1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 02 (Physics Part 2,Class XI)
Module Name/Title	Unit 10, Module13, Standing waves in pipes Chapter 15, Waves
Module Id	keph_201505_eContent
Pre-requisites	Students should have knowledge of standing waves
Objectives	After going through this module the learners will be able to: • Understand and explain formation of Standing waves in pipes closed at one end, • Understand and explain formation of Standing waves in pipes open at both ends • Differentiate between Fundamental mode and overtones • Establish Relation between fundamental mode and harmonics for stationary waves in pipes
Keywords	Nodes, antinodes, stationary waves in closed pipe, stationary waves in open pipe, harmonics, overtones, standing waves, stationary waves

2. Development Team

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1. UNIT SYLLABUS Unit: 10

Oscillations and waves

Chapter 14: oscillations

Periodic motion, time period, frequency, displacement as a function of time , periodic functions Simple harmonic motion (S.H.M) and its equation; phase; oscillations of a loaded spring-restoring force and force constant; energy in S.H.M. Kinetic and potential energies; simple pendulum derivation of expression for its time period.

Free forced and damped oscillations (qualitative ideas only) resonance

Chapter 15: Waves

Wave motion transverse and longitudinal waves, speed of wave motion, displacement, relation for a progressive wave, principle of superposition of waves , reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, beats, Doppler effect

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 15 MODULES

Module 1	• **Periodic motion** • **Special vocabulary** • **Time period, frequency,** • **Periodically repeating its path** • **Periodically moving back and forth about a point** • **Mechanical and non-mechanical periodic physical quantities**
Module 2	• **Simple harmonic motion** • **Ideal simple harmonic oscillator** • **Amplitude** • **Comparing periodic motions phase,** • **Phase difference** **Out of phase**

	In phase not in phase
Module 3	• Kinematics of an oscillator • Equation of motion • Using a periodic function (sine and cosine functions) • Relating periodic motion of a body revolving in a circular path of fixed radius and an Oscillator in SHM
Module 4	• Using graphs to understand kinematics of SHM • Kinetic energy and potential energy graphs of an oscillator • Understanding the relevance of mean position • Equation of the graph • Reasons why it is parabolic
Module 5	• Oscillations of a loaded spring • Reasons for oscillation • Dynamics of an oscillator • Restoring force • Spring constant • Periodic time spring factor and inertia factor
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Module 7	• Using a simple pendulum plot its L-T^2graph and use it to find the effective length of a second's pendulum • To study variation of time period of a simple pendulum of a given length by taking bobs of same size but different masses and interpret the result • Using a simple pendulum plot its L-T^2graph and use it to calculate the acceleration due to gravity at a particular place

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Module13	• **Standing waves in pipes closed at one end,** • **Standing waves in pipes open at both ends** • **Fundamental and overtones**

	• Relation between fundamental mode and overtone frequencies • Harmonics
Module 14	• Beats • Beat frequency • Frequency of beat • Application of beats
Module 15	• Doppler effect • Application of Doppler effect

MODULE 13

3. WORDS YOU MUST KNOW

Let us remember the words we have been using in our study of this physics course

- Displacement the distance an object has moved from its starting position moves in a particular direction.SI unit: m, this can be zero, positive or negative
- Non mechanical displacement periodically changing electric, magnetic, pressure of gases, currents, voltages are non-mechanical oscillations. They are represented by sin and cosine functions like mechanical displacements

  For a vibration or oscillation, the displacement could ne mechanical, electrical magnetic. Mechanical displacement can be angular or linear.

