1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 02 (Physics Part 2 ,Class XI)
Module Name/Title	Unit 10, Module 14, Beats Chapter 15, Waves
Module Id	keph_201506_eContent
Pre-requisites	Sound waves, superposition of waves, conditions of superposition and prediction of result, Interference, stationary waves
Objectives	After going through this module, the learner will be able to: • Understand the meaning of Beats • Differentiate between Beat frequency and frequency of beat • Appreciate application of beats in daily life
Keywords	Beats, beat frequency, frequency of beats, application of beats , superposition of waves

2. Development Team

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1. UNIT SYLLABUS Unit 10:

Oscillations and waves

Chapter 14: Oscillations

Periodic motion, time period, frequency, displacement as a function of time , periodic functions Simple harmonic motion (S.H.M) and its equation; phase; oscillations of a loaded spring-restoring force and force constant; energy in S.H.M. Kinetic and potential energies; simple pendulum derivation of expression for its time period.

Free forced and damped oscillations (qualitative ideas only) resonance

Chapter 15: Waves

Wave motion transverse and longitudinal waves, speed of wave motion , displacement , relation for a progressive wave, principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, beats, Doppler effect

2. MODULE WISE DISTRIBUTION OF UNIT SYLLABUS 15 MODULES

Module 1	• **Periodic motion** • **Special vocabulary** • **Time period, frequency,** • **Periodically repeating its path** • **Periodically moving back and forth about a point** • **Mechanical and non-mechanical periodic physical quantities**
Module 2	• **Simple harmonic motion** • **Ideal simple harmonic oscillator** • **Amplitude** • **Comparing periodic motions phase,**

	• **Phase difference** **Out of phase** **In phase** **not in phase**
Module 3	• **Kinematics of an oscillator** • **Equation of motion** • **Using a periodic function (sine and cosine functions)** • **Relating periodic motion of a body revolving in a circular path of fixed radius and an Oscillator in SHM**
Module 4	• **Using graphs to understand kinematics of SHM** • **Kinetic energy and potential energy graphs of an oscillator** • **Understanding the relevance of mean position** • **Equation of the graph** • **Reasons why it is parabolic**
Module 5	• **Oscillations of a loaded spring** • **Reasons for oscillation** • **Dynamics of an oscillator** • **Restoring force** • **Spring constant** • **Periodic time spring factor and inertia factor**
Module 6	• **Simple pendulum** • **Oscillating pendulum** • **Expression for time period of a pendulum** • **Time period and effective length of the pendulum** • **Calculation of acceleration due to gravity** • **Factors effecting the periodic time of a pendulum** • **Pendulums as 'time keepers' and challenges** • **To study dissipation of energy of a simple pendulum by plotting a graph between square of amplitude and time**
Module 7	• **Using a simple pendulum plot its L-T^2graph and use it to find the effective length of a second's pendulum** • **To study variation of time period of a simple pendulum of a given length by taking bobs of same size but different masses**

	and interpret the result • Using a simple pendulum plot its L-T^2graph and use it to calculate the acceleration due to gravity at a particular place
Module 8	• Free vibration natural frequency • Forced vibration • Resonance • To show resonance using a sonometer • To show resonance of sound in air at room temperature using a resonance tube apparatus • Examples of resonance around us
Module 9	• Energy of oscillating source, vibrating source • Propagation of energy • Waves and wave motion • Mechanical and electromagnetic waves • Transverse and longitudinal waves • Speed of waves
Module 10	• Displacement relation for a progressive wave • Wave equation • Superposition of waves
Module 11	• Properties of waves • Reflection • Reflection of mechanical wave at i)rigid and ii)nonrigid boundary • Refraction of waves • Diffraction
Module 12	• Special cases of superposition of waves • Standing waves • Nodes and antinodes • Standing waves in strings • Fundamental and overtones • Relation between fundamental mode and overtone frequencies, harmonics • To study the relation between frequency and length of a given wire under constant tension using sonometer • To study the relation between the length of a given wire and tension for constant frequency using a sonometer

Module13	• **Standing waves in pipes closed at one end,** • **Standing waves in pipes open at both ends** • **Fundamental and overtones** • **Relation between fundamental mode and overtone frequencies** • **Harmonics**
Module 14	• **Beats** • **Beat frequency** • **Frequency of beat** • **Application of beats**
Module 15	• **Doppler effect** • **Application of Doppler effect**

