1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 02 (Physics Part 2,Class XI)
Module Name/Title	Unit 10, Module 4, Mechanical energy of a harmonic oscillator Chapter14,Oscillations
Module Id	keph_201404_eContent
Pre-requisites	Periodic motion, periodic sine and cosine function, simple harmonic motion, phase, equations of motion of SHM, kinetic energy, potential energy
Objectives	After going through the module the learners will be able to: • Use graphs to understand kinematics of SHM • Understand kinetic energy and potential energy graphs of an oscillator • Appreciate the relevance of the mean position in SHM • Explain mechanical energy graphs of an oscillator
Keywords	Mechanical energy of an oscillator, potential energy displacement graph, kinetic energy displacement graph, energy in mechanical simple harmonic motion

2. Development Team

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1. UNIT SYLLABUS Unit 10:

Oscillations and waves

Chapter 14: oscillations

Periodic motion, time period, frequency, displacement as a function of time , periodic functions Simple harmonic motion (S.H.M) and its equation; phase; oscillations of a loaded spring-restoring force and force constant; energy in S.H.M. Kinetic and potential energies; simple pendulum derivation of expression for its time period.

Free forced and damped oscillations (qualitative ideas only) resonance

Chapter 15: Waves

Wave motion transverse and longitudinal waves, speed of wave motion, displacement, relation for a progressive wave, principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, beats, Doppler effect.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 15 modules

Module 1	• **Periodic motion** • **Special vocabulary** • **Time period, frequency,** • **Periodically repeating its path** • **Periodically moving back and forth about a point** • **Mechanical and non-mechanical periodic physical quantities**

Module 2	• **Simple harmonic motion** • **Ideal simple harmonic oscillator** • **Amplitude** • **Comparing periodic motions phase,** • **Phase difference** **Out of phase** **In phase**
Module 3	• **Kinematics of an oscillator** • **Equation of motion** • **Using a periodic function (sine and cosine functions)** • **Relating periodic motion of a body revolving in a circular path of fixed radius and an Oscillator in SHM**
Module 4	• **Using graphs to understand kinematics of SHM** • **Kinetic energy and potential energy graphs of an oscillator** • **Understanding the relevance of mean position** • **Equation of the graph** • **Reasons why it is parabolic**
Module 5	• **Oscillations of a loaded spring** • **Reasons for oscillation** • **Dynamics of an oscillator** • **Restoring force** • **Spring constant** • **Periodic time spring factor and inertia factor**
Module 6	• **Simple pendulum** • **Oscillating pendulum** • **Expression for time period of a pendulum** • **Time period and effective length of the pendulum** • **Calculation of acceleration due to gravity** • **Factors effecting the periodic time of a pendulum** • **Pendulums as 'time keepers' and challenges** • **To study dissipation of energy of a simple pendulum by plotting a graph between square of amplitude and time**
Module 7	• **Using a simple pendulum plot its L-T^2graph and use it to find the effective length of a second's pendulum** • **To study variation of time period of a simple pendulum of a given length by taking bobs of same size but different**

	masses and interpret the result • Using a simple pendulum plot its L-T^2graph and use it to calculate the acceleration due to gravity at a particular place
Module 8	• Free vibration natural frequency • Forced vibration • Resonance • To show resonance using a sonometer • To show resonance of sound in air at room temperature using a resonance tube apparatus • Examples of resonance around us
Module 9	• Energy of oscillating source, vibrating source • Propagation of energy • Waves and wave motion • Mechanical and electromagnetic waves • Transverse and longitudinal waves • Speed of waves
Module 10	• Displacement relation for a progressive wave • Wave equation • Superposition of waves
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Module13	• **Standing waves in pipes closed at one end,** • **Standing waves in pipes open at both ends** • **Fundamental and overtones** • **Relation between fundamental mode and overtone frequencies** • **Harmonics**
Module 14	• **Beats** • **Beat frequency** • **Frequency of beat** • **Application of beats**
Module 15	• **Doppler effect** • **Application of Doppler effect**

MODULE 4

3. WORDS YOU MUST KNOW

Let us remember the words we have been using in our study of this physics course

- Rigid body: an object for which individual particles continue to be at the same separation over a period of time
- Point object: if the position of an object changes by distances much larger than the dimensions of the body the body may be treated as a point object
- Frame of reference any reference frame the coordinates(x, y, z), which indicate the change in position of object with time
- Inertial frame is a stationary frame of reference or one moving with constant speed
- Observer someone who is observing objects
- Rest a body is said to be at rest if it does not change its position with surroundings
- Motion a body is said to be in motion if it changes its position with respect to its surroundings

- Time elapsed time interval between any two observations of an object
- Motion in one dimension. when the position of an object can be shown by change in any one coordinate out of the three (x, y ,z), also called motion in a straight line
- Motion in two dimension when the position of an object can be shown by changes any two coordinate out of the three (x, y ,z), also called motion in a plane
- Motion in three dimension when the position of an object can be shown by changes in all three coordinate out of the three (x, y ,z)
- Distance travelled the distance an object has moved from its starting position SI unit m, this can be zero, or positive
- Displacement the distance an object has moved from its starting position moves in a particular direction.SI unit: m, this can be zero, positive or negative

  For a vibration or oscillation, the displacement could ne mechanical, electrical magnetic. Mechanical displacement can be angular or linear.

