1. Details of Module and its structure

Subject Name	Physics
Course Name	Physics 02 (Physics Part 2.Class XI)
Module Name/Title	Unit 10, Module 6, Simple Pendulum Chapter 14, Oscillations
Module Id	keph_201406_eContent
Pre-requisites	Simple harmonic motion, kinematics of SHM, equations of motion of an oscillator, Dynamics of a simple harmonic oscillator, period time of an oscillator, frequency of an oscillator, restoring force, spring constant, inertia factor
Objectives	After going through this module, the learners will be able to: • Understand the oscillatory motion of a simple pendulum • Derive and expression for time period of a pendulum • Relate time period and effective length of the pendulum • Know a method to determine acceleration due to gravity • Appreciate Pendulums as 'time keepers' • Study dissipation of energy of a simple pendulum by plotting a graph between square of amplitude and time
Keywords	Simple Pendulum, time period, effective length, Amplitude, acceleration due to gravity, dissipation of energy

2. Development team

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TABLE OF CONTENTS

1. UNIT SYLLABUS Unit 10

Oscillations and waves

Chapter 14 oscillations

Periodic motion, time period, frequency, displacement as a function of time , periodic functions Simple harmonic motion (S.H.M) and its equation; phase; oscillations of a loaded spring-restoring force and force constant; energy in S.H.M. Kinetic and potential energies; simple pendulum derivation of expression for its time period.

Free, forced and damped oscillations (qualitative ideas only) resonance

Chapter 15 Waves

Wave motion transverse and longitudinal waves, speed of wave motion , displacement , relation for a progressive wave, principle of superposition of waves , reflection of waves , standing waves in strings and organ pipes , fundamental mode and harmonics ,beats ,Doppler effect

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 15 MODULES

Module 1	• **Periodic motion** • **Special vocabulary** • **Time period, frequency,** • **Periodically repeating its path** • **Periodically moving back and forth about a point** • **Mechanical and non-mechanical periodic physical quantities**
Module 2	• **Simple harmonic motion** • **Ideal simple harmonic oscillator** • **Amplitude** • **Comparing periodic motions phase,** • **Phase difference** **Out of phase**

	In phase not in phase
Module 3	• Kinematics of an oscillator • Equation of motion • Using a periodic function (sine and cosine functions) • Relating periodic motion of a body revolving in a circular path of fixed radius and an Oscillator in SHM
Module 4	• Using graphs to understand kinematics of SHM • Kinetic energy and potential energy graphs of an oscillator • Understanding the relevance of mean position • Equation of the graph • Reasons why it is parabolic
Module 5	• Oscillations of a loaded spring • Reasons for oscillation • Dynamics of an oscillator • Restoring force • Spring constant • Periodic time spring factor and inertia factor
Module 6	• Simple pendulum • Oscillating pendulum • Expression for time period of a pendulum • Time period and effective length of the pendulum • Calculation of acceleration due to gravity • Factors effecting the periodic time of a pendulum • Pendulums as 'time keepers' and challenges • To study dissipation of energy of a simple pendulum by plotting a graph between square of amplitude and time
Module 7	• Using a simple pendulum plot its L-T^2graph and use it to find the effective length of a second's pendulum • To study variation of time period of a simple pendulum of a given length by taking bobs of same size but different masses and interpret the result • Using a simple pendulum plot its L-T^2graph and use it to calculate the acceleration due to gravity at a particular place
Module 8	• Free vibration natural frequency • Forced vibration • Resonance

	• To show resonance using a sonometer • To show resonance of sound in air at room temperature using a resonance tube apparatus • Examples of resonance around us
Module 9	• Energy of oscillating source, vibrating source • Propagation of energy • Waves and wave motion • Mechanical and electromagnetic waves • Transverse and longitudinal waves • Speed of waves
Module 10	• Displacement relation for a progressive wave • Wave equation • Superposition of waves
Module 11	• Properties of waves • Reflection • Reflection of mechanical wave at i)rigid and ii)non-rigid boundary • Refraction of waves • Diffraction
Module 12	• Special cases of superposition of waves • Standing waves • Nodes and antinodes • Standing waves in strings • Fundamental and overtones • Relation between fundamental mode and overtone frequencies, harmonics • To study the relation between frequency and length of a given wire under constant tension using sonometer • To study the relation between the length of a given wire and tension for constant frequency using a sonometer
Module13	• Standing waves in pipes closed at one end, • Standing waves in pipes open at both ends • Fundamental and overtones • Relation between fundamental mode and overtone frequencies • Harmonics
Module 14	• Beats • Beat frequency

	• Frequency of beat • Application of beats
Module 15	• Doppler effect • Application of Doppler effect

MODULE 6

3. WORDS YOU MUST KNOW

Let us remember the words we have been using in our study of this physics course

- Rigid body: an object for which individual particles continue to be at the same separation over a period of time
- Point object: if the position of an object changes by distances much larger than the dimensions of the body the body may be treated as a point object
- Frame of reference any reference frame the coordinates(x, y, z), which indicate the change in position of object with time
- Inertial frame is a stationary frame of reference or one moving with constant speed
- Observer someone who is observing objects
- Rest a body is said to be at rest if it does not change its position with surroundings
- Motion a body is said to be in motion if it changes its position with respect to its surroundings
- Time elapsed time interval between any two observations of an object
- Distance travelled the distance an object has moved from its starting position SI unit m , this can be zero , or positive
- Displacement the distance an object has moved from its starting position moves in a particular direction.SI unit: m, this can be zero, positive or negative

  For a vibration or oscillation, the displacement could ne mechanical, electrical magnetic. Mechanical displacement can be angular or linear.