MODULE 14

3. WORDS YOU MUST KNOW

Let us remember the words we have been using in our study of this physics course

- Displacement the distance an object has moved from its starting position moves in a particular direction.SI unit: m, this can be zero, positive or negative
- Non mechanical displacement periodically changing electric, magnetic, pressure of gases, currents, voltages are non-mechanical oscillations. They are represented by sin and cosine functions like mechanical displacements

  1. For a vibration or oscillation, the displacement could ne mechanical, electrical magnetic. Mechanical displacement can be angular or linear.

  - **Energy:** In equilibrium position $y = 0$, we have

    Potential energy of the body, $U = 0(zero)$

    And kinetic energy of the body, $K = \frac{1}{2} m\,\omega^2\ a^2 = E_{max}$

    In maximum displace position $(y = a)$, we have

    Potential energy of the body, $U = \frac{1}{2} m\,\omega^2\ a^2 = E_{max}$

    And kinetic energy of the body, $K = O(zero)$

- **Wave:** A wave is a disturbance in the medium which causes the particles of the medium to undergo vibratory motion about their mean position.
- Wave motion: method of energy transfer from a vibrating source to any observer.
- Mechanical wave energy transfer by vibration of material particles in response to a vibrating source examples water waves, sound waves , waves in strings
- The speed of wave in medium depends upon elasticity and density
- Longitudinal mechanical wave a wave in which the particles of the medium vibrate along the direction of propagation of the wave
- Transverse mechanical wave a wave in which the particles of the medium vibrate perpendicular to the direction of propagation of the wave
- **A progressive wave:** The propagation of a wave in a medium means the particles of the medium perform simple harmonic motion without moving from their positions, then the wave is called a simple harmonic **progressive wave**

  In progressive wave, the disturbance produced in the medium travels onward, it being handed over from one particle to the next. Each particle executes the same type of vibration as the preceding one, though not at the same time. In this wave, energy propagates from one point in space to the other.

- **Displacement relation for a Progressive wave:** The displacement of the particle at an instant t is given by,

$$y = a\sin\,(\omega\,t - \phi\,)\ \text{ OR }\ y\ = a\ \sin\,(\omega\,t - k\,x\,)$$

If ϕ be the phase difference between the above wave propagating along the +X direction and another wave, then the equation of that wave will be

$$y = a\sin\left\{2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \phi\right\}$$

$$y = a\sin(\omega\,t - k\,x + \phi)$$

The displacement could also be expressed in terms of the cosine function without affecting any of the subsequent relation.

- **Particle Velocity:** The equation of a plane progressive wave propagating in the positive direction of X-axis is given by

$$v = \frac{dy}{dt} = \omega\,a\,cos\,(\omega\,t - k\,x)$$

The maximum particle velocity is given by,

$v_{max} = \omega\,a$, this is known as velocity amplitude of particle.

- **Particle Acceleration:** The instantaneous acceleration f of a particle is

$$f = \frac{du}{dt} = \omega^2 a \sin(\omega t - k x) = -\omega^2 y$$

The maximum value of the particle displacement y is a. Therefore, acceleration amplitude is $f_{max} = -\omega^2 a$

- **Principle of Superposition:** The net displacement of the medium / particles (through which waves travel) due to the superposition is equal to the sum of individual displacements (produced by each wave).
- **Standing wave:** When two identical waves of the same amplitude and frequency travel in opposite directions with the same speed along the same path superpose each other, the resultant wave does not travel in the either direction and is called a stationary or standing wave. It is called a standing wave because it does not appear to move. In such a wave, energy is confined in the space of the wave.
- **Nodes:** The points at which the amplitude is zero (i.e., where there is no motion of particles at all) are nodes.

