P on the axis of the dipole 15 cm away from its center O on the side of the positive charge, as shown in Fig. (a)

(b) a point Q, 15 cm away from O on a line passing through O and normal to the axis of the dipole, as shown in Fig (b)



SOLUTION

$$\begin{aligned}
 \text{(a) Field at P due to charge } +10 \mu\text{C} &= \frac{10^{-5}\text{C}}{4\pi(8.854 \times 10^{-12}\text{C}^2\text{N}^{-1}\text{m}^{-2})} \times \frac{1}{(15-0.25)^2 \times 10^{-4}\text{m}^2} \\
 &= 4.13 \times 10^6 \text{NC}^{-1} \text{ along BP}
 \end{aligned}$$

$$\begin{aligned}
 \text{Field at P due to charge } -10 \mu\text{C} &= \frac{10^{-5}\text{C}}{4\pi(8.854 \times 10^{-12}\text{C}^2\text{N}^{-1}\text{m}^{-2})} \times \frac{1}{(15+0.25)^2 \times 10^{-4}\text{m}^2} \\
 &= 3.86 \times 10^6 \text{NC}^{-1} \text{ along PA}
 \end{aligned}$$

The resultant electric field at P due to the two charges at A and B is $= 2.7 \times 10^5 \text{ N C}^{-1}$ along BP.

In this example, the ratio OP/OB is quite large ($= 60$). Thus, we can expect to get approximately the same result as above by directly using the formula for electric field at a far-away point on the axis of a dipole.

For a dipole consisting of charges $\pm q$, $2a$ distance apart, the electric field at a distance r from the centre on the axis of the dipole has a magnitude:

$$E = \frac{2P}{4\pi\epsilon_0 r^3} \quad (r/a \gg 1)$$

Where $p = 2a q$ is the magnitude of the dipole moment.

The direction of electric field on the dipole axis is always along the direction of the dipole moment vector (i.e., from $-q$ to q). Here,

$$p = 10^{-5} \text{ C} \times 5 \times 10^{-3} \text{ m} = 5 \times 10^{-8} \text{ C m}$$

Therefore,

$$E = \frac{2 \times 5 \times 10^{-8} Cm}{4\pi(8.854 \times 10^{-12} C^2 N^{-1} m^{-2})} \times \frac{1}{(15)^3 \times 10^{-6} m^3} = 2.6 \times 10^5 N C^{-1}$$

Along the dipole moment direction AB, which is close to the result obtained earlier.

$$\begin{aligned} \text{(b) Field at Q due to charge } +10 \mu C \text{ at B} &= \frac{10^{-5} C}{4\pi(8.854 \times 10^{-12} C^2 N^{-1} m^{-2})} \times \frac{1}{((15)^2 + (0.25)^2) \times 10^{-4} m^2} \\ &= 3.99 \times 10^6 N C^{-1} \text{ along BQ} \end{aligned}$$

$$\begin{aligned} \text{Field at Q due to charge } -10 \mu C \text{ at A} &= \frac{10^{-5} C}{4\pi(8.854 \times 10^{-12} C^2 N^{-1} m^{-2})} \times \frac{1}{((15)^2 + (0.25)^2) \times 10^{-4} m^2} \\ &= 3.99 \times 10^6 N C^{-1} \text{ along QA} \end{aligned}$$

Clearly, the components of these two forces with equal magnitudes cancel along the direction OQ but add up along the direction parallel to BA. Therefore, the resultant electric field at Q due to the two charges at A and B is:

$$\begin{aligned} &= 2 \times \frac{0.25}{\sqrt{(15)^2 + (0.25)^2}} \times 3.99 \times 10^6 N C^{-1} \text{ along BA} \\ &= 1.33 \times 10^5 N C^{-1} \text{ along BA} \end{aligned}$$

As in (a), we can expect to get approximately the same result by directly using the formula for dipole field at a point on the normal to the axis of the dipole:

$$\begin{aligned} E &= \frac{\rho}{4\pi\epsilon_0 r^3} \quad \left(\frac{r}{a} \gg 1\right) \\ &= \frac{5 \times 10^{-8} Cm}{4\pi(8.854 \times 10^{-12} C^2 N^{-1} m^{-2})} \times \frac{1}{(15)^3 \times 10^{-6} m^3} = 1.33 \times 10^5 N C^{-1} \end{aligned}$$

The direction of electric field in this case is opposite to the direction of the dipole moment vector. Again the result agrees with that obtained before.