- Path length actual distance is called the path length
- Position time, distance time, displacement time graph these graphs are used for showing at a glance the position , distance travelled or displacement versus time elapsed
- Speed Rate of change of distance is called speed its SI unit is m/s
- Average speed = total path length divided total time taken for the change in position
- Velocity Rate of change of position in a particular direction is called velocity, it can be zero , negative and positive , its SI unit is m/s
- Velocity time graph - graph showing change in velocity with time , this graph can be obtained from position time graphs
- Acceleration Rate of change of speed in a particular direction is called velocity, it can be zero , negative and positive, its SI unit is m/s^2
- Acceleration- time graph : graph showing change in velocity with time , this graph can be obtained from position time graphs
- Instantaneous velocity

  Velocity at any instant of time

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- Instantaneous acceleration

  Acceleration at any instant of time

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

- kinematics study of motion without considering the cause of motion
  - **Oscillation: one complete to and fro motion about the mean position** *Oscillation* refers to any periodic motion of a body moving about the equilibrium position and repeats itself over and over for a period of time.
  - **Vibration:** It is a to and fro motion about a mean position. The periodic time is small. So we can say oscillations with small periodic time are called vibrations. The displacement from the mean position is also small.
  - **Inertia:** *Inertia* is the tendency of an object in motion to remain in motion, or an object at rest to remain at rest unless acted upon by a force.
  - **Frequency:** The number of vibrations / oscillations in unit time.
  - **Time Period:** Time *period* is the time needed for one complete vibration
  - **Sinusoidal: like a sin θ $vs\theta$** A sine wave or sinusoid is a curve that describes a smooth periodic oscillation.
  - **Simple harmonic motion (SHM):** repetitive movement back and forth through an equilibrium position, so that the maximum displacement on one side of this position is equal to the maximum displacement on the other side. The time interval of each complete vibration is the same.
  - **Harmonic oscillator:** A harmonic oscillator is a physical system that, when displaced from equilibrium, experiences a restoring force proportional to the displacement.
  - **Equation of motion** The equations that relate velocity at an instant of time to acceleration and displacement

$$x(t) = A\cos\omega t,$$
$$v(t) = -\omega A\sin\omega t,$$
$$a(t) = -\omega^2 A\cos\omega t$$

- **Mechanical energy:** is the sum of potential **energy** and kinetic **energy**. It is the **energy** associated with the motion and position of an object.
- **Restoring force:** is a force exerted on a body or a system that tends to move it towards an equilibrium state.
- **Conservative force:** is a force with the property that the total work done in moving a particle between two points is independent of the taken path. When an object moves from one location to another, the force changes the potential energy of the object by an amount that does not depend on the path taken.
- **Bob:** A bob is the weight on the end of a pendulum

4. INTRODUCTION

Periodic motion occurs in nature. Systems and particles, big and small, can execute periodic motion with linear or angular displacements. Simple harmonic motion is a to and fro periodic motion about an equilibrium position. This position is also called the mean position. The vibration or oscillation of a system is due to an external force causing the initial displacement. This, in turn, causes a restoring force to bring the body back to its mean position. The particle does not give up the kinetic energy as it reaches the mean motion. Due to inertia, it overshoots the mean position and continues till its velocity becomes zero. The potential energy it gains allows it to again move towards the mean position, resulting in the to and fro periodic motion. This means the particle undergoing simple harmonic motion must have elasticity, so that it can develop the restoring force (F = $-kx$, spring constant k) and mass (m) to provide the inertia.

We have learnt that for an ideal oscillator, we can use the sine and cosine functions, both show periodicity with angle.

The displacement-time relation can be derived easily. This also helped us in understanding the periodic change in velocity during one oscillation. The velocity changes continuously and instantaneous velocity can be obtained using differentiation of displacement with respect to time.

In the previous module, we also learnt that the instantaneous acceleration is always directed towards the mean position, and it was proportional to the displacement. Hence, the acceleration was maximum at the extreme position. Acceleration was also dependent on amplitude.

We will now learn how the mechanical energy of the oscillator can be represented graphically.

5. EQUATION OF MOTION

The equations of motion for simple harmonic motion are

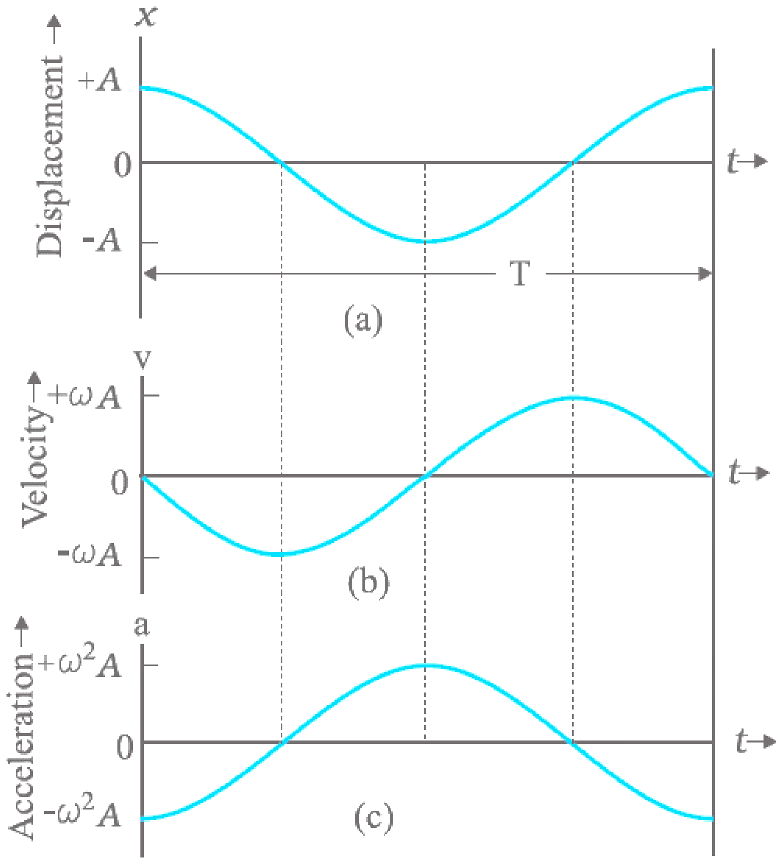
$$x(t) = A\cos\omega t,$$
$$v(t) = -\omega A\sin\omega t,$$
$$a(t) = -\omega^2 A\cos\omega t$$

Or

$$y(t) = A\cos\omega t,$$
$$v(t) = -\omega A\sin\omega t,$$
$$a(t) = -\omega^2 A\cos\omega t$$

The phase relation between displacement, velocity and acceleration was seen on plotting their graphs.