- **Antinodes:** The points at which the amplitude of the particle is the largest are called antinodes.
- **Conditions for special cases of superposition:**

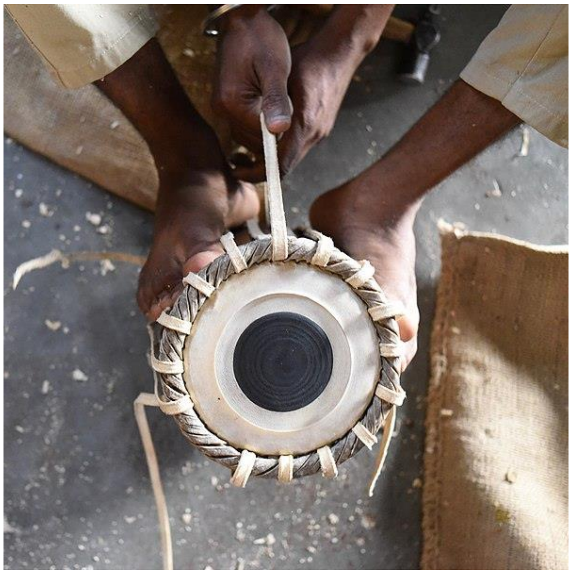
Interference: the process in which two or more light, sound, or electromagnetic waves of the same frequency combine to reinforce or cancel each other, the amplitude of the resulting wave being equal to the sum of the amplitudes of the combining waves.

Stationary waves: is a wave which oscillates in time but whose peak amplitude profile does not move in space. The locations at which the amplitude is minimum are called nodes, and the locations where the amplitude is maximum are called antinodes.

4. INTRODUCTION

You must have been or may have attended or seen on TV a music program.

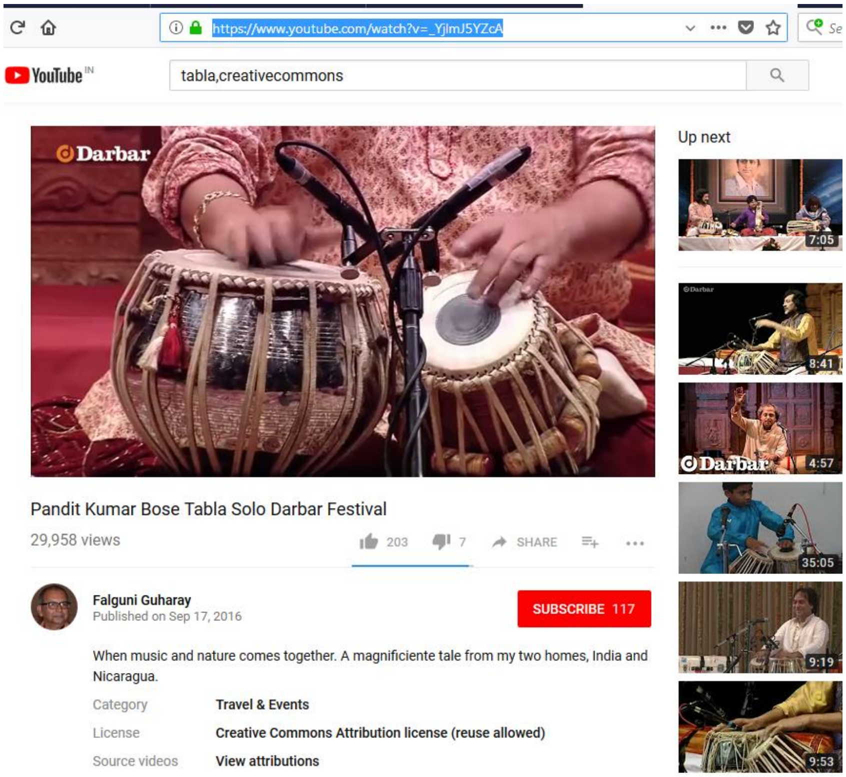
Many times the musicians tune their musical instruments on stage. The audience wonders why they are not starting the program. By tuning, we mean setting operating frequencies on the instrument. The string instruments such as guitar, tanpura, sitar, santoor and sarod have knobs which can change the frequency of vibration of the string by appropriately changing the tension in the wire . Also parchments of tabla and drums are often adjusted using small hammer.

