1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 02 (Physics Part 2 ,Class XI)
Module Name/Title	Unit 10, Module 3, Kinematics of an oscillator Chapter14, Oscillations
Module Id	keph_201403_eContent
Pre-requisites	Periodic motion, periodic sine and cosine function, simple harmonic motion, phase
Objectives	After going through this module the learners will be able to: • Interpret Kinematics of an oscillator • Explain the use of a periodic function for equations of motion (sine and cosine functions) • Deduce Equations of motion for a harmonic oscillator • Derive a relation between displacement, velocity and acceleration for an oscillator in SHM • Redefine SHM in terms of acceleration of the particle directed towards the mean position
Keywords	Kinematics of a simple harmonic oscillator, equations of motion of an oscillator

2. Development Team

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1. UNIT SYLLABUS

Unit 10: Oscillations and waves

Chapter 14: oscillations

Periodic motion, time period, frequency, displacement as a function of time, periodic functions Simple harmonic motion (S.H.M) and its equation; phase; oscillations of a loaded spring- restoring force and force constant; energy in S.H.M. Kinetic and potential energies; simple pendulum derivation of expression for its time period.

Free forced and damped oscillations (qualitative ideas only) resonance

Chapter 15: Waves

Wave motion transverse and longitudinal waves, speed of wave motion, displacement, relation for a progressive wave, principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, beats, Doppler effect

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 15 MODULES

Module 1	• **Periodic motion** • **Special vocabulary** • **Time period, frequency,** • **Periodically repeating its path** • **Periodically moving back and forth about a point** • **Mechanical and non-mechanical periodic physical quantities**
Module 2	

	• Simple harmonic motion • Ideal simple harmonic oscillator • Amplitude • Comparing periodic motions phase, Phase difference Out of phase In phase
Module 3	• Kinematics of an oscillator • Equation of motion • Using a periodic function (sine and cosine functions) • Relating periodic motion of a body revolving in a circular path of fixed radius and an Oscillator in SHM
Module 4	• Using graphs to understand kinematics of SHM • Kinetic energy and potential energy graphs of an oscillator • Understanding the relevance of mean position • Equation of the graph • Reasons why it is parabolic
Module 5	• Oscillations of a loaded spring • Reasons for oscillation • Dynamics of an oscillator • Restoring force • Spring constant • Periodic time spring factor and inertia factor
Module 6	• Simple pendulum • Oscillating pendulum • Expression for time period of a pendulum • Time period and effective length of the pendulum • Calculation of acceleration due to gravity • Factors effecting the periodic time of a pendulum • Pendulums as 'time keepers' and challenges • To study dissipation of energy of a simple pendulum by plotting a graph between square of amplitude and periodic time
Module 7	• Using a simple pendulum plot its L-T^2graph and use it to

	find the effective length of a second's pendulum • To study variation of time period of a simple pendulum of a given length by taking bobs of same size but different masses and interpret the result • Using a simple pendulum plot its L-T^2graph and use it to calculate the acceleration due to gravity at a particular place
Module 8	• Free vibration natural frequency • Forced vibration • Resonance • To show resonance using a sonometer • To show resonance of sound in air at room temperature using a resonance tube apparatus • Examples of resonance around us
Module 9	• Energy of oscillating source, vibrating source • Propagation of energy • Waves and wave motion • Mechanical and electromagnetic waves • Transverse and longitudinal waves • Speed of waves
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Module 11	• Properties of waves • Reflection • Reflection of mechanical wave at i)rigid and ii)non-rigid boundary • Refraction of waves • Diffraction
Module 12	• Special cases of superposition of waves • Standing waves • Nodes and antinodes • Standing waves in strings • Fundamental and overtones

	• **Relation between fundamental mode and overtone frequencies, harmonics** • **To study the relation between frequency and length of a given wire under constant tension using sonometer** • **To study the relation between the length of a given wire and tension for constant frequency using a sonometer**
Module13	• **Standing waves in pipes closed at one end,** • **Standing waves in pipes open at both ends** • **Fundamental and overtones** • **Relation between fundamental mode and overtone frequencies** • **Harmonics**
Module 14	• **Beats** • **Beat frequency** • **Frequency of beat** • **Application of beats**
Module 15	• **Doppler effect** • **Application of Doppler effect**

