1. Details of Module and its structure

Subject Name	Physics
Course Name	Physics 02 (Physics Part 2,Class XI)
Module Name/Title	Unit 10, Module 10, Propagation of Waves Chapter 15, Waves
Module Id	keph_201502_eContent
Pre-requisites	Previous module: Wave motion, types of waves, mechanical and electromagnetic waves, longitudinal and transverse waves, wave speed, factors effecting speed of wave in a medium
Objectives	After going through this module, the learners will be able to: • Establish a relation for the displacement for progressive waves • Graphically represent waves • Know the Principle of superposition of waves
Keywords	Equation of progressive wave, wave motion, particle velocity , particle acceleration, wave velocity, principle of superposition of waves

2. Development team

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1. UNIT SYLLABUS Unit: 10

Oscillations and waves
Chapter 14: oscillations

Periodic motion, time period, frequency, displacement as a function of time, periodic functions, Simple harmonic motion (SHM) and its equation; phase; oscillations of a loaded spring-restoring force and force constant; energy in SHM. Kinetic and potential energies, simple pendulum, derivation of expression for its time period
Free forced and damped oscillations (qualitative ideas only); resonance

Chapter 15: Waves

Wave motion transverse and longitudinal waves, speed of wave motion, displacement, relation for a progressive wave, principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, beats, Doppler effect

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 15 MODULES

Module 1	• **Periodic motion** • **Special vocabulary** • **Time period, frequency,** • **Periodically repeating its path** • **Periodically moving back and forth about a point** • **Mechanical and non-mechanical periodic physical**

	quantities
Module 2	• Simple harmonic motion • Ideal simple harmonic oscillator • Amplitude • Comparing periodic motions phase, • Phase difference Out of phase In phase not in phase
Module 3	• Kinematics of an oscillator • Equation of motion • Using a periodic function (sine and cosine functions) • Relating periodic motion of a body revolving in a circular path of fixed radius and an Oscillator in SHM
Module 4	• Using graphs to understand kinematics of SHM • Kinetic energy and potential energy graphs of an oscillator • Understanding the relevance of mean position • Equation of the graph • Reasons why it is parabolic
Module 5	• Oscillations of a loaded spring • Reasons for oscillation • Dynamics of an oscillator • Restoring force • Spring constant • Periodic time spring factor and inertia factor
Module 6	• Simple pendulum • Oscillating pendulum • Expression for time period of a pendulum • Time period and effective length of the pendulum • Calculation of acceleration due to gravity • Factors effecting the periodic time of a pendulum • Pendulums as ‘time keepers’ and challenges • To study dissipation of energy of a simple pendulum by plotting a graph between square of amplitude and time

Module 7	• **Using a simple pendulum plot its L-T^2graph and use it to find the effective length of a second's pendulum** • **To study variation of time period of a simple pendulum of a given length by taking bobs of same size but different masses and interpret the result** • **Using a simple pendulum plot its L-T^2graph and use it to calculate the acceleration due to gravity at a particular place**
Module 8	• **Free vibration natural frequency** • **Forced vibration** • **Resonance** • **To show resonance using a sonometer** • **To show resonance of sound in air at room temperature using a resonance tube apparatus** • **Examples of resonance around us**
Module 9	• **Energy of oscillating source, vibrating source** • **Propagation of energy** • **Waves and wave motion** • **Mechanical and electromagnetic waves** • **Transverse and longitudinal waves** • **Speed of waves**
Module 10	• **Displacement relation for a progressive wave** • **Wave equation** • **Superposition of waves**
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	• **Relation between fundamental mode and overtone frequencies, harmonics** • **To study the relation between frequency and length of a given wire under constant tension using sonometer** • **To study the relation between the length of a given wire and tension for constant frequency using a sonometer**
Module13	• **Standing waves in pipes closed at one end,** • **Standing waves in pipes open at both ends** • **Fundamental and overtones** • **Relation between fundamental mode and overtone frequencies** • **Harmonics**
Module 14	• **Beats** • **Beat frequency** • **Frequency of beat** • **Application of beats**
Module 15	• **Doppler effect** • **Application of Doppler effect**

MODULE 10

3. WORDS YOU MUST KNOW

Let us remember the words we have been using in our study of this physics course

- Displacement the distance an object has moved from its starting position moves in a particular direction.SI unit: m, this can be zero, positive or negative
- Non mechanical displacement periodically changing electric, magnetic, pressure of gases, currents, voltages are non-mechanical oscillations. They are represented by sin and cosine functions like mechanical displacements

  For a vibration or oscillation, the displacement could ne mechanical, electrical magnetic. Mechanical displacement can be angular or linear.
- Acceleration- time graph: graph showing change in velocity with time , this graph can be obtained from position time graphs
- Instantaneous velocity