https://commons.wikimedia.org/wiki/File:Tabla_tuning.jpg

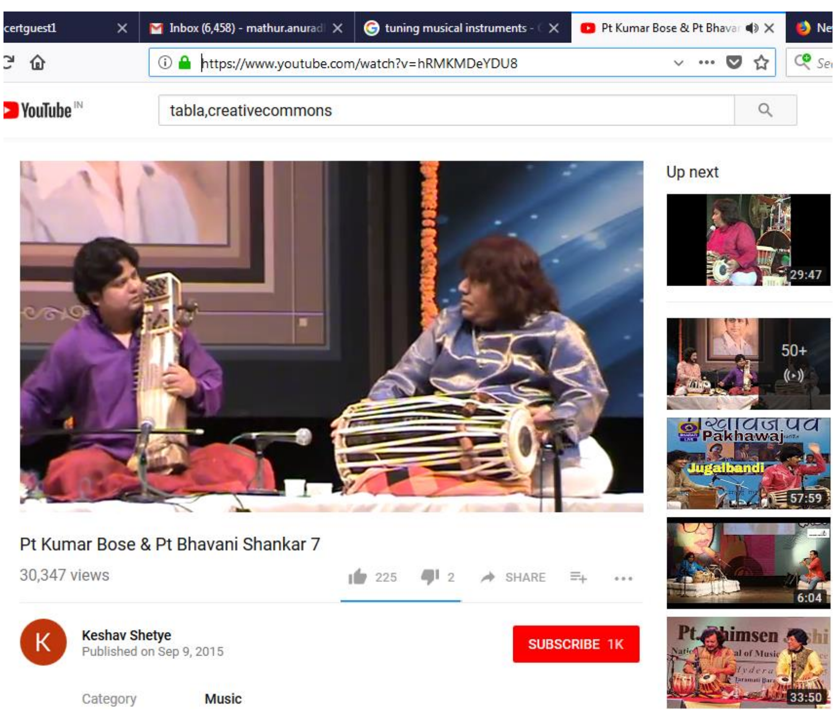
https://no.wikipedia.org/wiki/Gitar

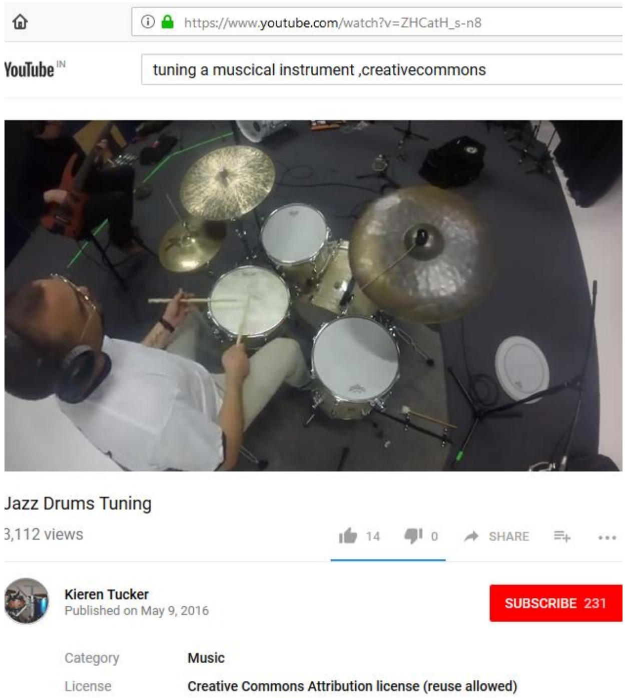
In music, **tuning an instrument** means getting it ready so that when the instruments are played it will sound with desired pitch: not too high or too low. When two or more **instruments** play together it is particularly important that they are in **tune** with one another.



https://www.youtube.com/watch?v=_YjlmJ5YZcA

Tabla and Pakhwaj





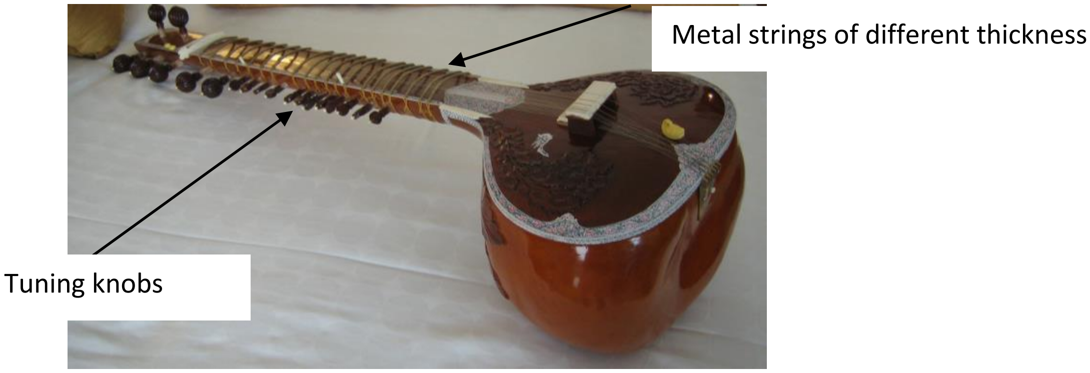
https://www.youtube.com/watch?v=ZHCatH_s-n8