Displacement, velocity and acceleration of a particle in simple harmonic motion have the same period T, but they differ in phase.



Displacement, velocity and acceleration of a particle in simple harmonic motion have the same period T, but they differ in phase, the graphs are not drawn to the same scale. Note the phase relation only

We can say

- **All quantities (displacement, velocity, acceleration) vary sinusoidally with time;**
- **Only their maxima differ and**

- **The different plots differ in phase.**
- **x varies between $-A$ to A;**
- **$v(t)$ varies from $-\omega A$ to ωA and**
- **$a(t)$ from $-\omega^2 A$ to $\omega^2 A$.**
- **With respect to displacement plot, velocity plot has a phase difference of $\pi/2$ and Acceleration plot has a phase difference of π.**

- **In addition, we had derived a relation between velocity and displacement**

$$v = \omega\sqrt{A^2 - x^2}$$

6. SOME INTERESTING GRAPHS

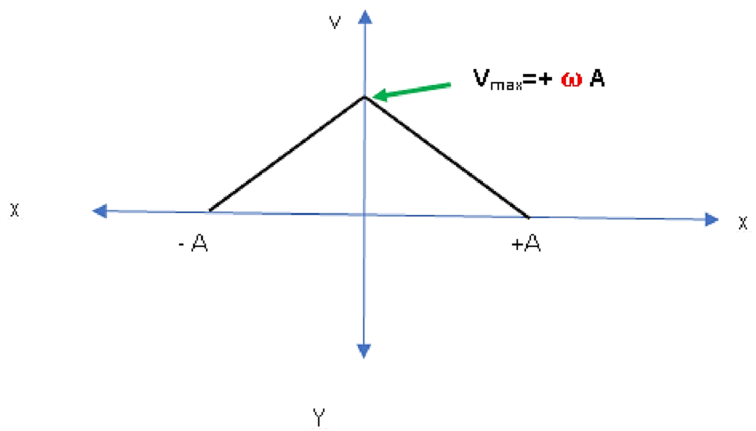
i) GRAPH of v – x

Velocity at any instant can be obtained by the slope of the tangent at that instant from a displacement-time graph

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Also,

$$v = \omega\sqrt{A^2 - x^2}$$



Consider the following v-A graph. This graph also shows that

- The velocity is maximum at the mean position
- At extreme position, when x =A, the velocity is zero

Yet this graph is not correct graph for an SHM because v does not vary as

$$v = \omega\sqrt{A^2 - x^2}.$$

The correct graph is discussed below with the help of an example

EXAMPLE

An oscillator has amplitude of 5 cm and a periodic time of 2 s.
a) Calculate its maximum speed.
b) What will be the speed at 1cm, 2 cm, 3cm and 4 cm from the mean position?
c) Plot a graph to show the velocity change with time.

SOLUTION

a) $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \text{ rad } s^{-1}$

$$v_{max} = \pm A\omega$$

$$v_{max} = 5\,\pi\, cm/s$$

b)