  Velocity at any instant of time

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- Instantaneous acceleration
  Acceleration at any instant of time
  $$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$
- kinematics study of motion without considering the cause of motion
- **Frequency:** The number of vibrations / oscillations in unit time.
- **Angular frequency:** a measure of the *frequency* of an object varying sinusoidally equal to 2π times the *frequency* in cycles per second and expressed in radians per second.
- **Oscillation:** **one complete to and fro motion about the mean position** *Oscillation* refers to any periodic motion of a body moving about the equilibrium position and repeats itself over and over for a period of time.
- **Vibration:** It is a to and fro motion about a mean position. The periodic time is small, so we can say oscillations with small periodic time are called vibrations. The displacement from the mean position is also small.
- **Inertia:** *Inertia* is the tendency of an object in motion to remain in motion, or an object at rest to remain at rest unless acted upon by a force.
- **Sinusoidal:** **like a sin $\boldsymbol{\theta}$ $\boldsymbol{vs\theta}$** A sine wave or sinusoid is a curve that describes a smooth periodic oscillation.
- **Simple harmonic motion (SHM):** repetitive movement back and forth about an equilibrium (mean) position, so that the maximum displacement on one side of this position is equal to the maximum displacement on the other side. The time interval of each complete vibration is the same.
- **Harmonic oscillator:** A *harmonic oscillator* is a *physical* system that, when displaced from equilibrium, experiences a restoring force proportional to the displacement.
- **Mechanical energy:** is the sum of potential **energy** and kinetic **energy**. It is the **energy** associated with the motion and position of an object.
- **Restoring force:** is a *force* exerted on a body or a system that tends to move it towards an equilibrium state.
- **Conservative force:** is a *force* with the property that the total work done in moving a particle between two points is independent of the taken path. When an object moves from one location to another, the *force* changes the potential energy of the object by an amount that does not depend on the path taken.
- **Periodic motion:** **motion** repeated in equal intervals of time.
- Simple pendulum: If a heavy point-mass is suspended by a *weightless*, inextensible and perfectly flexible string from a rigid support, then this arrangement is called a 'simple pendulum'
- Restoring Force: No net force acts upon a vibrating particle in its equilibrium position. Hence, the particle can remain at rest in the equilibrium position. When it is displaced from its equilibrium position, then a periodic force acts upon it which is always directed towards

the equilibrium position. This is called the 'restoring force'. The spring gets stretched and, due to elasticity, exerts a restoring force F on the body directed towards its original position. By Hooke's law, the force F is given by

$$F = -kx$$

- **Displacement Equation for a SHM:**

  $$\boldsymbol{y = A \sin \omega t}$$

- **Amplitude:** The maximum value of $\boldsymbol{\sin \omega t}$ is 1. The maximum value of the displacement y will be "A". This maximum displacement (for a SHM) is called the 'amplitude' of motion. It is equal to the radius of the reference circle.

- **Periodic time:** The time taken to complete one vibration; it also equals the time to go once around a circle of reference ($T = 2\pi/\omega$).
- **Frequency:** The number of oscillations, completed by the oscillating particle in one second, is called as its 'frequency' (f).

  $$f = \frac{1}{T} = \frac{\omega}{2\pi}$$
- **Phase:** When a particle vibrates, its position and direction of motion vary with time. The general equation of displacement is

  $\mathbf{y = a \sin (\omega t + \phi)}$, ϕ is called the 'initial phase'. We usually take $\phi = 0$, when we are talking about the SHM of a single particle.
- **Velocity in SHM:** The velocity (v) of the particle P, executing a SHM, can be expressed as a function of its displacement (y) from its mean position:

  $$v = \omega \sqrt{a^2 - y^2}$$

- **Acceleration in SHM:** For the particle executing a SHM, the acceleration (α) is directly proportional to the displacement (y) from the mean position and is always directed opposite to the instantaneous displacement. Hence, $\alpha = -\omega^2 y$ and

  $$\alpha_{max} = -\omega^2 a$$
- **Energy:** In equilibrium position $y = 0$, we have

  Potential energy of the body, $U = 0(zero)$

  And kinetic energy of the body, $K = \frac{1}{2} m\,\omega^2\, a^2 = E_{max}$

  In maximum displace position $(y = a)$, we have

  Potential energy of the body, $U = \frac{1}{2} m\,\omega^2\, a^2 = E_{max}$

  And kinetic energy of the body, $K = O(zero)$
- Wave motion: method of energy transfer from a vibrating source to any observer.
- Mechanical wave energy transfers by vibration of material particles in response to a vibrating source examples water waves, sound waves, waves in strings

- The speed of wave in medium the speed with which the energy propagates through the medium. Speed of the wave depends upon elasticity and density
- Longitudinal mechanical wave a wave in which the particles of the medium vibrate along the direction of propagation of the wave example sound waves
- Transverse mechanical wave a wave in which the particles of the medium vibrate perpendicular to the direction of propagation of the wave example water waves

4. INTRODUCTION

Wave motion is a mode of transfer of energy or a disturbance travelling through an elastic medium due to periodically oscillating source

We have discussed wave motion; the nature of mechanical waves (transverse and longitudinal waves) and the wave equation in an elastic medium.

It should be clearly understood that the wave velocity v is determined only by the elastic and internal properties of the medium.