Watch the video and listen to the drums

We studied the motion of objects vibrating or oscillating in isolation.

What happens in a system, which is a collection of such objects, like in a sitar, guitar, flute etc or a set of 'tablas'

https://upload.wikimedia.org/wikipedia/commons/8/8d/Instrument_perkusyjny_TABLA_firmy_Meinl.jpg



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A material medium provides a connect between each string or parchment.

Here, elastic forces bind the constituents to each other and, therefore, the motion of one affects that of the other.

If you drop a little pebble in a pond of still water, the water surface gets disturbed. The disturbance does not remain confined to one place, but propagates outward along a circle.

If you continue dropping pebbles in the pond, you see circles rapidly moving outward from the point where the water surface is disturbed. It gives a feeling as if the water is moving outward from the point of disturbance. If you put some cork pieces on the disturbed surface, it is seen that the cork pieces move up and down but do not move away from the centre of disturbance. This shows that the water mass does not flow outward with the circles, but rather a disturbance is created that moves.

Similarly, when we speak, the sound waves move outward from us, without any flow of air from one part of the medium to another.

The disturbances produced in air are much less obvious and only our ears or a microphone can detect them. These patterns, which move without the actual physical transfer or flow of matter as a whole, are called waves.

Waves transport energy and the pattern of disturbance has information that propagates from one point to another. Speech means production of sound waves in air and hearing amounts to their detection. Often, communication involves different kinds of waves. For example, sound waves may be first converted into an electric current signal which in turn may generate an electromagnetic wave that may be transmitted by an optical cable or via a satellite. Detection of the original signal will usually involve these steps in reverse order.

The most familiar type of waves such as waves on a string, water waves, sound waves, seismic waves, etc. is the so-called mechanical waves.

These waves require a medium for propagation, they cannot propagate through vacuum. They involve oscillations of constituent particles and depend on the elastic properties of the medium.

Waves in elastic media are intimately connected with harmonic oscillations. (Stretched strings, coiled springs, air, etc., are examples of elastic media.)

The sound travels as longitudinal and progressive waves since they travel from one part of the medium to another. The material medium as a whole does not move, as already noted.

What happens when two or more waves / wave pulses travelling in opposite directions cross each other?

It turns out that wave pulses continue to retain their identities after they have crossed. However, during the time they overlap, the wave pattern is different from either of the pulses.

Principle of superposition of waves:

According to this principle,

- Each wave moves as if others are not present.
- The constituents of the medium therefore suffer displacements due to both and since displacements can be positive and negative, the net displacement is an algebraic sum of the two.

SPECIAL CASES OF SUPERPOSITION

Superposition results in algebraic sum of individual displacements due to waves.

The resultant may not be easy to predict, however there are three cases in which we can discuss the result.

INTERFERENCE-

Condition

Two waves of

- same nature,
- same amplitude and
- same frequency
- moving in the same direction

Result

Redistribution of energy in space, occurrence of maxima and minima.

STATIONARY WAVE

Condition

Two waves of

- same nature,
- same amplitude and
- same frequency
- moving in the opposite direction, this may be due to reflection of the wave at a) rigid boundary b) non-rigid boundary

Result

Redistribution of energy within a small section of the medium. There are points where no energy exists called nodes, others called antinode where we find maximum energy.

In this module we are going to consider the superposition of sound waves with slightly different frequency.

5. BEATS

'Beats' is an interesting phenomenon arising from superposition of waves.