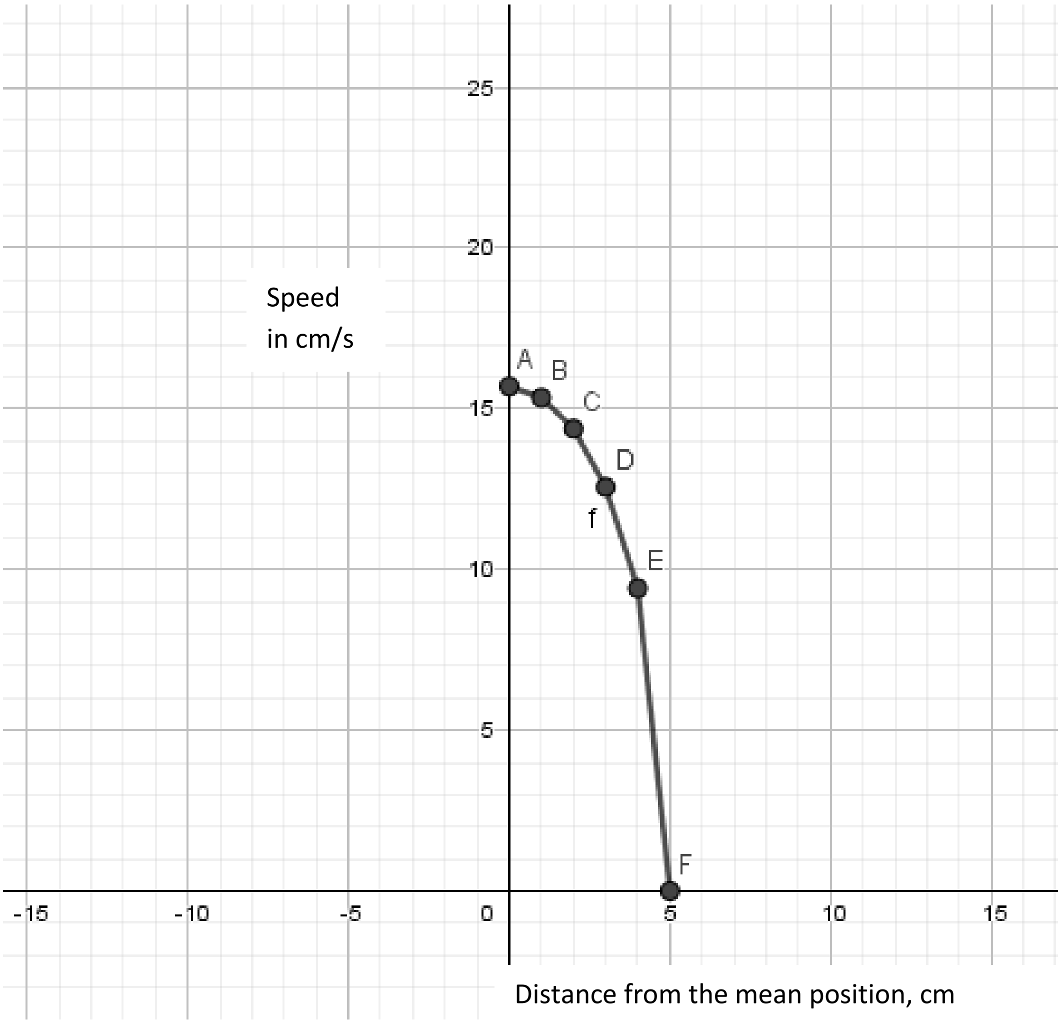
Position - x	$v = \omega\sqrt{A^2 - x^2}$	v
1 cm	$v = \pi\sqrt{5^2 - 1}$ $v = \pm\pi\sqrt{24}$	4.89 π cm/s 15.35 cm /s
2cm	$v = \pi\sqrt{5^2 - 2^2}$ $v = \pm\pi\sqrt{21}$	4.58 π cm/s 14.38 cm/s
3 cm	$v = \pi\sqrt{5^2 - 3^2}$ $v = \pm\pi\sqrt{16} = \pm 4\pi$	$4\pi cm/s$ 12.56 cm/s
4cm	$v = \pi\sqrt{5^2 - 4^2}$ $v = \pm\pi\sqrt{9} = \pm 3\pi$	$3\pi cm/s$ 9.42 cm/s
0 mean position	$V_{max} = 5\,\pi\, cm/s$	15.7 cm/s

Let us now draw v-x graph using Geo Gebra App

Use of geo gebra:

1. **Click on the geo gebra app, you need to download it, it's free**
2. **Go to view, click, choose spread sheet**
3. **Fill the data**

4. **Select data**
5. **Right click- choose- create, polyline**
6. **The graph generated may be zoomed in or out for better visibility**



Inference from the geo gebra graph

The (v –x) graph that we had drawn did not show the polyline as a section of a parabola, hence was not correct, though, we could predict the end points.

ii) GRAPH OF $v^2 - x^2$

$$v = \omega\sqrt{A^2 - x^2}$$

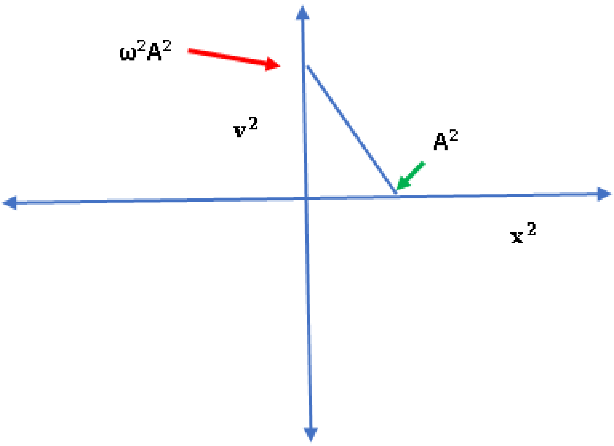
squaring both sides

$$v^2 = \omega^2(A^2 - x^2)$$

If we put $v^2 = p$ and $x^2 = q$, then this equation takes the form,

$$p = mq + c$$

Which is the equation of a straight line in p and q coordinates.



Can we extend the line beyond $x^2 = A^2$?

Give a reason for your answer.

Interpretation of the graph

Consider

$$v^2 = \omega^2(A^2 - x^2)$$

Or $$p = mq + c$$

For $x^2 = A^2$, $v^2 = 0$

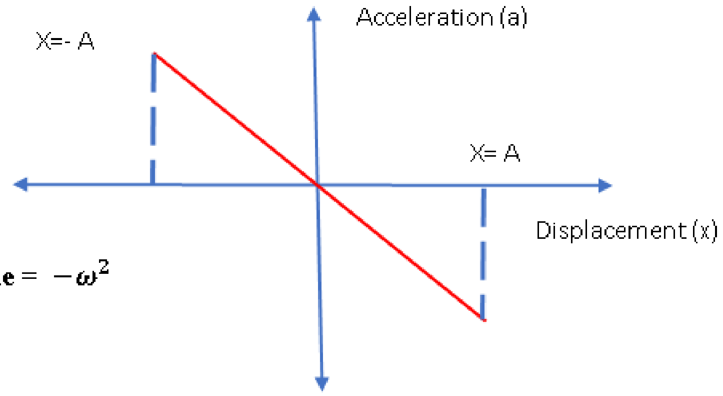
For x = 0, $v^2 = \omega^2 A^2$

This graph is correct as it can be matched with the equation of a straight line.