Therefore, v is constant for a medium as long as its physical properties like temperature, density and pressure do not change.

The frequency of the wave 'f' is characterized by the source, which produces the disturbance.

The wave-motion is a disturbance produced in a medium which advances in (a uniform and homogenous) medium with a **definite speed and frequency** without changing its form.

CHARACTERISTICS OF WAVE-MOTION:

i. For the propagation of mechanical waves, a material medium, having the properties of elasticity and inertia, is necessary.
ii. In wave-motion, the particles of the medium do not move from one place to another. They simply oscillate about their respective mean or equilibrium positions.
iii. The positions of the oscillating particles keep on changing during different states of the oscillation; however, the velocity of the disturbance remains constant (as it depends only upon the nature of the medium).
iv. The phase of oscillation of the consecutive particles of the medium goes on changing continuously.
v. In wave-motion, although the particles of the medium do not leave their respective mean positions, the energy does propagate from one part of the medium to the other.
vi. In a medium, two or more waves can propagate simultaneously without affecting each other's motion.

vii. Waves, reaching the interface of two media, are partly reflected and partly refracted.

viii. The phenomenon of interference and diffraction can occur in waves under appropriate conditions.

ix. The phenomenon of polarization does occur in transverse waves.

We will now derive a mathematical expression to describe a wave . we w ill also learn properties of waves.

5. PLANE PROGRESSIVE WAVE

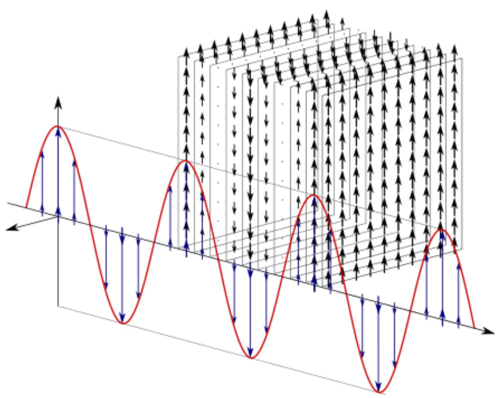
If, at any point *x* in a medium, we imagine a small plane perpendicular to the x-axis, all the particles on the plane will have the same displacement y at a given instant t. Such a wave is called a 'plane' wave.

When we produce waves in a medium continuously, the particles of the medium oscillate continuously with the same 'constant' amplitude but have different 'phase' values. Thus, when a plane progressive wave propagates in a medium then, at any instant, all the particles of the medium oscillate but their phases are different.

SIMPLE HARMONIC (PROGRESSIVE) WAVE

Let a wave propagate in a medium, in which the individual particles of the medium perform simple harmonic motion about their respective mean positions. Such a wave is called a 'simple harmonic (progressive) wave.

Suppose a simple harmonic progressive wave is advancing in a medium along the positive direction of the X-axis (from left to right). The displacement curve of this wave at any instant is shown in fig below.



Source Wikipedia

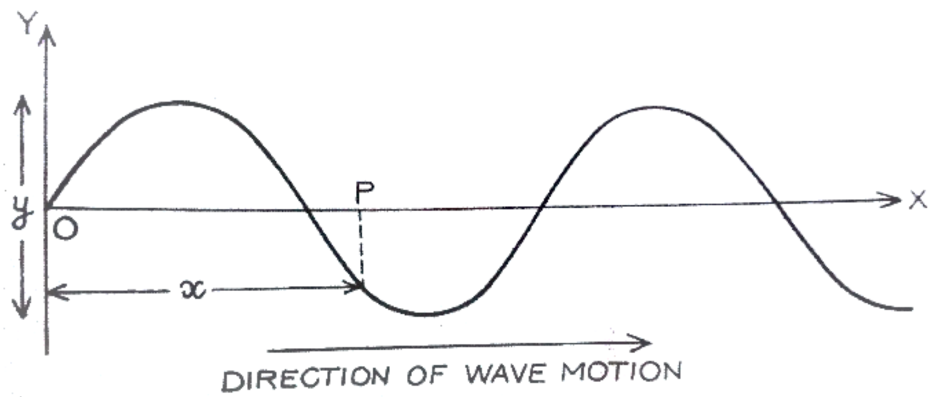
Suppose we count the time from the instant, when the particle at the origin O passes through its mean position in the positive direction of the Y-axis. Then, the displacement, y, of the particle at O , at any time t is given by

$$y = A \sin \omega t,$$

where A is the amplitude and ω is the angular frequency of the SHM executed by the particle. This is the displacement-time relation for a particle executing SHM.

6. DISPLACEMENT RELATION IN A PROGRESSIVE WAVE: wave equation

We know that in wave-motion, the successive particles start oscillating about their mean positions, a definite time later than their respective preceding particles. Therefore, as we move away from **a source O**, the **phase lag of the oscillation of particles with respect to the particle at O goes on increasing,**



If ϕ is the phase lag of a particle P**, at a distance x** from O, the **displacement of this particle at P, at an instant t,** is given by,

$$\boldsymbol{y = A \sin\ (\omega\, t - \phi\,)}$$

We know that for **a distance equal to λ, the phase-change is 2 π radians**.