  - **Energy:** In equilibrium position $y = 0$, we have

    Potential energy of the body, $U = 0(zero)$

    And kinetic energy of the body, $K = \frac{1}{2} m\, \omega^2\, a^2 = E_{max}$

    In maximum displace position $(y = a)$, we have

    Potential energy of the body, $U = \frac{1}{2} m\, \omega^2\, a^2 = E_{max}$

    And kinetic energy of the body, $K = O(zero)$

  - **Wave:** A wave is a disturbance in the medium which causes the particles of the medium to undergo vibratory motion about their mean position.
  - **Wave motion:** method of energy transfer from a vibrating source to any observer.
  - **Mechanical** wave energy transfer by vibration of material particles in response to a vibrating source examples water waves, sound waves , waves in strings
  - **The speed of wave in medium** depends upon elasticity and density
  - **Longitudinal mechanical wave** a wave in which the particles of the medium vibrate along the direction of propagation of the wave

- **Transverse mechanical wave** a wave in which the particles of the medium vibrate perpendicular to the direction of propagation of the wave
- **A progressive wave:** The propagation of a wave in a medium means the particles of the medium perform simple harmonic motion without moving from their positions, then the wave is called a simple harmonic **progressive wave**

  In progressive wave, the disturbance produced in the medium travels onward, it being handed over from one particle to the next. Each particle executes the same type of vibration as the preceding one, though not at the same time. In this wave, energy propagates from one point in space to the other.
- **Displacement relation for a Progressive wave:** The displacement of the particle at an instant t is given by,

  $$y = a \sin\ (\omega\, t - \phi\ )\ \text{ OR }\ y\ = a\ \sin\ (\omega\, t - k\ x\ )$$

  If ϕ be the phase difference between the above wave propagating along the +X direction and another wave, then the equation of that wave will be

  $$y = a \sin \left\{2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \ \phi\right\}$$

  $$y = a \sin(\omega\, t - k\ x + \ \phi)$$

  The displacement could also be expressed in terms of the cosine function without affecting any of the subsequent relation.
- **Particle Velocity:** The equation of a plane progressive wave propagating in the positive direction of X-axis is given by

  $$v = \frac{dy}{dt} = \ \omega\ a\ cos\ (\omega\ t - k\ x)$$

  The maximum particle velocity is given by,

  $v_{max}\ =\ \omega\ a$, this is known as velocity amplitude of particle.
- **Particle Acceleration:** The instantaneous acceleration f of a particle is

  $$f = \frac{du}{dt}\ =\ \omega^2\ a \sin\ (\omega\ t - k\ x)\ =\ -\omega^2\ y$$

  The maximum value of the particle displacement y is a. Therefore, acceleration amplitude is $f_{max}\ =\ -\ \omega^2\ a$
- **Principle of Superposition:** The net displacement of the medium / particles (through which waves travel) due to the superposition is equal to the sum of individual displacements (produced by each wave).

- **Standing wave:** When two identical waves of the same amplitude and frequency travel in opposite directions with the same speed along the same path superpose each other, the resultant wave does not travel in the either direction and is called a stationary or standing wave. It is called a standing wave because it does not appear to move. In such a wave, energy is confined in the space of the wave.
- **Nodes:** The points at which the amplitude is zero (i.e., where there is no motion of particles at all) are nodes.
- **Antinodes:** The points at which the amplitude of the particle is the largest are called antinodes.

4. INTRODUCTION

Many musical instruments consist of an air column enclosed inside of a hollow metal tube. Sometimes the metal tube may be more than a meter in length; it is often curved upon itself one or more times in order to conserve space.

https://www.tes.com/lessons/e4r83B6Swr_MoQ/copy-of-as-standing-waves
recap of difference between progressive and standing waves

1. Compare progressive waves to standing waves in terms of energy transferred

2. What is the wavelength in a progressive wave

3. In a progressive wave, do all parts of the wave have the same amplitude? Is this the same or different in a standing wave?