We cannot extend the graph beyond $x^2 = A^2$ as it would not have any physical meaning x can have a maximum value of A, the amplitude of the oscillator.

iii) GRAPH OF acceleration (a) - displacement x

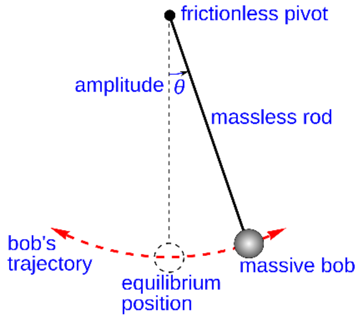
Acceleration = $-\omega^2 x$



7. ENERGY TRANSFORMATION IN A PENDULUM

The system executing to and fro motion possess mechanical energy
The fact is that the object starts oscillation once an external agency gives energy to it.

Take the example of a pendulum



The bob slows down and stops at the extreme position. It then, retraces its path to reach the extreme position on the other side of the mean position.

Notice the bob must have kinetic energy when it is moving, as it slows down this energy must be converting to potential energy and then at the extreme position, where it momentarily stops it must have potential energy only.

The situation is somewhat similar when a ball is thrown up vertically, its kinetic energy reduces as it goes up and coverts totally to potential energy when it stops at the highest point.

The ball goes higher if we throw it up with greater energy. This is the reason that the pendulum will have greater amplitude if more energy is initially given to it by taking the bob to a larger distance before letting it go.

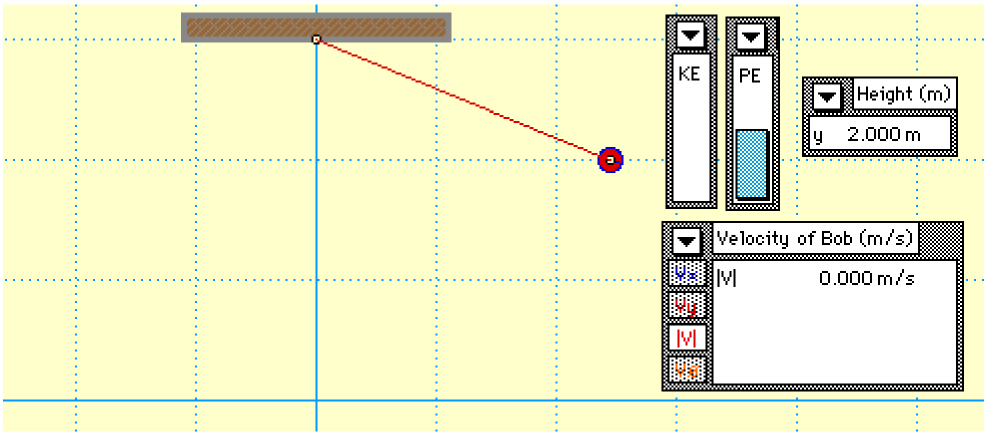
THINK ABOUT THESE

- **Do you think the same is true for any oscillating particle? Does the mechanical energy oscillate between kinetic and potential?**

- **Do you think the same would also be true for non-mechanical displacement say electric or magnetic, where the energy may be electrical and magnetic?**

The motion of a pendulum is a **classic example of mechanical energy conservation**. A pendulum consists of a mass (known as a *bob*) attached by a string to a pivot point. As the pendulum moves, it sweeps out a circular arc, moving back and forth in a periodic fashion. **Neglecting air resistance**, there are only two forces acting upon the pendulum bob. One force is gravity. The **force of gravity** acts in a downward direction and does work upon the pendulum bob, when it moves up.

However, **gravity is conservative force** and thus does not serve to change the total amount of mechanical energy of the bob as its motion is cyclic. The other force acting upon the bob is the **force of tension**. Tension is an external force and if it did do work upon the pendulum bob it would indeed serve to change the total mechanical energy of the bob. However, the force of tension does not do work since it always acts in a direction perpendicular to the motion of the bob. At all points in the trajectory of the pendulum bob, the angle between the force of tension and its direction of motion is 90 degrees. Thus, the force of tension does not do work upon the bob.
Check out the animation



http://www.physicsclassroom.com/mmedia/energy/pe.gif

Observe that the falling motion of the bob is accompanied by an increase in speed. As the bob loses height and PE, it gains speed and KE; yet the total of the two forms of mechanical energy is conserved
Since there are no forces doing work, the total mechanical energy of the pendulum bob is conserved. The conservation of mechanical energy is demonstrated in the animation. (Observe the KE and PE bars of the bar chart; their sum is a constant value in the animation)

Both kinetic and potential energies of a particle in SHM vary between zero and their maximum values, assuming that potential energy of the bob in its mean position is zero.

We have seen that the velocity of a particle executing SHM is a periodic function of time. It is zero at the extreme positions of displacement.

Therefore, **the kinetic energy (*K*) of such a particle, which is defined as**

$$\mathrm{KE} = \frac{1}{2}\mathrm{mv}^2$$

$$= \frac{\mathbf{1}}{\mathbf{2}}\mathbf{m\omega^2 A^2 sin^2(\omega t)}s$$

is also a periodic function of time, being zero when the displacement is maximum and maximum when the particle is at the mean position?

Note, since the sign of v is immaterial in ***K*E, the period of *KE* is *T*/2.**

What is the value of potential energy (*U*) of a particle executing simple harmonic motion?

We have seen that the concept of potential energy is possible only for conservative forces.

The spring force **F = – k x** is a conservative force, with associated potential energy

8. POTENTIAL ENERGY OF A SPRING

The spring force is an example of a variable force which is conservative.