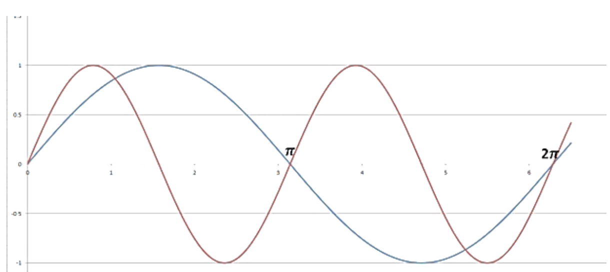
When two harmonic sound waves of close (but not equal) frequencies are heard at the same time, we hear a sound of similar frequency (the average of two close frequencies), but we hear something else also.

We **hear audibly distinct waxing and waning of the intensity of the sound, at a frequency equal to the difference in the two close frequencies**.

See the two graphs the blue shows y = sin θ, and the red y = sin 2θ

They have almost same amplitude

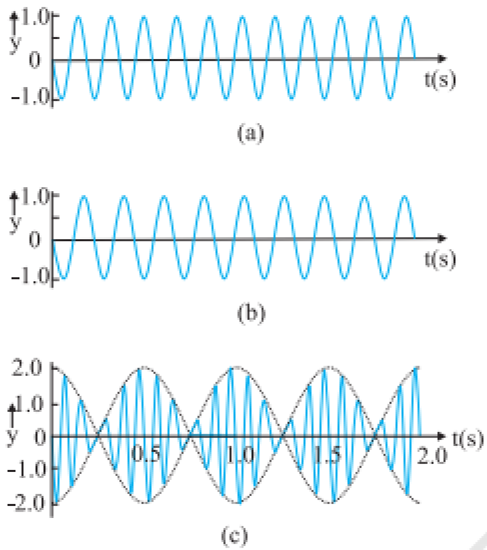
The overlap shows that the maximum would occur at points other than for any individual wave



Maxima will occur depending on the way the waves overlap

Musicians use this phenomenon, while tuning their musical instruments with each other. They go on tuning until their sensitive ears do not detect any beats.

Study the given graphs



- **Superposition of two harmonic waves,**
- **one of frequency 11 Hz [fig (a)], and**
- **the other of frequency 9Hz [fig (b)],**
- **Giving rise to beats of frequency 2 Hz, as shown in [fig (c)].**

Notice:

There are two large loops in the same duration.

This means the amplitude increases two times and lowers two times during the interval.

Physically amounting to 'two loud and two weak' sounds in the same duration.

This is called waxing and waning of sound.

The result is intermittent loud and soft sound.

6. MATHEMATICAL TREATMENT OF BEATS

Let us now consider the superposition using suitable equations for the waves

Suppose that y_1 and y_2 are the individual displacements of two sinusoidal waves whose frequencies are f_1 and f_2 respectively.

If the amplitude of each wave is A, then it can be shown that, at a single point, the displacements vary with time according to

$$y_1 = A\sin(2\pi f_1\ t)$$

$$y_2 = A\sin(2\pi f_2\ t)$$

If the two waves are superposed, the resultant displacement y, according to principle of superposition

$$y = y_1 + y_2$$

$$y = A\sin(2\pi f_1\ t) + A\sin(2\pi f_2\ t)$$

Consider the trigonometric identity

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

Hence $y = 2A\sin\left(\frac{2\pi f_1 t + 2\pi f_2 t}{2}\right)\cos\left(\frac{2\pi f_1 t - 2\pi f_2 t}{2}\right)$

We can rearrange and write this as

$$y = 2A\ \cos[\pi(f_1 - f_2)]t\ \ .\ \ \sin\left[2\pi\left(\frac{f_1+f_2}{2}\right)t\right]$$

The amplitude of the wave is

$$2A\ \cos[(f_1 - f_2)\pi t]$$

Comparing the equation with

$$y_1 = A\sin(2\pi f t)$$

The resultant is a **wave of frequency** $\frac{f_1+f_2}{2}$ **but with variable amplitude**.

What does that mean?

- The resultant wave does not have the frequency of f_1 or f_2
- The new frequency that one will hear would be the average of the two superposing frequency of

$$\frac{f_1 + f_2}{2}$$

- The amplitude varies in time as

$$2A \cos[(f_1 - f_2)\pi t]$$

This value will periodically be maximum, minimum or maximum **dependent on time t**
The amplitude is the largest when the term $\cos[[(f_1 - f_2)\pi t]]$ takes its limit +1 or –1.