4. Between nodes and on either side of the nodes, are the particles in phase or out of phase? (see video in a short while)

https://ccsearch.creativecommons.org/image/detail/lBdiygNmpwH2O3FMCGTw-g=

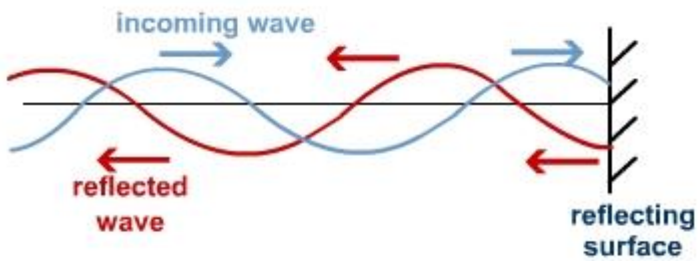
If the end of the tube is uncovered such that the air at the end of the tube can freely vibrate when the sound wave reaches it, then the end is referred to as **an open end.**

If **both** ends of the tube are uncovered or open, the musical instrument is said to contain an **open-end air column.** A variety of instruments operate on the basis of open-end air columns; examples include the flute.

A musical instrument has a set of natural frequencies at which it vibrates at when a disturbance is introduced into it. These natural frequencies are known as the harmonics of the instrument.

Each harmonic is associated with a standing wave pattern.

A **standing wave pattern is defined as a vibrational pattern created within a medium when the vibrational frequency of the source causes reflected waves from one end of the medium to interfere with incident waves from the source in such a manner that specific point along the medium appear to be standing still**.



https://www.s-cool.co.uk/assets/learn_its/alevel/physics/progressive-waves/standing-waves/image1.jpg

In the case of **stringed instruments** (as discussed earlier), standing wave patterns were drawn to depict the amount of movement of the string at various locations along its length.

Such patterns show nodes - points of no displacement or movement - at the two fixed ends of the string.

In the case of air columns, **a closed end in a column of air** is analogous **to the fixed end on a vibrating string**.

That is, at the closed end of an air column, air is not free to undergo movement and thus is forced into assuming the nodal positions of the standing wave pattern.

However, **air is free to undergo its back-and-forth longitudinal motion at the open end of an air column,** and as such, the standing wave patterns will depict **antinodes at the open ends of air columns.**

So the basis for drawing the standing wave patterns for air columns is that **vibrational antinodes will be present at any open end** and **vibrational nodes will be present at any closed end**.

If this principle is applied to open-end air columns, then the pattern for the fundamental frequency (the lowest frequency and longest wavelength pattern) will have antinodes at the two open ends and a single node in between.
For this reason, the standing wave pattern for the fundamental frequency (or first harmonic) for an **open-end air column is unique**

The distance between antinodes on a standing wave pattern is equivalent to one-half of a wavelength.

https://i.ytimg.com/vi/aIbIH5ZkmEs/hqdefault.jpg

Musical instruments depend on standing waves for their characteristic sounds.

In a Jal Tarang, the bowls are filled with different volumes of water. The height of air column above the water level is different. Hence the natural frequency in a particular bowl is unique.

We have learnt in last module that when a string is plucked the string resonates in such a way as to create a standing wave.

A good sitar player learns the art of creating harmonics and overtones in a rhythmic, appealing way, which is music to our ears- in case it was not so it would only be noise.

In this module, we will learn that if air is blown across the opening of a pipe, a standing wave is set up inside the pipe and a sound is generated.

FLUTE **CLARINET**

Musical wind instruments like flute, clarinet etc. are based on the principle of vibrations of air columns. Due to the superposition of the incident wave and the reflected wave, longitudinal stationary waves are formed in the pipe.

Before we proceed, let us see what is an organ pipe ?

In days, when microphone was not invented, pipes were placed in public spaces like the church. The organ pipes would resonate with the same frequencies and produce a loud clear sound. These can be seen in many old churches throughout the world.

Notice -The metal pipes of dissimilar lengths and dissimilar diameters

https://p1.pxfuel.com/preview/387/536/53/church-organ-music-organ-whistle.jpg

Organ pipe is the musical instrument in which sound is produced by setting an air column into vibrations.