- Acceleration- time graph : graph showing change in velocity with time , this graph can be obtained from position time graphs
- Instantaneous velocity

  Velocity at any instant of time

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- Instantaneous acceleration

  Acceleration at any instant of time

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

- kinematics study of motion without considering the cause of motion
  - **Oscillation: one complete to and fro motion about the mean position** *Oscillation* refers to any periodic motion of a body moving about the equilibrium position and repeats itself over and over for a period of time.
  - **Vibration:** It is a to and fro motion about a mean position. The periodic time is small, so we can say oscillations with small periodic time are called vibrations. The displacement from the mean position is also small.
  - **Frequency:** The number of vibrations / oscillations in unit time.
  - **Angular frequency:** a measure of the *frequency* of an object varying sinusoidally equal to 2π times the *frequency* in cycles per second and expressed in radians per second.
  - **Inertia:** *Inertia* is the tendency of an object in motion to remain in motion, or an object at rest to remain at rest unless acted upon by a force.
  - **Sinusoidal: like a sin $\boldsymbol{\theta}$ $\boldsymbol{vs\theta}$** A sine wave or sinusoid is a curve that describes a smooth periodic oscillation.
  - **Simple harmonic motion (SHM):** repetitive movement back and forth about an equilibrium (mean) position, so that the maximum displacement on one side of this position is equal to the maximum displacement on the other side. The time interval of each complete vibration is the same.
  - **Harmonic oscillator:** A *harmonic oscillator* is a *physical* system that, when displaced from equilibrium, experiences a restoring force proportional to the displacement.
  - **Mechanical energy:** is the sum of potential **energy** and kinetic **energy**. It is the **energy** associated with the motion and position of an object.
  - **Restoring force:** is a *force* exerted on a body or a system that tends to move it towards an equilibrium state.
  - **Conservative force:** is a *force* with the property that the total work done in moving a particle between two points is independent of the taken path. When an object moves from one location to another, the *force* changes the potential energy of the object by an amount that does not depend on the path taken.
  - **Bob:** A *bob* is the weight on the end of a pendulum
  - **Periodic motion: motion** repeated in equal intervals of time.
  - Simple pendulum: If a heavy point-mass is suspended by a *weightless,* inextensible and perfectly flexible string from a rigid support, then this arrangement is called a 'simple pendulum'

  - Restoring Force: No net force acts upon a vibrating particle in its equilibrium position. Hence, the particle can remain at rest in the equilibrium position. When it is displaced from its equilibrium position, then a periodic force acts upon it which is always directed towards the equilibrium position. This is called the 'restoring force'. The spring gets stretched and, due to elasticity, exerts a restoring force F on the body directed towards its original position. By Hooke's law, the force F is given by

$$F = -kx,$$

Displacement Equation of SHM:

$$y = a \sin \omega t$$

Time period: The time taken by an oscillating system to complete one oscillation,

$$T = 2\pi/\omega.$$

Frequency: The number of oscillations in one second is called the 'frequency' (n) of oscillation system.

$$n = \frac{1}{T} = \frac{\omega}{2\pi}$$

Phase: When a particle vibrates, its position and direction of motion vary with time. The general equation of displacement is

$$y = a \sin(\omega t + \phi),$$

ϕ is called the 'initial phase' we usually we have $\phi = 0$ when we are talking about the SHM of a single particle.

Velocity in SHM: v in terms of **a** and $\boldsymbol{y}$ as

$$v = \omega\sqrt{a^2 - y^2}$$

Acceleration in SHM: Acceleration of a moving particle is

$\therefore\ \alpha = -\left(\frac{v^2}{a^2}\right) y.$ Or $\alpha = -\omega^2 y.$

4. INTRODUCTION:

We have learnt that a particle can execute vibratory or oscillatory motion. This is a to and fro motion about a mean position. We learnt the kinematics and the equations of motion for simple harmonic motion and our study was limited.

We also considered why such a motion should occur at all, hence the cause of oscillatory motion or dynamics of an oscillator. A mechanical oscillation will occur in a body or system when displaced from its position of equilibrium, provided a restoring force (F = -kx) sets up in it and the displacement is not too large. The time period and frequency of an oscillating system depend both on the spring constant (k) and the mass oscillating.

These are given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\boldsymbol{\omega = \sqrt{\frac{k}{m}}\ \ or\ 2\pi f = \sqrt{\frac{k}{m}}}$$

Examples in the modules have helped us understand the basic features of oscillatory motion.

We will now study the simple pendulum

5. SIMPLE PENDULUM AND EXPRESSION FOR THE TIME PERIOD

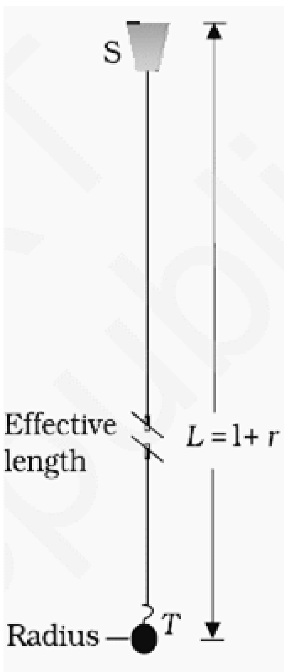
If a heavy point-mass is suspended by a *weightless,* inextensible and perfectly flexible string from a point on a rigid support, then this arrangement is called a 'simple pendulum'.

In practice, however, all these conditions are difficult to meet. Therefore, this is the definition of an ideal simple pendulum.

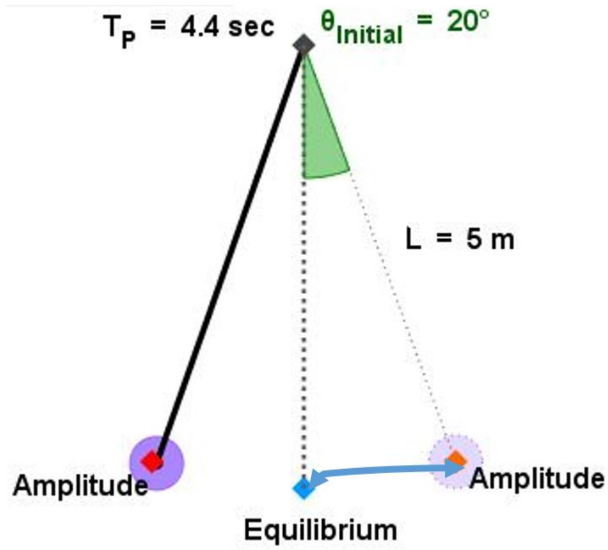
A metallic solid sphere is suspended by a inextensible cotton thread from a point rigid support.

This is the practical simple pendulum which is nearest to the ideal pendulum.