Hence, the phase-change, for a distance x, will be

$$\boldsymbol{\phi = \frac{2\,\pi}{\lambda}\ x\ = k\,x}$$

Here, $\frac{2\pi}{\lambda}$ **is called the propagation constant** and is denoted by **k**

$$\therefore\ \ \boldsymbol{y\ = A\ \sin\ (\omega\, t - k\, x\ )}$$

If the wave is propagating along the – **x** direction, then inside the bracket in the above equation, there will be a plus sign instead of the minus sign. So we have,

$$\boldsymbol{y\ = A\ \sin\ (\omega\, t + k\, x\ )}$$

These equations can be written as:

$\boldsymbol{y\ = a\ \sin\ (\omega\, t - k\, x\ )}$ **, Propagation along +ve x direction**

$\boldsymbol{y = a \sin(\omega t + k x)}$ **, Propagation along -ve x direction**

Now, we can write, $\boldsymbol{\omega = 2\pi/T}$**,**

where T is the period of oscillation of the particles also, $k = \frac{2\pi}{\lambda}$.

Hence, for the wave, propagating along the positive direction of x-axis, we have

$$\therefore\ \boldsymbol{y = A \sin\left(\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right)}$$

$$\textbf{Or}\ \boldsymbol{y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)}$$

For the wave is propagating along – x direction, we would have

$$\boldsymbol{y = A \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right)}$$

In wave motion, $\mathbf{f = \frac{v}{\lambda}}$ **or** $\mathbf{\frac{1}{T} = \frac{v}{\lambda}}$ therefore, above equation can also be written as

$$\boldsymbol{y = a \sin 2\pi \left(\frac{v}{\lambda} t - \frac{x}{\lambda}\right)}$$

$$\boldsymbol{y = A \sin \frac{2\pi}{\lambda}(v t - x)}$$

If the wave is propagating along –x direction, then

$$\boldsymbol{y = A \sin \frac{2\pi}{\lambda}(v t + x)}$$

In equation, $y = A \sin(\omega t - k x)$, using $\omega = 2\pi f$ and $k = \frac{2\pi}{\lambda} = \frac{2\pi}{v/f} = \frac{2\pi f}{v}$, we get

$$y = A \sin 2\pi n f \left(t - \frac{x}{v}\right)$$

For the wave propagating along – X direction,

$$y = A \sin 2\pi f \left(t + \frac{x}{v}\right)$$

Hence, equations

$$\boldsymbol{y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)}$$

$$\boldsymbol{y = A \sin \frac{2\pi}{\lambda}(f t - x)}$$

$$\boldsymbol{y(x,t) = A \sin 2\pi f \left(t - \frac{x}{v}\right)}$$

y*(*x*,*t*) : displacement as a function of position x and time *t
***A*: amplitude of a wave**
$\omega = 2\pi f$: angular frequency of the wave
***k* : angular wave number**

are the different forms for the displacement relations of a progressive harmonic wave propagating along the positive direction of the x-axis.

If the wave is propagating along –x direction, then equations

$$\mathbf{y = A\ sin\ (\omega\, t + k\, x\ )}$$

$$\mathbf{y = A \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right)}$$

$$\mathbf{y = A \sin 2\pi f \left(t - \frac{x}{v}\right)}$$

$$\mathbf{y = A \sin 2\pi f \left(t + \frac{x}{v}\right)}$$

are the different forms of displacement relation for a progressive's harmonic wave?

If ϕ be the phase difference between the above wave, propagating along the +X direction, and another harmonic wave, the equation of that wave will be

$$y = A \sin \left\{2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) \pm \phi\right\}$$

$$\mathbf{y = A \sin(\omega\, t - k\, x\ \pm\ \phi)}$$

7. GRAPHICAL REPRESENTATION OF SIMPLE HARMONIC WAVE

The **equation of a simple harmonic wave contains three variables:**

- **particle displacement y**
- **particle position x**
- **time** t.

The graph of y can be drawn in one plane, when, of the two independent variables (x and t) one is kept constant and the other is varied. We shall study how y varies with t at a particular position of x; and how y varies with x at a particular time t.

Time-Displacement Graph of a Particle:

Suppose a simple harmonic progressive wave is advancing in a medium in the positive direction of the x-axis. At any instant t, the displacement y of particle of the medium, at a distance x from the source, is represented by the following equation:

$$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Here, A is the amplitude of particle-oscillating, T is time-period and λ is the wavelength. It is evident from this equation that the displacement y **changes periodically** with the **position x** of **the particle** and also **with time t**.