We will discuss the two types of Organ pipes which are

(1) **Closed organ pipes- closed at one end while open at the other end.**

(2) **Open organ pipe -open at both ends.**

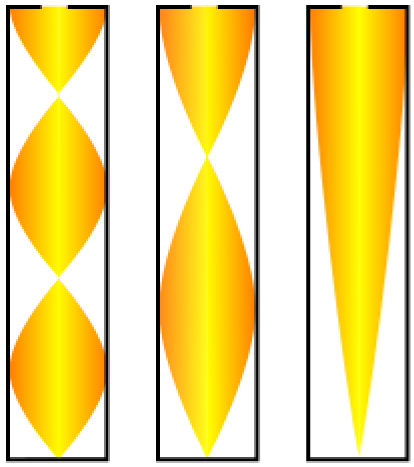
In this module, we will discuss the formation of standing waves in closed and open organ pipes.

5. STANDING WAVES IN CLOSED ORGAN PIPE

Closed organ pipe- one end of the pipe is open and the other end is closed.

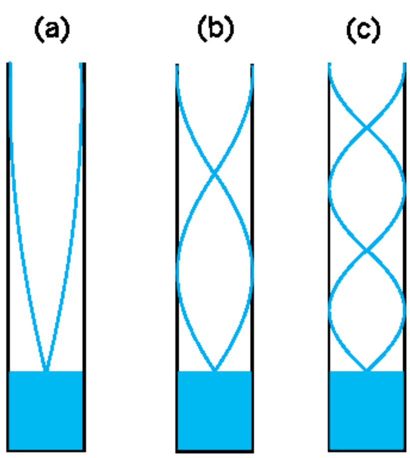
If air is blown lightly at the open end of the closed organ pipe,

- **the air column sets into vibrations and**
- **wave thus generated gets reflected from the closed end,**
- **the direction of motion of particles change,**
- **the displacement is zero at the closed end.**
- **The displacement is maximum at the open end, that means there is a node at the closed end and an antinode at the open end.**



https://commons.wikimedia.org/wiki/File:Onde_stationnaire_vitesse_tuyau_ferme_trois_modes.svg

We draw these as

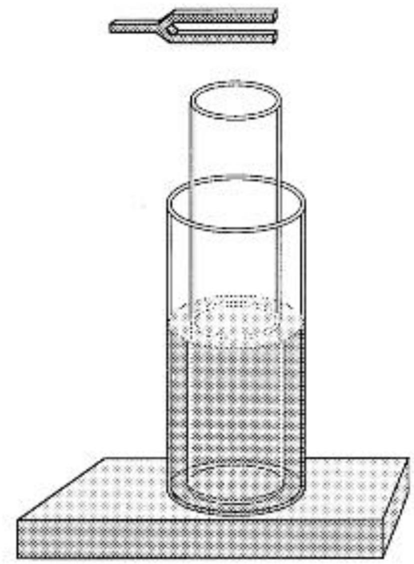


See the drawing carefully and check out whether the conditions as mentioned above are represented.

Analytical treatment of Standing waves in closed organ pipe

Let us next consider normal modes of oscillation of an air column with one end closed and the other open.

A glass tube partially filled with water can illustrate this system.



You can amplify a tuning fork by holding it over a pipe and changing the length of the pipe. At certain pipe lengths, the pitch made by the tuning fork sounds very loud as it resonates with the air column in the pipe

- **The end in contact with water is a node, while the open end is an antinode.**
- **At the node the pressure changes are the largest, while the displacement is minimum (zero).**
- **At the open end - the antinode, it is just the other way - least pressure change and displacement is maximum.**

Consider a cylindrical pipe of length L placed vertically with its closed end (node) at x = 0 and antinode at x = L

Now consider a sound wave travelling along the tube and a reflected wave of the same amplitude and wavelength in the opposite direction.