The heavy metal sphere is called the 'bob' and the distance from the **point of suspension** to the **centre of gravity (center of mass) of the bob** is called the **'effective length'** of the pendulum.



When the bob is displaced slightly to one side from its mean position and released, then it oscillates about the mean position.



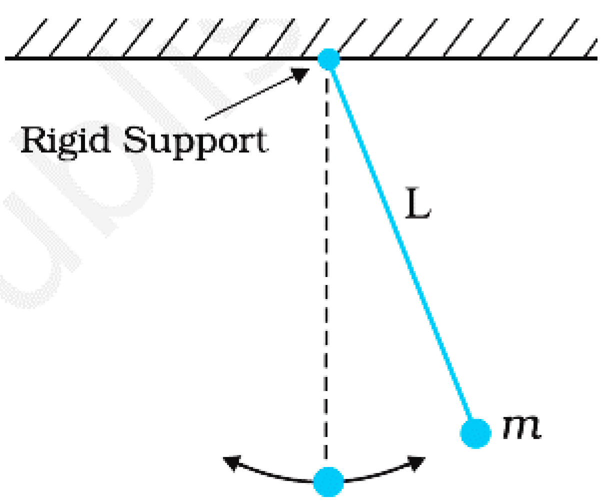
TIME - PERIOD OF SIMPLE PENDULUM:

Suppose, l is the effective length of a simple pendulum and m the mass of its bob. The bob is suspended from a fixed point

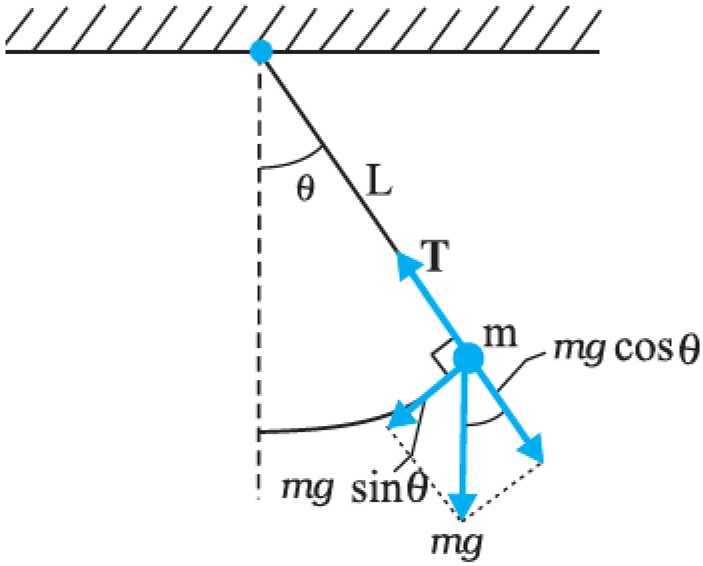
Suppose at any instant during oscillation, the bob is in a position A, when its displacement is
$OA = x$
and the thread makes an angle θ with the vertical.

In this position of the bob, two forces act upon it.

i) **Gravitational force**(mg), which acts at the centre of gravity and is directed vertically downward.



A bob oscillating about its mean position.



The radial force T-mg cos θ provides centripetal force.
The tangential force mg sin θ provides the restoring force

Thus we see the role of gravity , in turn acceleration due to gravity for providing the restoring force for the pendulum system

ii) **Tension 'T'** in the thread, which acts along the direction as shown

The force mg can be resolved into two components

- the component $\boldsymbol{mg}$ **cos** $\boldsymbol{\theta}$ in line with the thread and directed opposite to T and
- the component $\boldsymbol{mg}$ **sin** $\boldsymbol{\theta}$ perpendicular to the thread. The component mg $\cos\theta$ becomes the tension T' and the component mg $\sin\theta$ tends to bring the bob back to its mean position. This is the restoring force F (say) on the bob. Thus,

$$\mathbf{F} = -\mathbf{mg}\sin\boldsymbol{\theta}$$

The **negative sign** indicates that the force F is opposite to the displacement that is, directed towards the mean position.

If the angular displacement θ of the bob be small and measured in radian, then

$$\sin\theta = \frac{OA}{SA} = \frac{x}{l}$$

$$\left[\therefore \quad \text{angle} = \frac{\text{arc}}{\text{radius}}\right]$$

$$\therefore \quad \mathbf{F} = -\frac{\boldsymbol{mg}}{\boldsymbol{l}}\mathbf{x}$$

But, according to **Newton's law of motion,**

Force = mass × acceleration.

Therefore, if the acceleration of the bob be α, then

$$F = m\,\alpha = \frac{\boldsymbol{mg}}{\boldsymbol{l}}\,x$$

$$\therefore \quad \alpha = -\frac{g}{l}x$$

$\frac{g}{l}$ is constant for a given pendulum at a given place., l is fixed and acceleration due to gravity is constant at a place

Therefore,

$$\boldsymbol{\alpha \propto -x}$$

Thus**, the acceleration α of the bob is directly proportional to its displacement x and is directed opposite to the displacement**.

Therefore, **the motion of the bob is simple harmonic.**

Its time period is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\mathbf{F = m\,\alpha = \frac{mg}{l}\,x = kx}$$

$$\mathbf{k = \frac{mg}{l}}$$

$$T = 2\pi\sqrt{\frac{m}{mg/l}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

This is the expression for the periodic-time of a simple pendulum of length 'l' at a place where the acceleration due to gravity is 'g'

THINK ABOUT THESE

Notice that the expression

$$T = 2\pi\sqrt{\frac{l}{g}}$$

does not contain the term m.

So,

The time period of the pendulum does not depend upon the mass of the bob.

- **How would the time period change, if the pendulum was in an aircraft?**
- **How would the time period change, if the pendulum was in a moving train?**
- **How would the time period change, if the pendulum was in a warm room?**
- **How would the time period change, if the pendulum was in a spacecraft?**
- **How would the time period change, if the pendulum was displaced by small amplitude?**
- **How would the time period change, if the pendulum bob was heavier?**
- **How would the time period change, if the pendulum was made using an extensible thread/elastic strip?**

FACTORS AFFECTING THE TIME-PERIOD OF SIMPLE PENDULUM:

(i) **Dependence of the time period of a Pendulum on the Amplitude:**

It is evident from the formula for the time period of a pendulum that it does not depend upon the amplitude. But, this is true only for small amplitudes. If the angular amplitude, starting from 5^0, be increased to 10^0 , 15^0,… up to 60^0 and the time period be measured in each case then we shall see that up to 10^0………..15^0 the time period remains unchanged, but after this it increases slightly.