In other words, **the intensity of the resultant wave waxes (becomes loud) and wanes (becomes faint) with time**

$\cos(n\pi)$ should be ± 1 , where n = 0,1,2,3,…

$cos(2n+1)\frac{\pi}{2}$ will be 0 when n = 0,1,2,3,4….

In terms of time

$$[(f_1 - f_2)\pi t] = n\pi$$

Or,

$$t = \frac{n}{f_1 - f_2},$$

n	t
0	0
1	$\frac{1}{f_1 - f_2}$
2	$\frac{2}{f_1 - f_2}$
3	$\frac{3}{f_1 - f_2}$

So at equal interval of time the amplitude is maximum
The time interval between two consecutive maxima and minima is

$$\Delta t = \frac{1}{f_1 - f_2}$$

We will hear maximum sound or the beat period is

$$\frac{1}{f_1 - f_2}$$

or in 1 secod the sound will be maximum or minimum $(f_1 - f_2)$ times

- Intensity ∝ square of amplitude or

  $$I \propto 4A^2 \cos^2\pi(f_1 - f_2)t$$

  Using the identity $2\cos^2\theta = 1 + \cos 2\theta$

  We can write
  Intensity ∝ $\mathbf{2A^2[1 + \cos 2\pi(f_1 - f_2)t]}$

  This means that the amplitude is not constant, but varies with time periodically. **The maximum value of amplitude is 2 A.**

7. BEAT FREQUENCY AND FREQUENCY OF BEAT

The number of times we hear the beat per second is called beat frequency

$$f_1 - f_2$$

Or difference in frequency of the two waves.

Human ear is sensitive to detecting beats up to 10 s apart.

The **frequency of beat** (produced by two waves of frequency f_1 **and** f_2) $= \frac{f_1+f_2}{2}$

EXAMPLE

Two sound notes of frequency 256 Hz and 260Hz are sounded together

Calculate the beat frequency and frequency of beat

SOLUTION

Beat frequency = $\Delta f = f_1 - f_2$ = 260 - 256 = 4 Hz, so 4 loud sound per second.

Frequency of beat =

$$\frac{f_1 + f_2}{2}$$
$$= \frac{260 + 256}{2} = 258\ Hz$$
the frequency any observer will hear

EXAMPLE

A tuning fork A makes 5 beats per second with a tuning fork B of frequency 255 Hz. A is filled and the beats occur at shorter interval. Find the frequency of A.

SOLUTION

Fork A can have the frequency 255 $\pm$ 5 Hz. That is either 5Hz higher or 5Hz lower than fork B 260 Hz or 250 Hz

When A is filled, beats are occurring at shorter intervals which means ,there are more beats per second., implies the difference between the frequency of the two forks increases.

Therefore, original frequency of A can only be 260 Hz.

On filing the frequency of the fork increases, on loading it,say making it thicker using wax, cello-tape, piece of paper the frequency decreases.

EXAMPLE

Tuning fork of unknown frequency when sounded with one of frequency 210 Hz gives 4 beats per second. When loaded with a little wax, it again gives four beats per second.

How do you account for this?

What is the unknown frequency?

SOLUTION

Unknown frequency can be 210 $\pm$ 4 Hz that is 214 or 206 Hz

If unknown frequency is 206 hertz then on loading the number of beats must be greater than four, because the loaded fork frequency will become smaller. However if the unknown frequency is 214 Hz then on loading it may decrease to a value of 206 Hz so as to again produce 4 beats per second, with the known frequency of 210 Hz .

Therefore unknown frequency is 214 Hz

Measuring an unknown frequency

The phenomenon of beats can be used to determine the unknown frequency of some other source by causing the same to sound with a source of known frequency

In general a frequency generator is used for mechanical tuning of musical instruments.

EXAMPLE

Two sitar strings A and B playing the note 'Dha' are slightly out of tune and produce beats of frequency 5 Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease to 3 Hz. What is the original frequency of B if the frequency of A is 427 Hz?

SOLUTION

Increase in the tension of a string increases its frequency.

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

where f is the frequency of the vibrating string of

length l

mass per unit length = μ

tension =T

If the original frequency of B (F_B) were greater than that of A (F_A), further increase in F_B should have resulted in an increase in the beat frequency.