The wave travelling down can be represented as

$$\boldsymbol{y_1(x,t) = a\sin(\omega t - kx)}$$

The wave travelling along negative direction can be represented as

$$y_2(x,t) = -a\sin(\omega t + kx)$$

The resultant wave on the string is, according to the principle of superposition:

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$$= a\,[\sin(\omega t - kx) - \sin(\omega t + kx)]$$

Using the familiar trigonometric identity

sin (A+B) - sin (A–B) = 2 sin B cos A, we get,

y (x, t) = -(2a sin kx) cos ωt

This equation represents standing wave.

The amplitude of this wave is 2a sin kx.

Position of nodes

Nodes are the position of zero oscillation

Taking the end in contact with water to be x = 0, the node condition is amplitude = 0

2a sin kx = 0

sin kx = 0

kx= nπ

Using $k = \frac{2\pi}{\lambda}$

$x = \frac{n\lambda}{2}$ **where n = 0, 1, 2**

The node condition is satisfied.

Position of antinodes

Antinodes are the position of maximum displacement

If the other end, x = L is an antinode,

Then amplitude is max

2a sin k L = 1

sin k L=1
$L = (n' + \frac{1}{2})\frac{\lambda}{2}$, for n ′= 0, 1, 2, 3, …

The possible wavelengths are then restricted by the relation:

$\lambda = \frac{2L}{\left(n' + \frac{1}{2}\right)}$, for n ′= 0, 1, 2, 3,... and the corresponding frequencies can be calculated as

Frequency = $\frac{\text{velocity}}{\text{wavelength}}$

The normal modes – the natural frequencies – of the system are

$\nu = \left(n' + \frac{1}{2}\right)\frac{v}{2L}$ n ′= 0, 1, 2, 3, ...

$\nu = n\,.\frac{v}{4L}$ n = 1, 3, 5, 7………..

This video shows the relation between closed pipe and wavelength

https://youtu.be/kA-8IniFmDw

Here n is known as harmonic number. n = 1 corresponds to first harmonic or fundamental mode.
In a closed pipe, only odd harmonics are possible.

Fundamental mode and harmonics

- **Any system in which standing waves can form has numerous natural frequencies.**
- **The set of all possible standing waves are known as the harmonics of a system.**
- **The simplest of the harmonics is called the fundamental or first harmonic.**
- **Subsequent standing waves are called the second harmonic, third harmonic, etc.**
- **If air is blown strongly at the open end, frequencies higher than fundamental frequency can be produced. They are called harmonics of the fundamental frequency.**

In general, harmonics above the fundamental, especially in music theory, are sometimes also called overtones.

Difference between overtones and harmonics

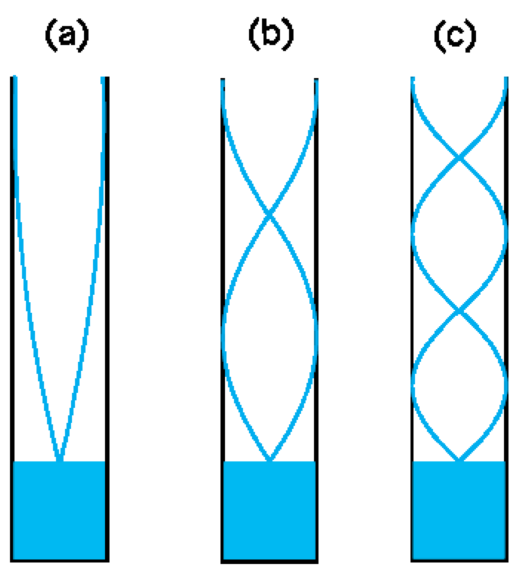
- **The harmonics are integral multiple of the fundamental frequency.**
- **But overtones are not necessarily integral multiples of fundamental frequency.**
- **In some musical instruments, an overtone is observed at 1.4 times fundamental frequency.**
- **Hence, words, ‘overtones’ and ‘harmonics’ should not use interchangeably.**