(ii) **Dependence of the time period of a pendulum on the Effective Length:**

It is evident from the pendulum's formula that the time period T of a pendulum is directly proportional to the square-root of its effective length $'l'$.

If the length of the pendulum be increased to four times, the periodic-time will become twice,

https://www.maxpixel.net/static/photo/1x/Balance-Child-Childrens-Day-Playground-556030.jpg

When a child sitting on a swing stands up then the centre of gravity rises up and so the **effective length of the swing decreases and the time period also correspondingly decreases**, (**if the swing is not being pushed_)**

EXAMPLE

A mother and daughter are swinging on two identical swings side by side in a park.

a) **Will the time period be the same?**
b) **Will the time period change if the daughter stands up and swings standing?**

SOLUTION

a) the time period for both will be the same because the effective length is nearly the same and the time period does not depend upon mass and the amplitude
b) Yes, because the effective length of the daughter will become lesser, making her time period lesser. They will no longer be in sync.

(iii) **Dependence of time period of a Pendulum on Acceleration due to Gravity:**

The time period of a **simple pendulum is inversely proportional to the square-root of the acceleration due to gravity g.**

THINK ABOUT THESE

- When a pendulum-clock is taken up to a **hill or down in a mine**, then due to a **decrease in the value of g. its time period increases, that is, the pendulum clock is slowed down**.

Suppose **a simple pendulum is suspended in a lift (elevator)**.

- If the lift is **going up** with accelerated motion, then for the pendulum the effective value of **g increases** and the **time-period of the pendulum decrease**.
- On the other hand, if the lift is **coming down** with **accelerated motion then the effective value of g decreases and the periodic-time of the pendulum increases**.
- If the **string of the lift breaks**, that is, the lift is falling like a free body, then the effective value of g becomes zero and so the time period of the pendulum becomes **infinite**, that is, the pendulum does not oscillate at all.
- If **the lift goes up or comes down with 'uniform' speed,** then there will be no effect on the time period.
- **If the pendulum is in a satellite** everybody inside a satellite remains in a state of weightlessness, that is, the effective value of g remains zero. Hence, a pendulum inside a satellite will not oscillate. Therefore, the pendulum-clock cannot work inside space satellites.

6. APPLICATION OF THE EXPRESSION FOR TIME PERIOD OF A PENDULUM

We understand the simple harmonic nature of oscillation of a simple pendulum.

We will now use the expression for time period

REPRESENTATION OF *L*-T^2 GRAPH

A graph shows the dependence of a physical quantity on another

Suppose we study the l -T^2 graph

So,

we experimentally obtain the periodic time for pendulums set up with known effective lengths

What is the relation between l and T^2?

If

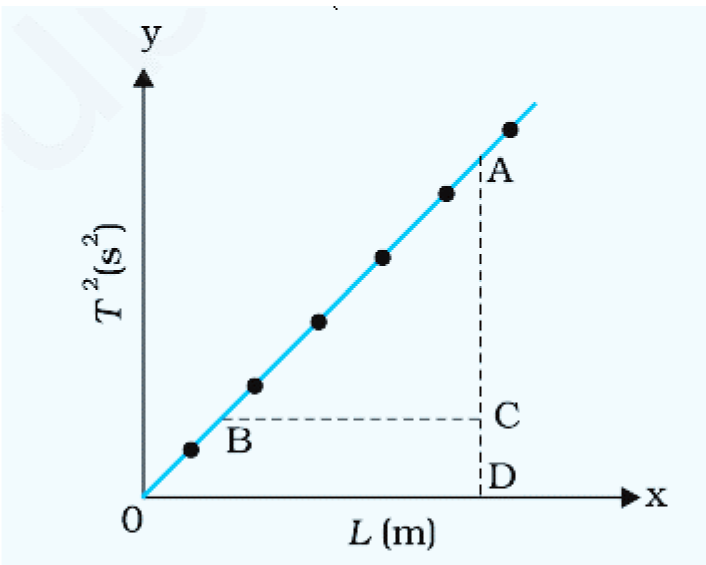
$$\mathbf{T = 2\pi\sqrt{\frac{l}{g}}}$$

Squaring both sides will give us a mathematical result

$$T^2 = \frac{4\pi^2 l}{g}$$

Taking
l along x-axis and
T ^{2} **along y-axis.**

We will find that the graph is a straight line passing through origin as shown



Value of time period for any length can be obtained directly from the graph also we can obtain the length of the pendulum if we know its time period

b) DETERMINATION OF ACCELERATION DUE TO GRAVITY 'g'

The equation of this line graph will **be** **y = mx**

$$T^2 = \frac{4\pi^2 l}{g}$$

The **slope of the l -T^2 graph** is

$$\frac{4\pi^2}{g}$$

We can calculate 'g' from the slope

$$\text{slope} = \frac{4\pi^2}{g} \text{ or } \qquad g = \frac{4\pi^2}{\text{slope}}$$

EXAMPLE

A student was asked by a teacher to find the length of a thick rope suspended from the ceiling of the room and almost reaching the floor without any thing to measure it. Suggest a suitable method

SOLUTION

The student may oscillate the rope if it is heavy enough and find its time period. This he can do by using his wrist watch and counting at least 10 oscillations. Time for one can be found.