But the beat frequency is found to decrease.

This shows that (F_B) < (F_A),

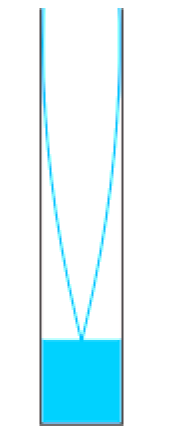
Since (F_A) - (F_B) = 5 Hz, and (F_A) = 427 Hz, we get (F_B) = 422 Hz

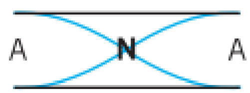
EXAMPLE

Consider a closed organ pipe of length 25 cm. Another open organ pipe gives 8 beats with it. What is the length of the second pipe?

Take speed of sound as 300 m/s.

SOLUTION





Let the length of the second pipe be l

$$F_{closed} = \frac{30000}{4 \times 25} = 300Hz$$

This gives 8 beats so the other pipe will have a frequency of either 308 Hz or 292Hz

$$F_{open} = \frac{30000}{2l}$$

Option 1

$$308 = \frac{30000}{2l} \quad or \quad l = \frac{30000}{2 \times 308} = 48.70\ cm$$

Option 2

$$292 = \frac{30000}{2l} \quad or \quad l = \frac{30000}{2 \times 292} = 51.37\ cm$$

The length of the second pipe could be 48.70 cm or 51.37 cm.

EXAMPLE

Two tuning forks A and B give 5 beats per second. A resounds with a closed air column 25.0 cm long and B with an open column 50.83 cm long.

Calculate their frequencies.

SOLUTION

Let f_A and f_B be the two frequencies

And let the velocity of sound be v cm/s

$$F_A = \frac{v}{4l} = \frac{v}{4 \times 25} = \frac{v}{100}$$

$$F_B = \frac{v}{2l} = \frac{v}{2 \times 50.83} = \frac{v}{101.66}$$

$$F_A - F_B = 5$$

$$\frac{v}{100} - \frac{v}{101.66} = 5$$

$$v = \frac{(5 \times 100 \times 101.66)}{1.66} = 30620.5\ cm\ s^{-1}$$

$$F_A = \frac{30620.5\ cm\ s^{-1}}{4\ \times 25} = 306\ Hz$$

And the other $F_B = \frac{30620.5\ cm\ s^{-1}}{2 \times 50.83} = 301 Hz$

THINK ABOUT THESE

- **Would beats take place with light waves?**
- **Why no beats are heard if the difference in frequency of two superposing sound waves is more than 10 Hz?**
- **The waves proceed as individual progressive waves after the region of overlap. So if the tabla player does not adjust the microphone well would you enjoy the beats?**
- **Would there be beats if the frequencies are infrasonic? Ultrasonic?**

8. APPLICATION OF THE PHENOMENON OF BEATS IN DAILY LIFE

Beats are used for determining unknown frequencies, and matching frequencies from different sources especially required in an orchestra

Piano, sarangi or even harmonium tuners use the *phenomenon of beats* when bringing strings into unison. They basically tune each and every chord to get the perfect frequency.

Piano and organ tuners even use a method involving counting *beats*, aiming at a particular number for a specific interval.

It is used in Police RADAR.

9. SUMMARY

- Superposition of waves takes place whenever two or more waves overlap in the same region
- Three cases of superposition have predictable effects
- Interference occurs when two waves of same frequency and amplitude travel in the same direction and superpose. The result is redistribution of energy in space
- Stationary waves are formed when two waves of same amplitude, same frequency travelling in the opposite direction overlap. The waves may arise due to reflection at a rigid or a non-rigid boundary. The result is formation of stationary waves trapping the energy in a localised area. Within the influenced region redistribution of energy takes place forming nodes and antinodes.
- Beats are a result of superposition of two waves of nearly the same amplitude, nearly same frequency travelling in the same direction. This results in redistribution of energy with time.
- The waxing and waning of sound is called beats.
- The difference between the frequencies should be equal to or less than 10 Hz.
- Beats are used for determining unknown frequencies, matching frequencies.

https://www.youtube.com/watch?v=eVIKrAdpuqU

Understanding the mathematics behind the beat frequency and how it applies to musical instruments