If we watch a particular particle of the medium for which $x = 0$, then, from the above equation, we get

$$y = A \sin 2\pi \frac{t}{T}$$

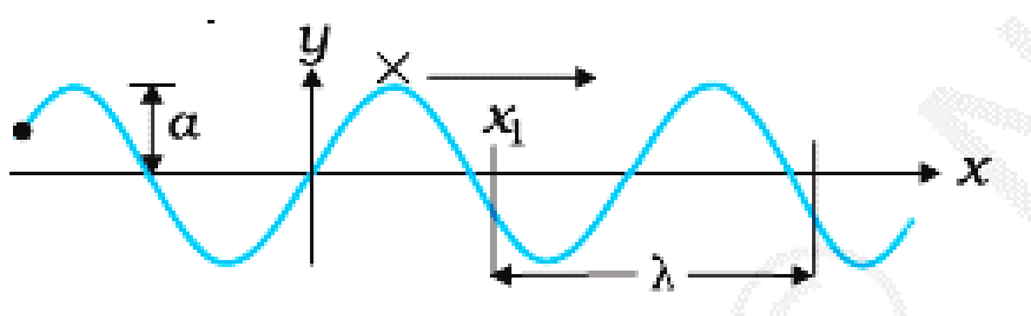
This shows that the displacement y of the particle is changing simple harmonically with time t; or we can say that, the particle is performing simple harmonic motion.

This is true for every particle.

The values of the displacements, y of particle (at $x = 0$) at different instants of time t, are tabulated below:

t	o	T/4	T/2	3T/4	T	5T/4	3T/2	7T/4
y	o	+ A	o	-A	0	A	0	-A

On the basis of these values, the $y - t$ graph is drawn as shown in Fig.



Distance-Displacement Graph of Particles: From the wave-equation

$$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right),$$

It is evident that the displacements y of a particle changes simple harmonically with the position x of the particle.

Suppose, at one particular instant, say at $\boldsymbol{t = 3T/2}$, we have to determine displacements of different particles. Substituting t = 3T/2 in the above equation, we get

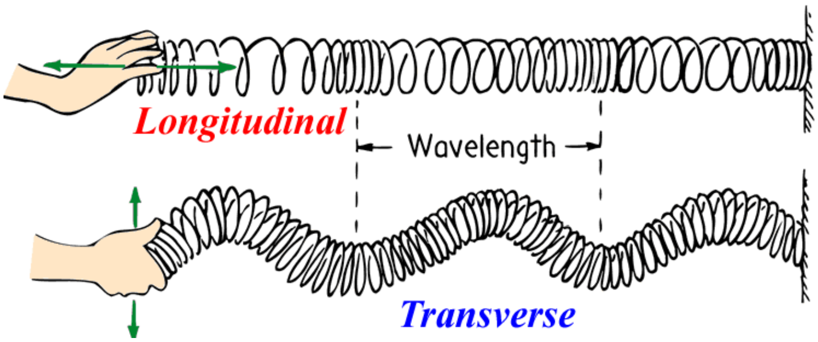
$$y = a \sin 2\pi\left(\frac{3}{2} - \frac{x}{\lambda}\right) = a \sin\left(3\pi - \frac{2\pi x}{\lambda}\right)$$

For different positions, x, of the particles, the displacement y, at t = 3T/2, are as given in the following table:

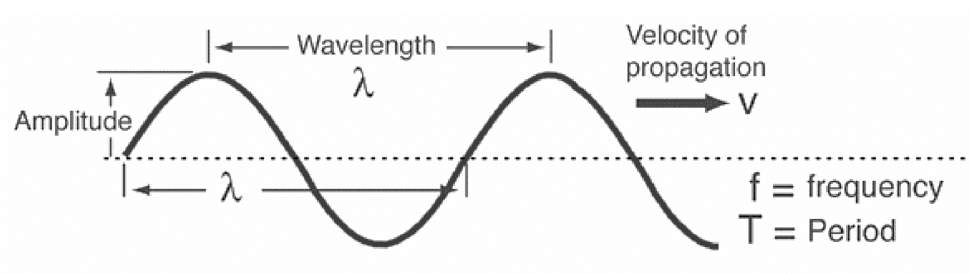
x	o	λ/4	λ/2	3 λ/4	λ	5 λ/4	3 λ/2	7 λ/4
y	o	+a	o	-a	o	+a	o	-a

On the basis of these values, the x-y graph is drawn as shown in fig. below.

Again, it is sine curve. To get an idea of the displacement of a particle, notice that when sound



waves propagate in air, the displacement y, of the particles of air is of the order of 10^{-6} m or less.



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Source: Wikipedia

THINK ABOUT THIS

- **Individual particles of the medium execute SHM**
- **The energy travels with a speed specific of the medium**
- **The particle velocity varies from 0 to maximum. the maximum velocity (amplitude of the wave) would be dependent on energy of the source**
- **The speed of propagation of energy does not depend on the energy of oscillation of the source**
- **Imagine a loud and a feeble sound note the speed of travel of sound is the**

same for both.

8. RELATION BETWEEN PHASE DIFFERENCE AND PATH DIFFERENCE FOR TWO PARTICLES

We have learnt that when a particle oscillates, the displacement of the particle from its equilibrium position and its direction of motion changes periodically.