The student could use **a l – T^2 graph or calculate**

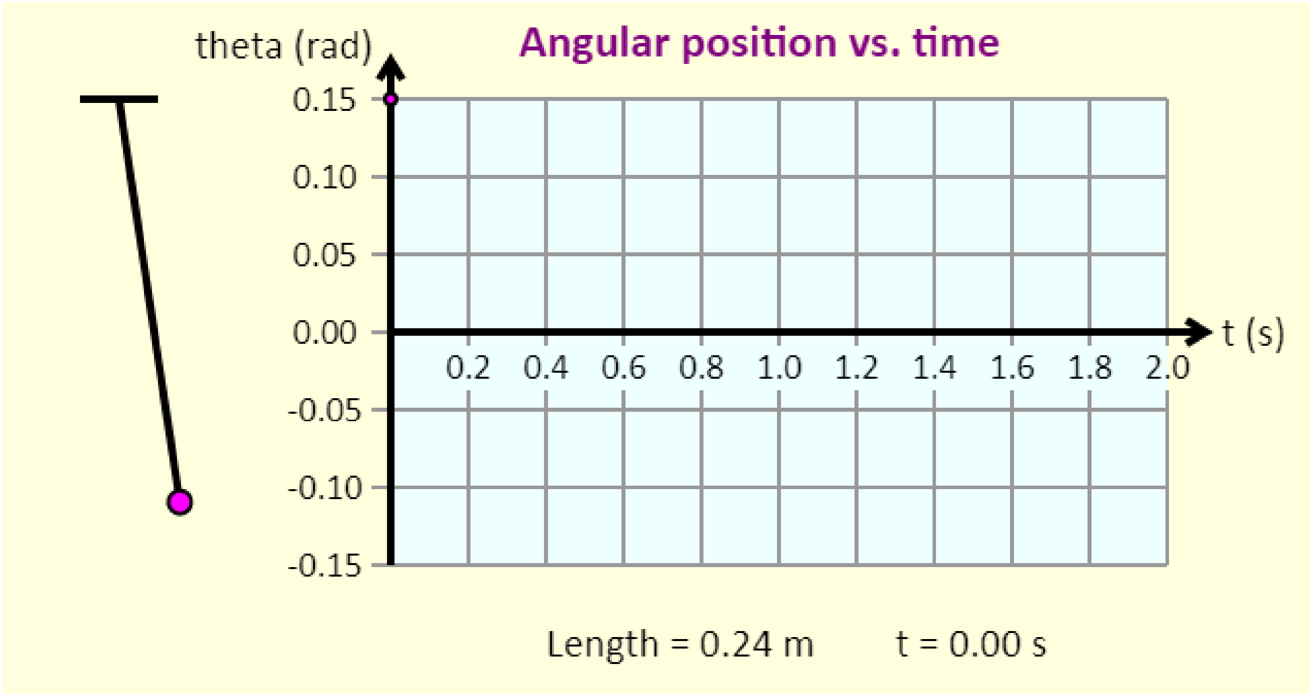
$$T^2 = \frac{4\pi^2 l}{g}$$

Though the answer will be approximate because

- the rope has weight, and
- is not a pendulum as per definition?
- the time period is an estimation it will give a fairly good estimation of length of the rope

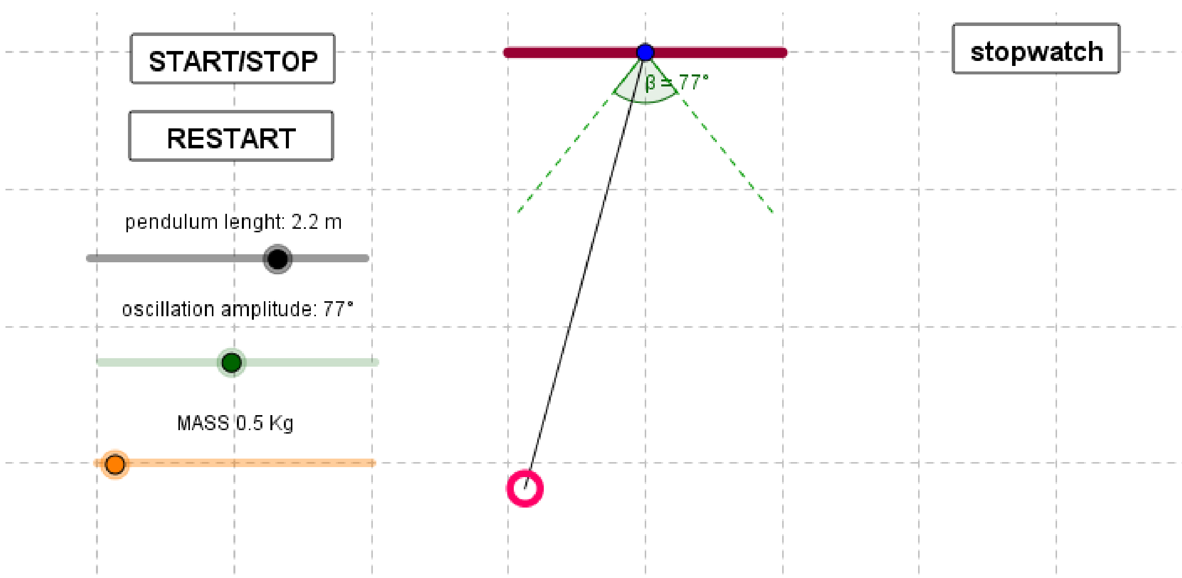
EXPERIMENTAL DEMONSTRATION OF TIME PERIOD OF A SIMPLE PENDULUM

You can study the factors effecting the time period of simple pendulum by using simulation



Source: physics.bu.edu
http://physics.bu.edu/~duffy/HTML5/pendulum.html

Watch the animation or view it in GeoGebra



https://www.geogebra.org/material/show/id/HrnNWHte#download-popup

d) DETERMINATION OF EFFECTIVE LENGTH OF A SECOND'S PENDULUM:

What is the length of a simple pendulum, with a periodic time of 2 seconds?

If the periodic-time of a pendulum is 2 seconds, then it is called a 'second's pendulum'. Hence, putting T = 2 seconds in the formula of time period T of simple pendulum, we get

$$2 = 2\,\pi\sqrt{\frac{l}{g}}$$

or

$$l = g/\pi^2$$

Hence, if we know the value of g at a place we can calculate the length of the second's pendulum.

Taking value of g = 9.8 meter/second^2 on earth's surface, length of second pendulum is

$$\mathbf{l = \frac{9.8}{(3.14)^2} = 0.99\ m \approx 1\ m}$$

Hence, length of second's pendulum on earth's surface is nearly 1 metre. On moon's surface value of g is $\frac{1}{6}^{th}$ that on the earth. Hence, the length of second pendulum on the moon ,will be nearly $\frac{1}{6}^{th}$ of 1 metre or 16.6 cm.