EXAMPLE:

State the direction of electric field intensity due to an electric dipole at a point in the

(1) End on position (along the axis)

(2) Broadside side on position (along a line perpendicular to the dipole axis)**SOLUTION-**

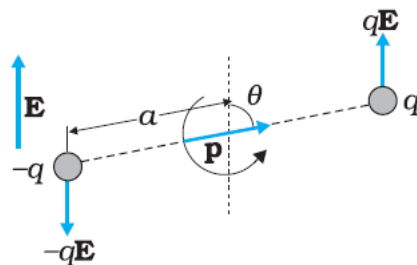
(1) For end on position i.e. on axial line, electric field is along electric dipole moment p (from $-q$ to $+q$).

(2) For broad side on position i.e. equatorial plane, electric field is opposite electric dipole moment p (from $+q$ to $-q$).

8. ELECTRIC DIPOLE IN A UNIFORM EXTERNAL ELECTRIC FIELD

If a point charge is placed in a uniform electrical field, it would move along the field lines depending upon the direction of the electric field and accelerate

If the charge was fixed by some means, the electrostatic force will still act on it even if it does not move from its position

What will happen to the dipole if it is placed in a uniform electric field?

A force $F_1 = qE$ will act on positive charge and $F_2 = -qE$ on negative charge.

Since, F_1 and F_2 is equal in magnitude but opposite in direction. Hence,

$$F_{net} = F_1 + F_2 = 0.$$

Thus, **net force on an electric dipole in a uniform field is zero.**

This means electric dipole cannot have any translatory motion in uniform field.

As shown in the above figure although forces are equal and opposite, their line of application of force is different. Hence a **couple** acts upon a dipole, which tends to set the dipole in the direction of the field

When the net force is zero, the torque (couple) is independent of the origin. Its magnitude equals the magnitude of each force multiplied by the arm of the couple (perpendicular distance between the two anti-parallel forces)

$$\text{Magnitude of torque} = q E \times 2 a \sin \theta = 2 q a E \sin \theta$$

Its direction is normal to the plane of the paper, coming out of it.

The magnitude of $p \times E$ is also $pE \sin \theta$ and its direction is normal to the paper, coming out of it.

Thus,

$$\boldsymbol{\tau} = \boldsymbol{P} \times \boldsymbol{E}$$

Further,

this torque is zero at $\theta = 0^\circ$ or $\theta = 180^\circ$, *i.e.*, when the dipole is parallel or anti parallel to E and maximum at $\theta = 90^\circ$.

This torque will tend to align the dipole with the field E. When p is aligned with E, the torque is zero.

So we can conclude that in a uniform electric field the dipole experiences no net force but experiences a torque which tries to align the dipole in the direction of the field ‘

EXAMPLE:

When do we say a dipole is in equilibrium?

SOLUTION

When an electric dipole is placed in a uniform electric field net force on it is zero for any position of the dipole in the electric field.

But torque acting on it is zero only at $\theta = 0^\circ$ and 180° .

Thus, we can say that at these two positions of the dipole, net force or torque on it is zero or the dipole is in equilibrium.

When it is displaced from this position, the torque has a tendency to rotate it in opposite direction to the displacement, and to finally bring it to the position $\theta = 0^\circ$, i.e., the position of stable equilibrium.

EXAMPLE

Why does water have a large dielectric constant ($k=80$) than say mica ($k=6$) ?

SOLUTION

Water molecule has a permanent dipole moment so it has a large dielectric strength

EXAMPLE

What is the angle between the direction of electric field at any point?

a) axial point

b) equatorial point due to an electric dipole

SOLUTION

a) The directions of electric field (E_{axial}) along axial line is in the same direction as the dipole moment which is from negative to positive.

b) The direction of electric field ($E_{\text{equatorial}}$) on points along the equatorial line is opposite to the dipole moment hence the angle between the two fields, electric field (E_{axial}) and electric field ($E_{\text{equatorial}}$) is 180°

EXAMPLE

An electric dipole with dipole moment $4 \times 10^{-8} \text{ C m}$ is aligned at 30° with the direction of a uniform electric field of magnitude $5 \times 10^4 \text{ NC}^{-1}$.