RELATION BETWEEN FUNDAMENTAL FREQUENCY AND HARMONICS FOR A CLOSED PIPE

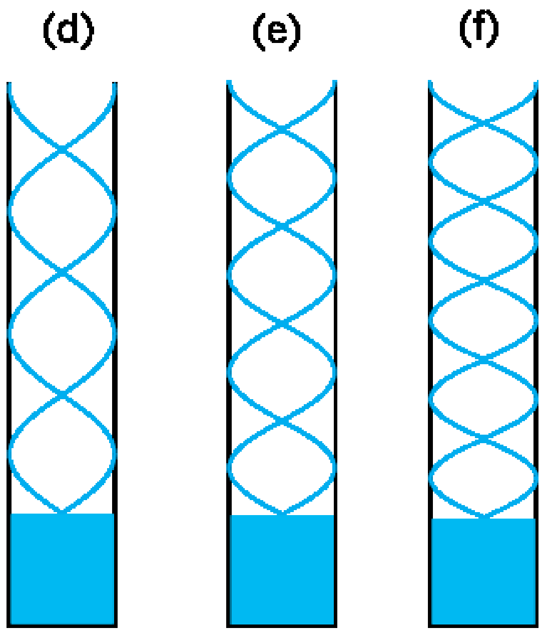
The fundamental frequency corresponds to n = 1, and is given by $\frac{v}{4L}$

The higher frequencies are **odd harmonics**, i.e., odd multiples of the fundamental frequency: $3\left(\frac{v}{4L}\right)$, $5\left(\frac{v}{4L}\right)$, etc.

The first six odd harmonics of air column with one end closed and the other open are shown in the figure.



Fundamental or first harmonic **3rd harmonic** **5th harmonic**

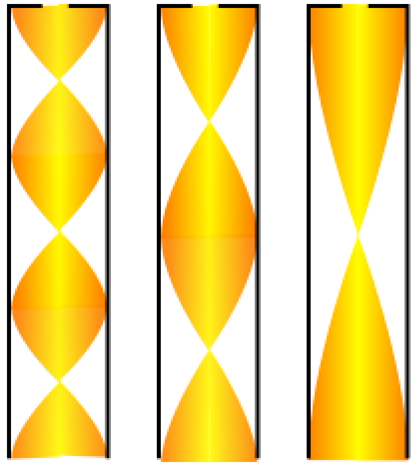


Normal modes of an air column open at one end and closed at the other end. Only the odd harmonics are seen to be possible.

6. STANDING WAVES IN OPEN ORGAN PIPE

In open organ pipe both the ends are open.

- **The waves are reflected from these ends are reflected from these ends without change of phase.**
- **The particles have maximum displacement at the open ends.**
- **As a result, at both the ends the antinodes are formed.**
- When air is blown into the open organ pipe, the air column vibrates and antinodes are formed at the ends and a **node is formed in the middle of the pipe.**
- **It is so symmetrical that the result can be drawn graphically and analyzed using simple mathematics**



https://commons.wikimedia.org/wiki/File:Onde_stationnaire_vitesse_tuyau_ouvert_trois_modes.svg

ANALYTICAL TREATMENT OF STANDING WAVE IN OPEN ORGAN PIPE

In case of an organ pipe, the reflecting surface is an open boundary at both the end of the pipe. Therefore, **no change of phase is observed in reflected wave.**

Let the equation of wave travelling along the positive direction of x axis be given by

$$y_1(x,t) = a\sin\left(\omega t - kx + \frac{\pi}{2}\right)$$
$$y_1 = a\cos(\omega t - kx)$$

The equation of reflected wave is given by

$$y_2(x,t) = a\cos(\omega t + kx)$$

Using principle of superposition

$$y(x,t) = y_1(x,t) + y_2(x,t)$$
$$y(x,t) = \cos(\omega t - kx) + \cos(\omega t + kx)$$

We get,

$$\boldsymbol{y(x,t) = 2a\cos(kx)\cos(\omega t)}$$

Using trigonometric identity , cos(A+B) + cos(A-B) = 2 cosA cosB

which is the general expression of standing wave.