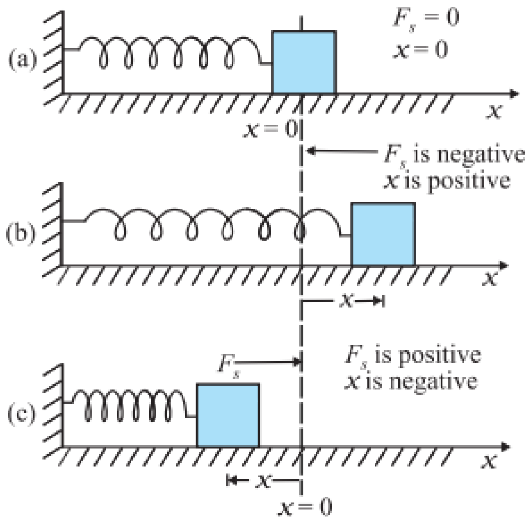


Figure shows a block attached to a spring and resting on a smooth horizontal surface. The other end of the spring is attached to a rigid wall. The spring is light and may be treated as massless.

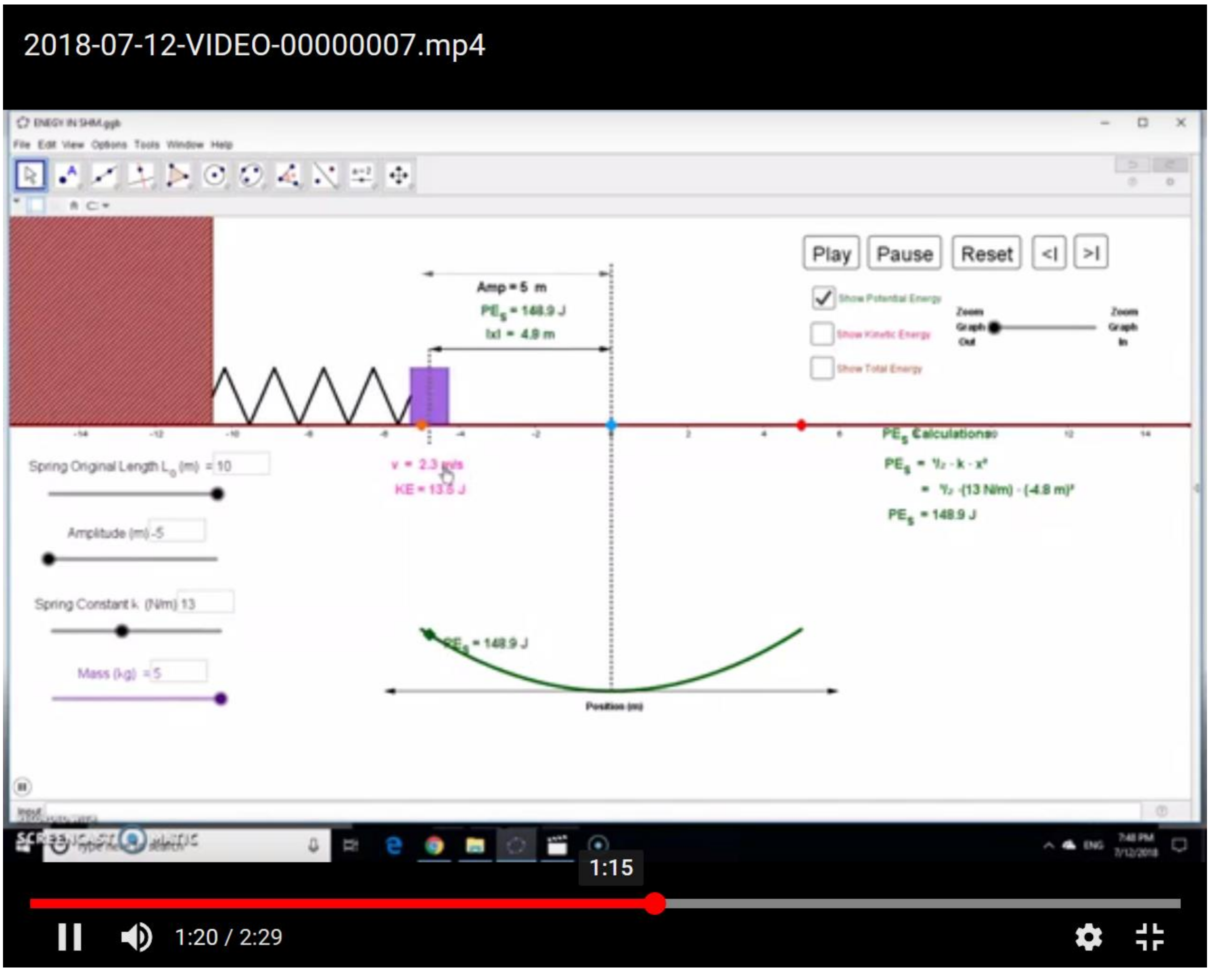
In an **ideal spring**, the spring force F_s is proportional to x, where x is the displacement of the block from the equilibrium position. The displacement could be either positive or negative.

This force law for the spring is called **Hooke's law** and is mathematically stated as

$F_s = -kx$

The constant k is called the **spring constant**.

Its unit is N m^{-1}.



Watch the animation, see how different parameters change the value of energy of the oscillating spring block system

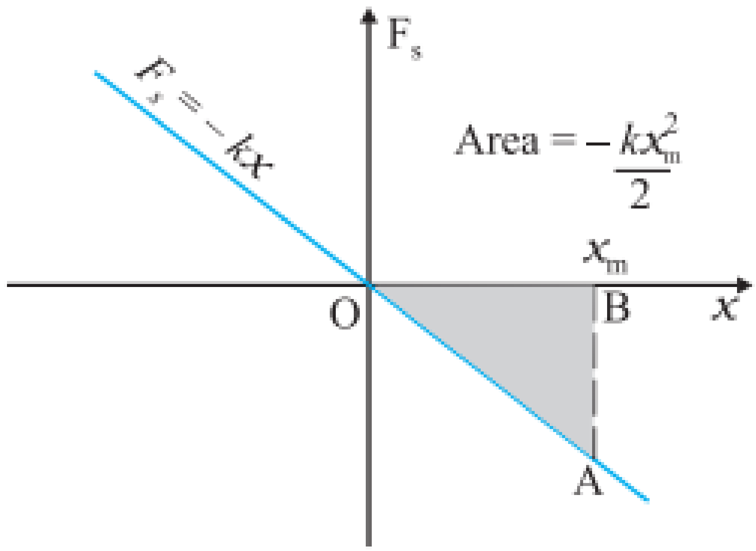
Suppose that we pull the block outwards. If the extension is x_m,