Calculate the magnitude of the torque acting on the dipole

SOLUTION

$$\text{Torque} = 2 q a E \sin \theta$$

Substituting the given values

$$= 2 \times 4 \times 10^{-8} \text{Cm} \times 5 \times 10^4 \text{NC}^{-1} \sin 30 = 2 \times 10^{-3} \text{Nm}$$

9. ELECTRIC DIPOLE IN A NON-UNIFORM EXTERNAL ELECTRIC FIELD

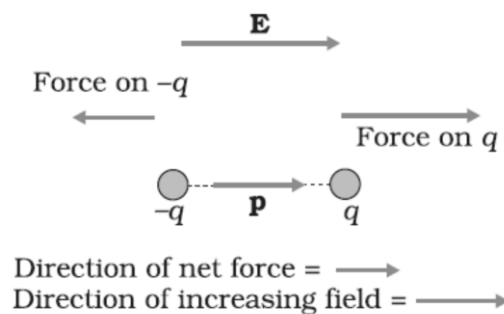
What happens if the field is not uniform?

In that case, the net force will evidently be non-zero. In addition, there will, in general, be a torque on the system as before.

Hence **dipole will have both translatory as well as rotational motion.**

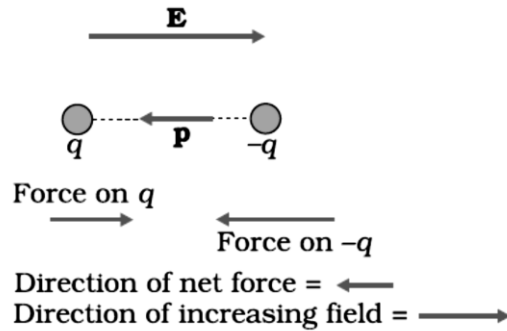
Let us consider a simple situation that is

i) **When \mathbf{p} is parallel to \mathbf{E}**



or

ii) **When \mathbf{p} is anti-parallel to \mathbf{E}**



In either case, the net torque is zero, but there is a net force on the dipole if E is not uniform.

It is easily seen that when p is parallel to E , the dipole has a net force in the direction of increasing field.

When p is anti-parallel to E , the net force on the dipole is in the direction of decreasing field. In general, the force depends on the orientation of p with respect to E .

THINK ABOUT THESE AND PLOT GRAPHS

- a) Sketch the variation of electric field of an electric dipole along points on its axial line at a point on the equatorial plane and that of point charge with r .

Hint

At a point on the dipole axis:

$$E = \frac{2p}{4\pi\epsilon_0 r^3} \quad \text{for } r \gg a$$

$$E_{\text{axial}} \propto \frac{2}{r^3}$$

At a point on the equatorial plane:

$$E = -\frac{p}{4\pi\epsilon_0 r^3} \quad \text{for } r \gg a$$

$$E_{\text{equatorial}} \propto \frac{1}{r^3}$$

Electric field due to a point charge

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$E \propto \frac{1}{r^2}$$

- b) **Draw the orientation of electric dipole in a uniform electric field where it experiences a half of maximum torque.**

Hint

$$\text{Torque} = 2 q a E \sin \theta$$

Maximum torque = $2 q a E$

$$\theta = 90^\circ$$

$$\text{For } \sin 30^\circ = \frac{1}{2}$$

Half Maximum torque = $q a E$

10.SUMMARY

In module we have learnt

- i) An electric dipole is a pair of equal and opposite charges q and $-q$ separated by some distance $2a$. Its dipole moment vector p has magnitude $2qa$ and is in the direction of the dipole axis from $-q$ to q .
- ii) Field of an electric dipole in its equatorial plane (i.e., the plane perpendicular to its axis and passing through its center) at a distance r from the center:

$$E = \frac{-p}{4\pi\epsilon_0} \frac{1}{(a^2+r^2)^{3/2}} = \frac{-p}{4\pi\epsilon_0 r^3} \quad \text{for } r \gg a$$

Dipole electric field on the axis at a distance r from the center:

$$E = \frac{2pr}{4\pi\epsilon_0(r^2 - a^2)^2} = \frac{2p}{4\pi\epsilon_0r^2} \quad \text{for } r \gg a$$

The $1/r^3$ dependence of dipole electric fields should be noted in contrast to the $\frac{1}{r^2}$ dependence of electric point charges.

- iii) In a uniform electric field E , a dipole experiences a torque τ given by $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$ but experiences no net force.