POSITION OF ANTINODES

Since both ends are open, antinodes will be formed at both the ends.
That means amplitude is maximum at both the ends.

At x = L, amplitude is maximum

$$\cos kL = 1$$

Implies that ,k L= n π

$$L = \frac{n\lambda}{2}$$

$$\lambda = \frac{2L}{n}$$

Corresponding frequencies are
(Frequency= velocity /wavelength)

$f = \frac{nv}{2L}$ **where n = 1,2,3 ……..**

FUNDAMENTAL MODE AND HARMONICS

In an open pipe, the frequency of the fundamental mode is given by $f = \frac{v}{2L}$. And all harmonics are allowed.

RELATION BETWEEN FUNDAMENTAL MODE AND HARMONICS FOR AN OPEN PIPE

a) **Fundamental mode or 1st harmonic**
For n =1

$f_1 = \frac{v}{2L}$

This is called fundamental mode or 1st harmonic

b) **2nd harmonic**
For n = 2

$f_2 = \frac{2v}{2L}$

$f_2 = 2f_1$

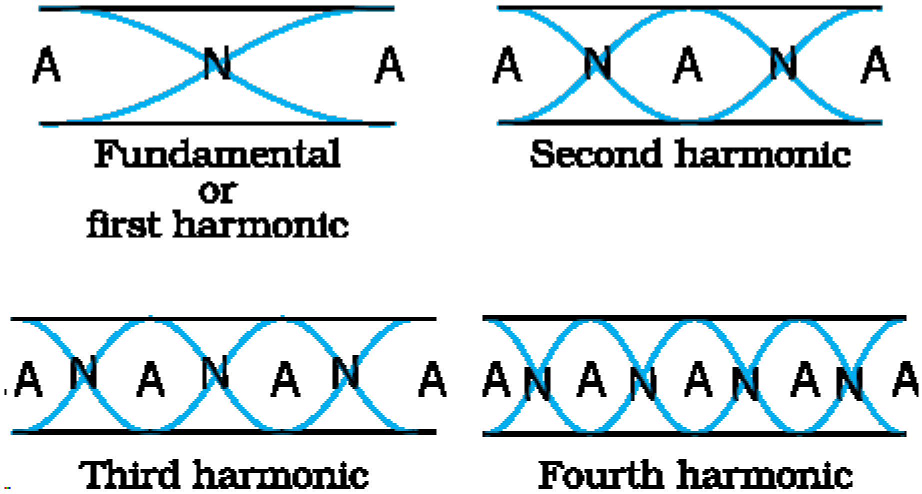
c) **3rd harmonic**
n = 3

$f_3 = \frac{3v}{2L}$

$f_3 = 3f_1$

Thus in general for the harmonic, in case of an open pipe,

$f_n = nf_1$, where n= 1,2 3 …….



Standing waves in an open pipe, first four harmonics are depicted.

This video shows Standing waves in open tubes
https://youtu.be/BhQUW9s-R8M

This video shows Fundamental frequency-harmonics and overtones- Standing waves in pipe
https://youtu.be/5Dw4rG7qRM8T

This video shows Traveling Waves, Standing Waves, and Musical Instruments
https://youtu.be/D_RIzl1uCxY

INTERESTING TO READ

Reflection of sound in an open pipe

When a high pressure pulse of air travelling down an open pipe reaches the other end, its momentum drags the air out into the open, where pressure falls rapidly to the atmospheric pressure.

As a result, the air following after it in the tube is pushed out. The low pressure at the end of the tube draws air from further up the tube. The air gets drawn towards the open end forcing the low pressure region to move upwards.