The quantity which expresses, at any instant, the instantaneous displacement of the particle as well as its direction of motion, is called the 'phase' of the particle.

When a wave propagates through a medium, all the particles of the medium oscillate in the same manner about their equilibrium positions. However, if we observe a few particles close to one another, we shall find that their instantaneous displacements and directions of motion are different.

Suppose a simple harmonic progressive wave is propagating in a medium in the + x-direction. The instantaneous displacement of a medium particle, at a distance x from the origin, is given by

$$y = a\sin(\omega\, t - k\, x)$$

In this equation, the argument of sine function is $(\omega\, t - k\, x)$.

This represents the phase (ϕ) of the particle in position x at any instant t.

We therefore, have

$$\phi = (\omega\, t - k\, x)$$

Suppose, at any instant t, ϕ_1 and ϕ_2 are the phase of two particles whose distances from the origin are x_1 and x_2 respectively.

Then from equation, $\phi = (\omega\, t - k\, x)$, we have

$$\phi_1 = (\omega\, t - k\, x_1) \text{ and } \phi_2 = (\omega\, t - k\, x_2)$$

Hence, $\phi_1 - \phi_2 = k\,(x_1 - x_2) = \frac{2\pi}{\lambda}\,(x_1 - x_2)$

$$\boldsymbol{\Delta\ \phi = \frac{2\pi}{\lambda} \times \Delta\, x}$$

This is the **phase difference between two particles whose path difference is Δ x.** If Δ x = λ, then

$$\Delta\ \phi = 2\pi$$

Thus, **the phase difference between two particles at a given time instant, for two particles of the medium, having a path difference λ (wavelength) is 2π. Clearly, these two particles are in the same phase of oscillation**.

Again

$$\phi_1 = (\omega\, t_1 - k\, x) \quad \text{and} \quad \phi_2 = (\omega\, t_2 - k\, x)$$

So, **at a given position x, for two different time instant t_1 *and* t_2, we have**

$$\phi_1 - \phi_2 = \omega(t_1 - t_2) = \frac{2\pi}{T}(t_1 - t_2)$$

$$\Delta\,\phi = \frac{2\pi}{T} \times \Delta\,t$$

This is **the phase- change for a (particular) particle (in a given position) in a time-interval $\Delta\,t$. *If* $\Delta\,t = T$,** we have, after one time period.

$$\Delta\,\phi = 2\pi$$

A phase difference 2π, implies that the particle regains its initial phase.

We can use the above results to give the following definitions;

Wavelength (λ) equals the path difference between two particles of the medium that are in the same phase at a given time instant.

Time period (T) equals the time interval after which a given particle of the medium regains its initial phase.

9. VELOCITY AMPLITUDE AND ACCELERATION AMPLITUDE OF A PARTICLE IN A PROGRESSIVE WAVE

Particle Velocity: The equation of a plane progressive wave propagating along the positive direction of X-axis is given by

$$y = a\sin(\omega\, t - k\, x)$$

Here, a is displacement amplitude of a particle of the medium. The instantaneous velocity v of a particle is obtained by differentiating y in above equation with respect to time t, that is

$$v = \frac{dy}{dt} = \omega\, a\, cos\,(\omega\, t - k\, x)$$

It follows that the maximum particle velocity is given by

$$v_{max} = \omega\, a$$

This is known as velocity amplitude of the particle.

Particle Acceleration: The instantaneous acceleration f (say) of a particle is obtained by differentiating v with respect to t. Hence

$$f = \frac{dv}{dt} = \omega^2 \, a \sin(\omega \, t - k \, x)$$

$$= -\omega^2 \, y$$

Since the maximum value of the particle displacement is a, the acceleration amplitude has a magnitude

$$f_{max} = -\,\omega^2 \, a$$

EXAMPLE

A wave travelling along a string is described by,
***y(x, t)* = 0.005 sin (80.0 *x* – 3.0 *t*),**
in which the numerical constants are in SI units (0.005 m, 80.0 rad m^{-1}, and 3.0 rad s^{-1}).
Calculate

a. the amplitude,
b. the wavelength, and
c. the period and frequency of the wave. Also,

calculate the displacement *y* of the wave at a distance *x* = 30.0 cm and time *t* = 20 s ?

SOLUTION

On comparing this displacement equation with equation
$y\,(x, t\,) = a \sin(k\,x - \omega\,t\,)$,
We find

a. the amplitude of the wave is 0.005 m = 5 mm.
b. the angular wave number k and angular frequency ω are

$k = 80.0\ m^{-1}$ and $\omega = 3.0\ s^{-1}$
We then relate the wavelength λ to k through
$\lambda = 2\pi/k$

$$= \frac{2\pi}{80.0m^{-1}}$$

= 7.85 cm

c. Now we relate T to ω by the relation
$T = 2\pi/\omega$

$$= \frac{2\pi}{3.0s^{-1}}$$

= 2.09 s

and frequency, $f = 1/T = 0.48$ Hz
The displacement y at x = 30.0 cm and time t = 20 s is given by

y = (0.005 m) sin (80.0 × 0.3 – 3.0 × 20)

= (0.005 m) sin (–36 + 12π)
= (0.005 m) sin (1.699)

= (0.005 m) sin (970) nearly =5 mm

TRY THESE

- A transverse harmonic wave on a string is described by

$$y(x, t) = 3.0 \sin (36\,t + 0.018\,x + \pi/4)$$

where x and y are in cm and t in s. The positive direction of x is from left to right.