the work done by the spring force is = Average force x displacement (when the displacement is small)

average force $= \frac{0 + kx_m}{2}$

displacement = x_m

work done = $\frac{1}{2}k{x_m}^2$



Now, if such a spring is stretched and released it will execute simple harmonic motion because the force is proportional to the displacement and is in direction opposite to it. Since the ideal simple harmonic oscillator has spring constant k, its

$$PE = \frac{1}{2}kx^2$$

Hence, the potential energy of a particle executing simple harmonic motion is,

$$\mathbf{PE = \frac{1}{2}kA^2\cos^2(\omega t)}$$

Thus,

The potential energy of a particle executing simple harmonic motion is also periodic, with period *T*/2, being zero at the mean position and maximum at the extreme displacements.

This is because the spring force is conservative.

9. TOTAL ENERGY OF THE SYSTEM

The total energy, ***E***, of the system is,

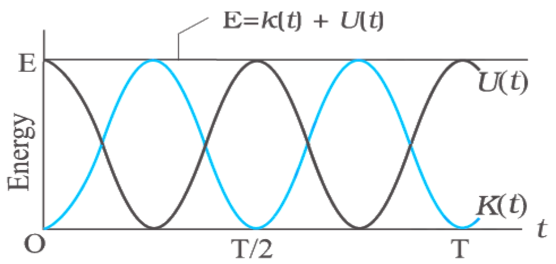
E =KE + PE

$$E = \frac{1}{2}m\omega^2A^2sin^2(\omega t) + \frac{1}{2}kA^2cos^2(\omega t)$$
$$= \frac{1}{2}kA^2\left(sin^2(\omega t) + cos^2(\omega t)\right)$$
$$= \frac{1}{2}\mathbf{kA^2} \qquad \because\ [sin^2(\omega t) + cos^2(\omega t)1]$$

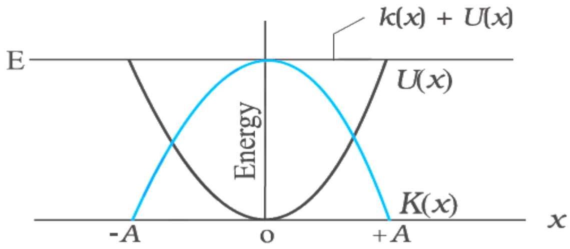
Thus,

The **total mechanical energy of a harmonic oscillator is thus independent of time** as expected for motion under any conservative force.

The time and displacement dependence of the potential and kinetic energies of a linear simple harmonic oscillator are shown in



(a)



(b)

- Kinetic energy, potential energy and total energy as a function of time [shown in (a)]
- Kinetic energy, potential energy and total energy as a function of displacement [shown in (b)] of a particle in SHM.
- The kinetic energy and potential energy both repeat after a period T/2.
- The total energy remains constant at all t or x.

Observe that

- Both kinetic energy and potential energy in SHM are seen to be always positive
- Both kinetic energy and potential energy peak twice during each period of SHM.
- For $x = 0$, the energy is only kinetic;
- At the extremes $x = \pm A$, it is all potential energy.
- In the course of motion between these limits, kinetic energy increases at the expense of potential energy and vice-versa.

Equation of the KE curve

$$\text{KE}=\frac{1}{2}\text{m}\omega^2\text{A}^2\sin^2(\omega\text{t})$$

$$\text{KE}=\frac{1}{2}\text{m}\omega^2[\text{A}^2-\text{A}^2\cos^2(\omega\text{t})]$$

$$\text{KE}=\frac{1}{2}m\omega^2[A^2-x^2]$$

KE - x curve is a parabola

KE max = $\frac{1}{2}m\omega^2A^2$ when x = 0

KE min = 0 when x = A

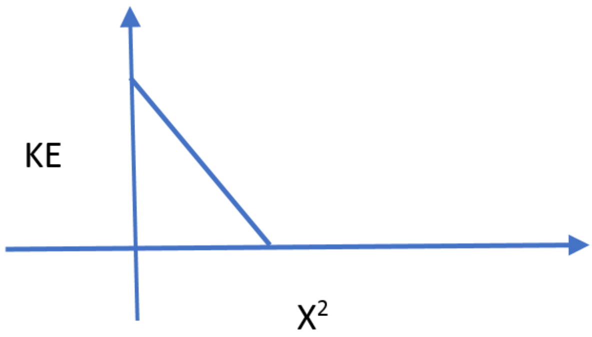
What is the shape of KE - x^2

KE = C ($\frac{1}{2}m\omega^2A^2$) $-x^2$

Y= -mx + C

A straight line with a negative slope intercept on y axis = $\frac{1}{2}m\omega^2A^2$

Intercept on x-axis = $\frac{1}{2}\omega^2A^2$



The values of slope and intercepts depend on mass, amplitude and frequency

10. REDEFINING MEAN POSITION IN TERMS OF ENERGY

- In an oscillator executing ideal simple harmonic motion, the oscillator moves to and fro about a point of **minimum potential energy**, **maximum velocity**, hence **maximum kinetic energy**.
- The amplitude on either side of mean position is equal.
- If the oscillator energy increases, the amplitude increases but the mean position remains the same.