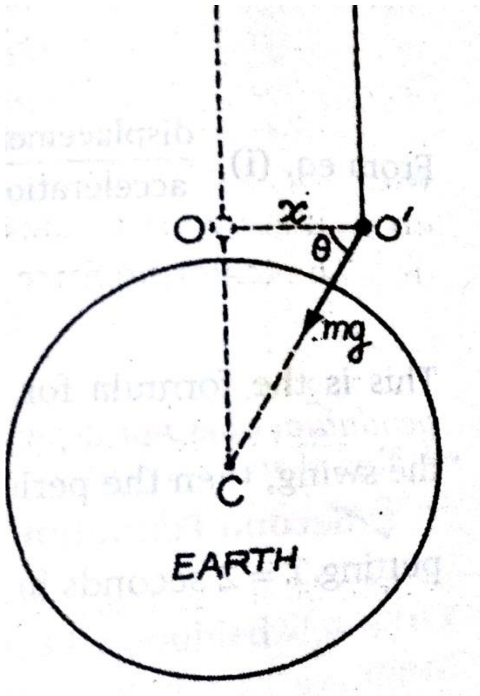
e) TIME-PERIOD OF A SIMPLE PENDULUM OF INFINITE LENGTH

From the formula

$$\mathbf{T = 2\pi\sqrt{\frac{l}{g}}}$$

for the time-period of a simple pendulum, it appears that when the length of the simple pendulum will become infinite, its time-period will also become infinite.

But it is not so, the time-period of a pendulum of infinite length is 84.6 minutes (not infinite). It is the maximum limit of the time-period of a simple pendulum.



We can prove this as follows:

When the length of the pendulum is finite, its bob will oscillate along a straight line.

Let O be the equilibrium position of the bob and O' its instantaneous position during oscillation.

Let OO' = x.

The force of gravity acting on the bob is $F = mg$, where mg is the weight of the bob acting towards the centre C of the earth.

The component of this force along the direction of motion of the bob is

$$F_x = -m\, g \cos\ \theta$$

The **negative sign** that the force F_x is directed opposite to the displacement, that is, towards the mean position O. This is the restoring force on the bob.

From the figure, we have

$$\cos\ \theta = \frac{OO'}{O'C} = \frac{x}{R_e},$$

where R_e is the distance of the bob from the centre of the earth, that is, the earth.

$$\therefore \; F_x = -mg\frac{x}{R_e}$$

The acceleration of the bob is

$$\alpha = \frac{F_x}{m} = -(g/R_e)x$$

In this equation, g/R_e is constant at a given place. Hence,

$$\alpha \propto -x$$

Thus, the acceleration α of the bob is directly proportional to its displacement x and the direction of acceleration is opposite to that of the displacement. Hence, the motion of the bob is simple harmonic, whose time-period is

$$\therefore \quad T = 2\pi\sqrt{\frac{R_e}{g}}.$$

Substituting $R_e = 6.4 \times 10^6 \; m \;\; and \; g = 9.8\, m\, s^{-2}$,

We get

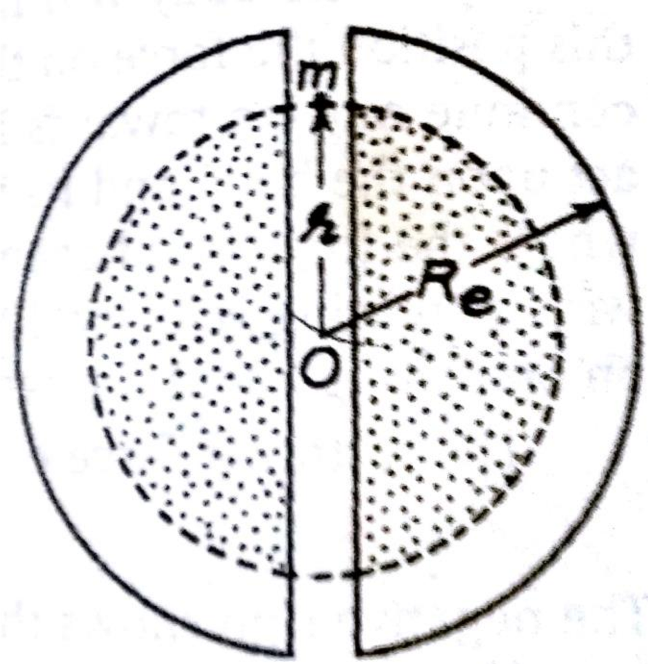
$T = 2 \times 3.14 \times \sqrt{\frac{6.4 \times 10^6}{9.8}} =$ 5075 second = **84.6 minutes.**

f) MOTION OF A BODY DROPPED IN AN IMAGINARY TUNNEL ACROSS EARTH

Let us consider an oscillation of a body through a tunnel right across the spherical earth this is hypothetical but allows us to understand how the time period of such an oscillation can be calculated.

WHEN TUNNEL IS PASSING ACROSS THE CENTRE OF THE EARTH:

Let a body of mass m be dropped at one end of the tunnel.



Gravitational force acts upon the body, whose direction is towards the centre of the earth.

- **Due to the variation in the value of 'g' the magnitude of this force is maximum at the surface of the earth**
- **As we go towards the centre of the earth it goes on decreasing and becomes zero at the centre of earth**

value of g is maximum at the surface of the earth and zero at the centre

Let R_e be the radius of the earth and ρ the density. Suppose, at any instant the body m in the tunnel is at a distance r from the centre O of the earth

Because the body is inside the earth, only the inner sphere of radius r will exert a gravitational force F upon the body. Thus

$$F = -G\,\frac{\left(\frac{4}{3}\,\pi\, r^3 \rho\right)\, m}{r^2}$$

Negative sign is taken because the force F is of attraction. Hence, the acceleration of the body will be

$$\alpha = \frac{F}{m} = -\frac{4}{3}\,\pi\, G\rho\, r = -\omega^2\, r,$$

where $\omega^2 = \left(\frac{4}{3}\pi G\rho\right)$ is a constant.
Thus, the acceleration α being proportional to the displacement r and oppositely directed, hence, the motion of the body is simple harmonic, whose time-period is

$$\mathbf{T} = \frac{\mathbf{2\pi}}{\boldsymbol{\omega}} = \sqrt{\frac{\mathbf{3\,\pi}}{\mathbf{G\,\rho}}}$$

This is the same expression that we derived earlier for the case of tunnel not passing through the centre of the earth.