As a result, a pulse of high pressure air travelling down the tube turns into a pulse of low pressure air travelling up the tube. We say a pressure wave has been reflected at the open end with a change in phase of 180^0.

Standing waves in an open pipe organ like the flute is a result of this phenomenon.

Compare this with what happens when a pulse of high pressure air arrives at a closed end: it collides and as a result pushes the air back in the opposite direction.

Here, we say that the pressure wave is reflected, with no change in phase.

EXAMPLE

A pipe, 30.0 cm long, is open at both ends. Which harmonic mode of the pipe resonates a 1.1 kHz source? Will resonance with the same source be observed if one end of the pipe is closed? Take the speed of sound in air as 330 m s^{-1}.

SOLUTION

The first harmonic frequency is given by

$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$ (Open pipe)

where L is the length of the pipe.

The frequency of its nth harmonic is:

$\nu_n = \frac{nv}{2L}$, for n = 1, 2, 3, ... (open pipe)

For L = 30.0 cm, v = 330 m s^{-1},

$\nu_n = \frac{n \times 330\ (m\ s^{-1})}{0.6\ (m)} = 550\ n\ s^{-1}$

Clearly, a source of frequency 1.1 kHz will resonate at ν_2, i.e. the second harmonic.

Now if one end of the pipe is closed $\nu_1 = \nu_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$ (pipe closed at one end) and only the odd numbered harmonics are present:

$\nu_3 = \frac{3v}{4L}$, $\nu_5 = \frac{5v}{4L}$, and so on.

For L = 30 cm and v = 330 m s^{-1}, the fundamental frequency of the pipe closed at one end is 275 Hz and the source frequency corresponds to its fourth harmonic. Since this harmonic is

not a possible mode, no resonance will be observed with the source, the moment one end is closed.

7. SUMMARY

- **A travelling wave, at a rigid boundary or a closed end, is reflected with a phase reversal but the reflection at an open boundary takes place without any phase change.**
  For an incident wave
  yi (x, t) = a sin (kx – ωt)
  the reflected wave at a rigid boundary is
  yr (x, t) = – a sin (kx + ωt)
  For reflection at an open boundary
  yr (x,t) = a sin (kx + ωt)

- The interference of two identical waves moving in opposite directions produces standing waves. For a string with fixed ends, the standing wave is given by
  y (x, t) = [2a sin kx] cos ωt

- **Standing waves are characterised by fixed locations of zero displacement called *nodes* and fixed locations of maximum displacements called *antinodes.* The separation between two consecutive nodes or antinodes is λ/2.**

- The oscillation mode with lowest frequency is called the fundamental mode or the first harmonic.

- **The *second harmonic* is the oscillation mode with *n* = 2 and so on.**

- A pipe of length *L* with one end closed and other end open (such as air columns) vibrates with frequencies given by

$$\nu = \left(n + \frac{1}{2}\right)\frac{v}{2L}$$

***n* = 0, 1, 2, 3, ...**

The set of frequencies represented by the above relation are the normal modes of oscillation of such a system. The lowest frequency given by *v*/4*L* is the fundamental mode or the first harmonic.

- **A string of length *L* fixed at both ends or an air column closed at one end and open at the other end, vibrates with frequencies called its normal modes. Each of these frequencies is a *resonant frequency* of the system.**

- A pipe of length L with one end closed and other end open (such as air columns) vibrates with frequencies given by
  $\nu = n.\frac{v}{4L}$, n = 1, 3,5,7, ...

- **The set of frequencies represented by the above relation are the normal modes of oscillation of such a system. The lowest frequency given by v/4L is the fundamental mode or the first harmonic.**

- A pipe of length L with both ends open (such as air column in a pipe with both ends open) vibrates with frequencies given by
  $\nu = n.\frac{v}{2L}$, n =1, 2, 3, 4………..

- **The fundamental mode has frequency $\nu_1 = \frac{v}{2L}$ in this case.**
  n is also known as harmonic number.