(a) Why is this a travelling wave?
(b) What are the speed and direction of its propagation?
(c) What are its amplitude and frequency?
(d) What is the initial phase at the origin?
(e) What is the least distance between two successive crests in the wave?
(f) plot the displacement (y) versus (t) graphs
(g) For x = 0, 2 and 4 cm. What are the shapes of these graphs?
(h) In which aspects does the oscillatory motion in travelling wave differ from one point to another: amplitude, frequency or phase?

- **For the travelling harmonic wave**

$$y(x, t) = 2.0 \cos 2\pi\,(10t - 0.0080\,x + 0.35)$$

where x and y are in cm and t in s.
Calculate the phase difference between oscillatory motions of two points separated by a distance of

(a) 4 m,
(b) 0.5 m,
(c) $\lambda/2$,
(d) $3\lambda/4$

- A travelling harmonic wave on a string is described by

$$y(x, t) = 7.5 \sin (0.0050x + 12t + \pi/4)$$

(a) what are the displacement and velocity of oscillation of a point at x = 1 cm, and t = 1 s? Is this velocity equal to the velocity of wave propagation?
(b) Locate the points of the string which have the same transverse displacements and velocity as the x = 1 cm point at t = 2 s, 5 s and 11 s.

- **A sound pulse (for example, a short pip by a whistle) is sent across a medium.**

(a) Does the pulse have a definite (i) frequency, (ii) wavelength, (iii) speed of propagation?
(b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s),
Is the frequency of the note produced by the whistle equal to 1/20 or 0.05 Hz ?

10. SUPERPOSITION OF WAVES:

What happens when two wave pulses travelling in opposite directions cross each other? It turns out that wave pulses continue to retain their identities after they have crossed. **However, during the time they overlap, the wave pattern is different from either of the pulses. When the pulses overlap, the resultant displacement is the algebraic sum of the displacement due to each pulse.**
This is known as the **principle of superposition of waves.**

According to this principle,

Each pulse moves as if others are not present.

The constituents of the medium therefore suffer displacements due to both and since displacements can be positive and negative,
the net displacement is an algebraic sum of the two

So we can have two or more progressive waves propagate simultaneously in a medium, they (generally) do so without affecting the motion of one another. Therefore, the resultant displacement of each particle of the medium, at any instant, is equal to the algebraic sum of the displacement produced by the two waves separately.

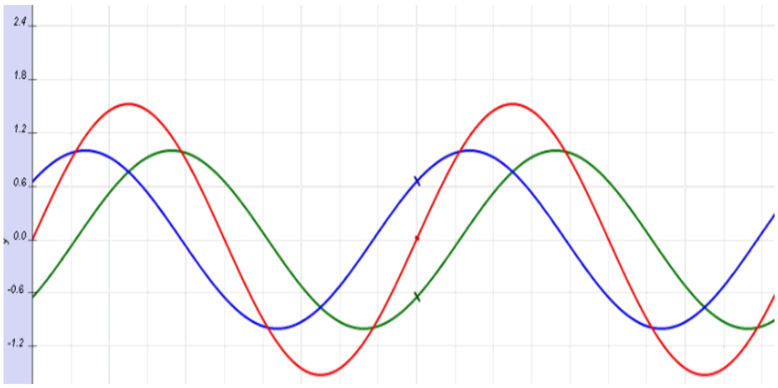
It holds for all types of waves, provided the waves are not of very large amplitude. [**If waves are of very large amplitude, (as laser waves), this principle does not hold.]**

Principle of Superposition:

According to principle of superposition:

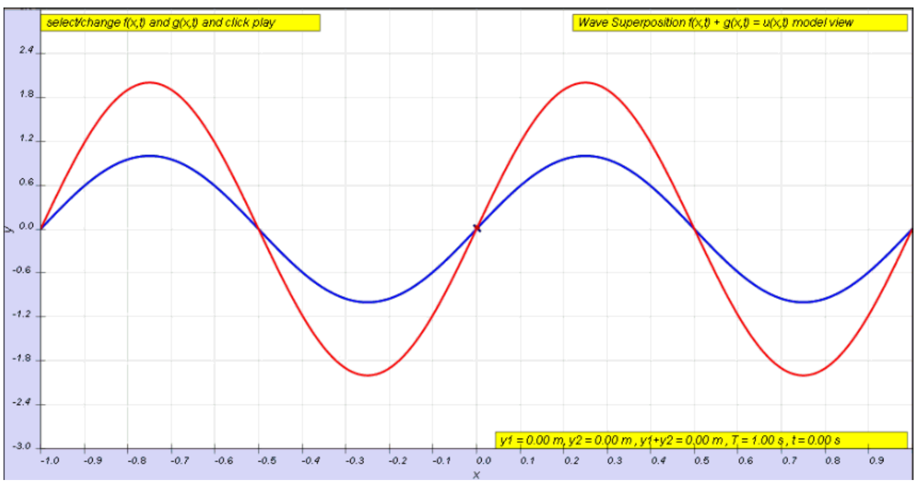
If y_1, y_2 *and* y_3 (and so on) are the displacements at a particular time of a particle at a particular position, due to the individual waves, its resultant displacement (at that time instant) will be given by