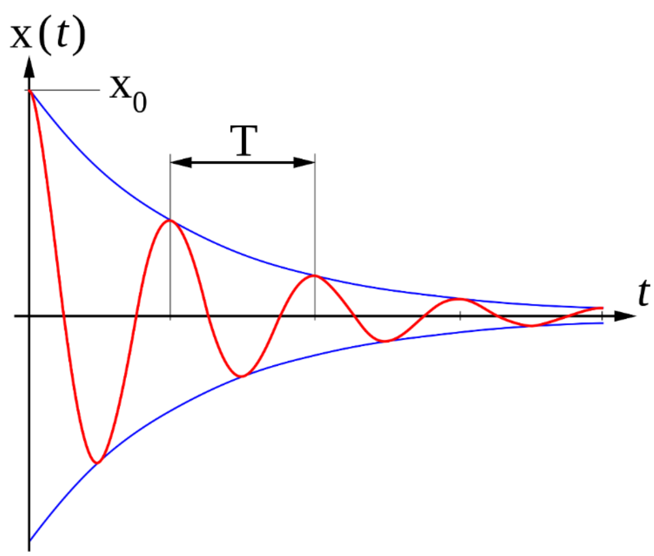
7. STUDY DISSIPATION OF ENERGY OF A SIMPLE PENDULUM BY PLOTTING A GRAPH BETWEEN SQUARE OF AMPLITUDE AND TIME

We know that **the motion of a simple pendulum, swinging in air, dies out eventually.**
Why does it happen?

This is because the air drag and the friction at the support oppose the motion of the pendulum and dissipate its energy gradually.

The pendulum is said to execute **damped oscillations**.

In damped oscillations, the energy of the system is lost continuously; but, for small damping, the oscillations remain approximately periodic.
The dissipating forces are generally the frictional forces. To understand the effect of such external forces on the motion of an oscillator,



A displacement time graph for a pendulum showing the time period remains the same but the amplitude decreases

https://upload.wikimedia.org/wikipedia/commons/thumb/d/d4/Damped_oscillation_graph2.svg/2000p x-Damped_oscillation_graph2.svg.png

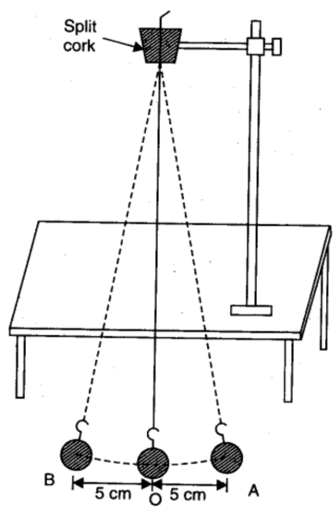
DO AN ACTIVITY

Damping of two pendulums of equal mass due to air: Set up two simple pendulums of equal length. The bob of one should be small in size say made of solid brass. The bob of the other should be of the same mass but larger in size — either of a lighter material like thermocole or a hollow rubber ball . Give them the same initial displacement and release simultaneously. Observe that in the pendulum with the larger bob the amplitude decreases more rapidly. Due to its larger area, air offers more resistance to its motion. Though both pendulums had the same energy to start with, the larger bob loses more energy in each oscillation.

The time period remains the same even when the amplitude decreases.

Or

Use a pendulum and find its amplitude at regular intervals, determine its periodic time and plot a graph



Experimental set up to study the dissipation of energy of an oscillating pendulum

The maximum energy E_{max} at maximum amplitude A_{max} is given by

$$E_{max} = \frac{1}{2} k A^2_{max}$$

$$k = \frac{mg}{l}$$

Observations you will need to take

Mass of the bob (m) = …. g x 10^{-3} = .kg

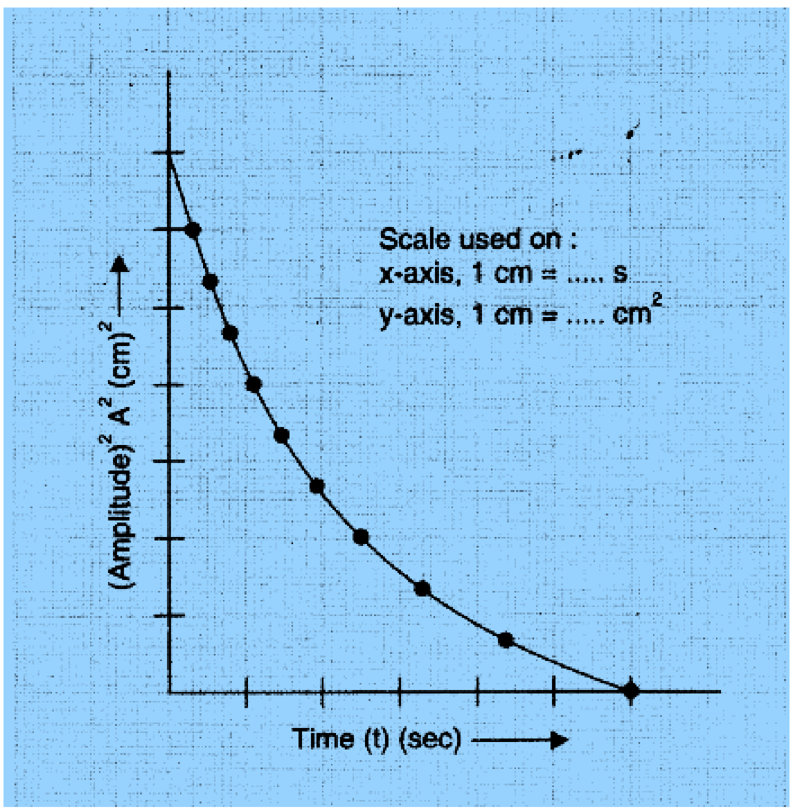
Effective length of the pendulum = length of thread + length of hook attached to the bob + radius of the bob

L = … m

Force constant $= k = \frac{mg}{l}$ =……. Nm^{-1} **take $g = 9.8\ ms^{-2}$**

sno	Amplitude (A) m	Sqaure of amplitude (A^2) m^2	Time (s) t	$\mathbf{E_t = \frac{1}{2}kA_t^2}$	Energy dissipated $E_{max} - E_t$ Joule
1					
2					
3					
4					

Take readings after 10 oscillations of time elepsed and amplitude. Plot a graph



The graph shows dissipation of energy of a pendulum with time

TRY THESE

(i) What is the length of a pendulum that has a period of 0.500 s? (*g=9.8 m/s²*)

Answer- 6.21 cm

(ii) What is the period of a 1.00m long pendulum?