11. DEFINING SIMPLE HARMONIC MOTION IN TERMS OF MECHANICAL ENERGY

A to and fro motion about a mean position, where the acceleration is directed towards the mean position. The mechanical energy oscillates with half the periodic time between potential energy and kinetic energy, maximizing at extreme position and mean positions respectively.

EXAMPLE

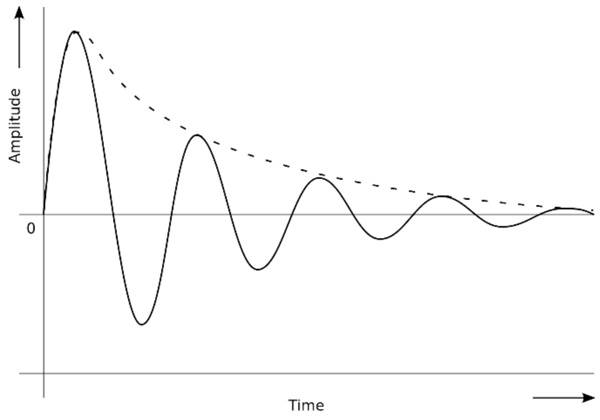
a) Show the variation of amplitude with time for an oscillator dissipating energy

b) Show energy time graph for an oscillator where energy is conserved

b) Where there is gradual loss in total energy?

SOLUTION

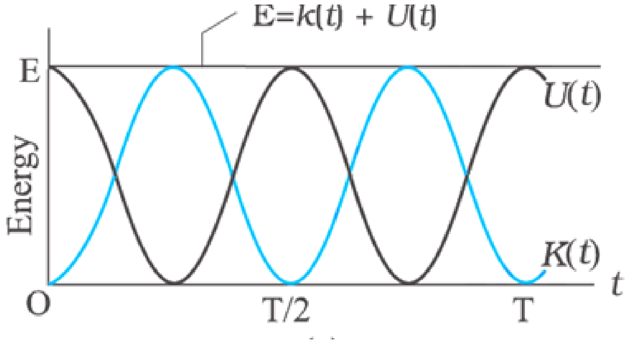
a)



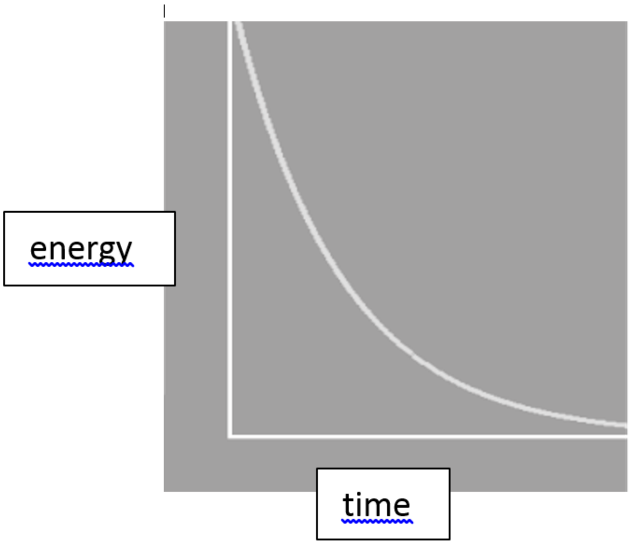
https://upload.wikimedia.org/wikipedia/commons/thumb/a/a2/Damped_sinewave.svg/2000px-Damped_sinewave.svg.png

b) Kinetic energy

KE max = $\frac{1}{2}m\omega^2A^2$ **is proportional to** v^2 **or** ω^2A^2



c) No system is perfect. Energy of the oscillator gradually dissipates.



The energy will diminish with time

TRY THESE

(i) Show graphically without any description, the variation of potential energy of a particle in SHM, with respect to its displacement from its mean position.

(ii) The amplitude of an oscillator is doubled, how will it affect

(a) The periodic time,

(b) Total energy,

(c) Maximum velocity

(iii)At what points, the energy of an oscillator will be entirely potential energy?

(iv) At what distance from the mean position is the kinetic energy in a simple harmonic oscillator equal to the potential energy?

Hint: (iv) KE= $\frac{1}{2}m\omega^2[A^2 - x^2] = PE = \frac{1}{2}m\omega^2x^2$

$$2x^2 = A^2$$

$$x = \frac{A}{\sqrt{2}}$$

12. SUMMARY

- **A particle will execute simple harmonic motion only if conditions for a restoring force can set up in it**
- **A particle will execute simple harmonic motion if it possess inertia**
- **The mechanical energy is conserved if the forces are conservative**
- **Potential energy of an oscillator is given by**

$$PE = \frac{1}{2}kx^2$$

Or

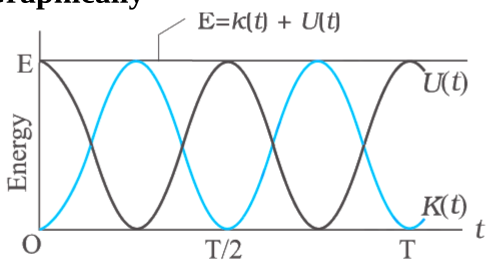
$$PE = \frac{1}{2}kA^2cos^2(\omega t)$$

- **Kinetic energy of an oscillator is given by**

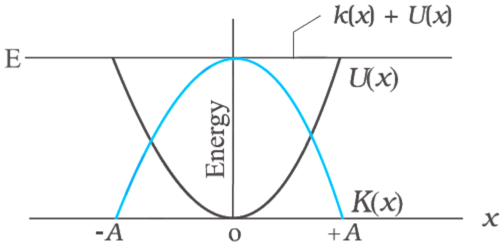
$$KE = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\omega^2A^2sin^2(\omega t)$$

- **Graphically**



(a)



(b)