$$\boldsymbol{y = y_1 + y_2 + y_3 \ldots\ldots\ldots\ldots}$$



Source Wikipedia

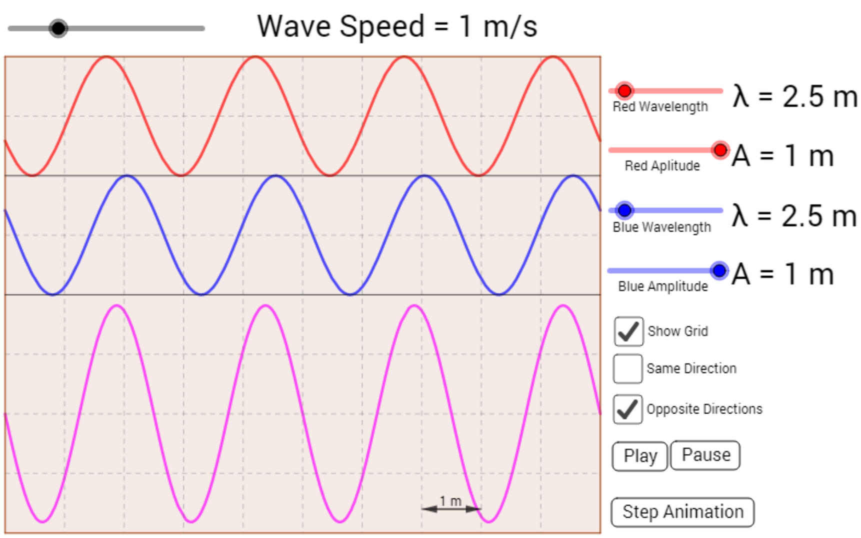
The effect of the (i) crest of one wave meeting the crest of other wave and (ii) trough of another wave would be as shown here with fig (a).



Source Wikipedia

The principle of superposition means that if a number of waves are propagating in a medium, then each one propagates independently as if the other waves were not present at all; the shape and other characteristics of any of the individual wave are not changed due to the presence of other waves. This can be seen in practice. When we listen to an orchestra, we receive a complex sound due to the superposition of sound waves of different characteristics produced by different musical instruments. Still we can recognize separately the individual sounds of different instruments. Similarly, our TV antenna receives the waves of different frequencies, transmitted simultaneously, by different TV stations. When we tune the TV to a particular channel, we receive the relay of programme from that channel only as if the other channels were silent.

Thus, **the principle of superposition holds not only for the mechanical waves but also for electromagnetic waves**.



Source Wikipedia

In the above fig., the third wave is the resultant of the two waves having its amplitude as of the sum of the two amplitude.

https://www.geogebra.org/m/szhn8Tdq
superposition wave.ggb
superposition 2.ggb

We may now conclude that

a) Wave motion preserves its form while it propagates through space without any dissipation of its energy.
b) The resultant of any two waves can be found by using the principle of superposition
c) The principle of superposition of waves is very much useful for study of the phenomena of

- Beat
- Interference of waves
- Stationary waves

11. SUMMARY:

Amplitude: The maximum displacement that an oscillating particle of the medium, undergoes on either side of its equilibrium position is called the 'amplitude'.

Time Period: The time taken by a medium particle to complete one oscillation is called the 'time period'. It is denoted by T.

Frequency: The number of oscillations made by a medium particle in a unit time (1 second) is called 'frequency'. It is denoted by f.

Phase: The phase of an oscillation particle at any instant describes the position and direction of motion of the particle at that instant.

Wavelength: The distance moved by the wave in a time of one complete oscillation of a particle of the medium is called the 'wavelength'.

Wave Speed: The distance traversed by a wave in a unit time (1 second) is called the 'wave speed'. It is denoted by v.

Displacement Equation of SHM: The displacement $\boldsymbol{y = a \sin \omega t}$. This is the displacement-equation of the simple harmonic motion for a progressive wave.

$$y = a \sin\left(\frac{2\pi}{\lambda T} t - \frac{2\pi}{\lambda} x\right)$$

Principle of Superposition: If y_1, y_2 and y_3 and so on are the displacements at a particular time at a particular position, due to the individual waves, the resultant displacement will be

$y = y_1 + y_2 + y_3$ ….………..

When two or more waves traverse the same medium, the displacement of any element of the medium is the algebraic sum of the displacements due to each wave. This is known as the ***principle of superposition*** **of waves**