Answer- 2.01 s

(iii)The pendulum on a cuckoo clock is 5.00 cm long. What is its frequency?

Answer- 2.23 Hz

(iv) What is the phase difference between the S.H.M's $y_1 = a\sin\omega t$ and $y_2 = a\cos\omega t$.

Answer- $\frac{\pi}{2}$

(v) Write down the formula for the time-period of S.H.M. of a particle in terms of its displacement and acceleration.

Answer. $T = 2\pi\sqrt{y/\alpha}$, where y is displacement and α is instantaneous acceleration

(vi)Which of the following examples represent periodic motion?
Give reasons for your answer.

a) A swimmer completing one (return) trip from one bank of a river to the other and back.
b) A freely suspended bar-magnet displaced from its N-S direction and released.
c) A hydrogen molecule rotating about its centre of mass.
d) An arrow released from a bow.

SOLUTION

a) It is a **non-periodic** motion. Although the motion of the swimmer is to and fro, but its nature is such that it cannot have a definite, period.

b) It is a **periodic**, simple harmonic motion.

c) It is a **periodic** motion.

d) It is a **non-periodic** motion, as the arrow never returns.

(vii) Which of the following examples represent S.H.M., and which represent periodic but not S.H.M.?

Give reason for your answer.

a) The rotation of the earth about its own axis.
b) Motion of an oscillating mercury column in a U-tube.
c) Motion of a ball-bearing inside a smooth curved bowl when released from a point slightly above the lowermost position.
d) General vibration of a polyatomic molecule about its equilibrium configuration.

SOLUTION

a) It is a periodic, but not simple harmonic, motion; because the earth does not have a to and fro motion about a fixed point.

b) Motion is S.H.M.

c) Motion is S.H.M.

d) A polyatomic molecule has a number of natural frequencies and hence its vibration is a superposition of S.H.M.'s of a number of different frequencies, this superposition is periodic, but not simple harmonic.

(viii) If instead of cosine function, we choose sine function to describe S.H.M.
$x = B \sin(\omega t + \alpha)$,

what are the amplitude and initial phase of the particle with the initial condition:

$\boldsymbol{x_0 = 1cm, v_0 = \pi\, cm/sec}$ **and** $\boldsymbol{\omega = \pi\, rad/sec^{-1}}$

SOLUTION

Displacement, $x = B\sin(\omega t + \alpha)$.

Velocity, $v = \frac{dx}{dt} = B\,\omega\cos(\omega t + \alpha)$

At t = 0, we have $x_0 = B\sin\alpha$

and $v_0 = B\,\omega\cos\alpha$.

From these two equations, we have

$$B^2 = x_0^2 + \frac{v_0^2}{\omega^2}$$

and
$$\tan\alpha = \frac{x_0}{v_0/\omega} = \frac{x_0\ \omega}{v_0}$$

Substituting x_0 = 1 cm, $v_0 = \pi\, cm\, s^{-1}$ and $\omega = \pi\, rad\, s^{-1}$, we have

$$B^2 = (1)^2 + \frac{\pi^2}{\pi^2} = 1 + 1 = 2$$

or
$$B = \sqrt{2}cm$$

and
$$\tan\alpha = \frac{x_0\,\omega}{v_0} = \frac{1\times\pi}{\pi} = 1$$

or
$$\alpha = \frac{\pi}{4}\ or\ \frac{5\,\pi}{4}.$$

(ix) A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released, the period of torsional oscillation is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant (torsional rigidity) of the wire.

SOLUTION

The torsional oscillations of the disc are angular simple harmonic oscillations whose time-period is given by

$$T = 2\pi\sqrt{\frac{I}{C}},$$

Where I is moment of inertia of the disc about the wire as axis and c is torsional rigidity of the wire. If M is the mass of the disc and R its radius, then

$$I = \frac{1}{2}\, M\, R^2 = \frac{1}{2} \times 10\, kg\ \times (0.15\, m)^2 = 0.1125\, kg\, m^2.$$

Now, from the above formula, we have

$$C = \frac{4\,\pi^2 I}{T^2} = \frac{4 \times (3.14)^2 \times (0.1125\ \mathrm{kg\ m^2})}{(1.5\ \mathrm{s})^2} = 1.7\ \mathrm{Nm}.$$

8. SUMMARY:

- If a heavy point-mass is suspended by a *weightless,* inextensible and perfectly flexible string from a rigid support, then this arrangement is called a 'simple pendulum'
- A metallic solid sphere is suspended by a cotton thread from a rigid support. This is the practical simple pendulum which is nearest to the ideal pendulum.
- The sphere is called the 'bob' and the distance from the point of suspension to the centre of gravity of the bob is called the 'effective length' of the pendulum.
- $T = 2\pi \sqrt{\frac{l}{g}}$, This is the formula for the time period of a simple pendulum.
- The expression for the time period of a pendulum does not contain m, so the time period of the pendulum does not depend upon the mass of the bob.
- If the time period of a pendulum is 2 seconds, then it is called a 'second's pendulum'. Putting T = 2 seconds in the formula of time period T of simple pendulum, at value of $g = 9.8\ m/s^2$ on earth's surface, length of second pendulum is $l = \frac{9.8}{(3.14)^2} = 0.99\, meter\ \approx 1\, meter.$

Reference:

Web and video link

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Books

- CONCEPT OF PHYSICS by HC VERMA
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- HANDBOOK OF PHYSICS BY NIPENDRA BHATNAGA