MODULE 3

3. WORDS YOU MUST KNOW

Let us remember the words we have been using in our study of this physics course

- Rigid body: an object for which individual particles continue to be at the same separation over a period of time
- Point object: if the position of an object changes by distances much larger than the dimensions of the body the body may be treated as a point object
- Frame of reference any reference frame the coordinates(x, y, z), which indicate the change in position of object with time
- Inertial frame is a stationary frame of reference or one moving with constant speed
- Observer someone who is observing objects
- Rest a body is said to be at rest if it does not change its position with surroundings
- Motion a body is said to be in motion if it changes its position with respect to its surroundings

- Time elapsed time interval between any two observations of an object

- Motion in one dimension: when the position of an object can be shown by change in any one coordinate out of the three (x, y ,z), also called motion in a straight line
- Motion in two dimension: when the position of an object can be shown by changes any two coordinate out of the three (x, y ,z), also called motion in a plane
- Motion in three dimension: when the position of an object can be shown by changes in all three coordinate out of the three (x, y ,z)
- Distance travelled: the distance an object has moved from its starting position SI unit m , this can be zero, or positive
- Displacement: the distance an object has moved from its starting position moves in a particular direction.SI unit: m, this can be zero, positive or negative

  For a vibration or oscillation, the displacement could ne mechanical, electrical magnetic. Mechanical displacement can be angular or linear.
- Path length: actual distance is called the path length
- Position time, distance time, displacement time graph these graphs are used for showing at a glance the position , distance travelled or displacement versus time elapsed
- Speed: Rate of change of distance is called speed its SI unit is m/s
- Average speed = total path length divided total time taken for the change in position
- Velocity: Rate of change of position in a particular direction is called velocity, it can be zero , negative and positive, its SI unit is m/s
- Velocity time graph: graph showing change in velocity with time , this graph can be obtained from position time graphs
- Acceleration: Rate of change of speed in a particular direction is called velocity, it can be zero, negative and positive, its SI unit is m/s^2
- Acceleration- time graph : graph showing change in velocity with time , this graph can be obtained from position time graphs
- Instantaneous velocity

  Velocity at any instant of time

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- Instantaneous acceleration

  Acceleration at any instant of time

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

- kinematics: study of motion without considering the cause of motion

- **Oscillation: one complete to and fro motion about the mean position**
  Oscillation refers to any periodic motion of a body moving about the equilibrium position and repeats itself over and over for a period of time.
- **Vibration:** It is a to and fro motion about a mean position. The periodic time is small. So we can say oscillations with small periodic time are called vibrations. The displacement from the mean position is also small.
- **Inertia:** *Inertia* is the tendency of an object in motion to remain in motion, or an object at rest to remain at rest unless acted upon by a force.
- **Frequency:** The number of vibrations / oscillations in unit time.
- **Time Period:** Time *period* is the time needed for one complete vibration
- **Sinusoidal: like a sin $\boldsymbol{\theta}$ $\boldsymbol{vs\theta}$** A sine wave or sinusoid is a curve that describes a smooth periodic oscillation.
- **Amplitude:** Amplitude of the vibrating body and wave is the maximum displacement from the mean position.
- **Simple harmonic motion (SHM):** repetitive movement back and forth through an equilibrium position, so that the maximum displacement on one side of this position is equal to the maximum displacement on the other side. The time interval of each complete vibration is the same.
- **Harmonic oscillator:** A harmonic oscillator is a physical system that, when displaced from equilibrium, experiences a restoring force proportional to the displacement.
- **Equation of motion:** The equations that relate velocity at an instant of time to acceleration and displacement

4. INTRODUCTION

We have considered **motion in a straight line, motion in a plane, motion in a circle and motion about a fixed axis or rotation.**
For each, we have established equations of motion.

The equations that relate velocity at an instant of time to acceleration and displacement.
These equations were

$$\boldsymbol{v(t) = v(\,0) + a(t - 0)}$$

$$\boldsymbol{x = x(0) + v(0)t + \frac{1}{2}at^2}$$

$$\boldsymbol{v(t)^2 - v(0)^2 = 2a[x(t) - x(0)]}$$

The **advantage of equations of motion,** is that they make it easy to determine any of the unknown quantities (x, v, t, a) if others are known.

You have done many problems applying them.

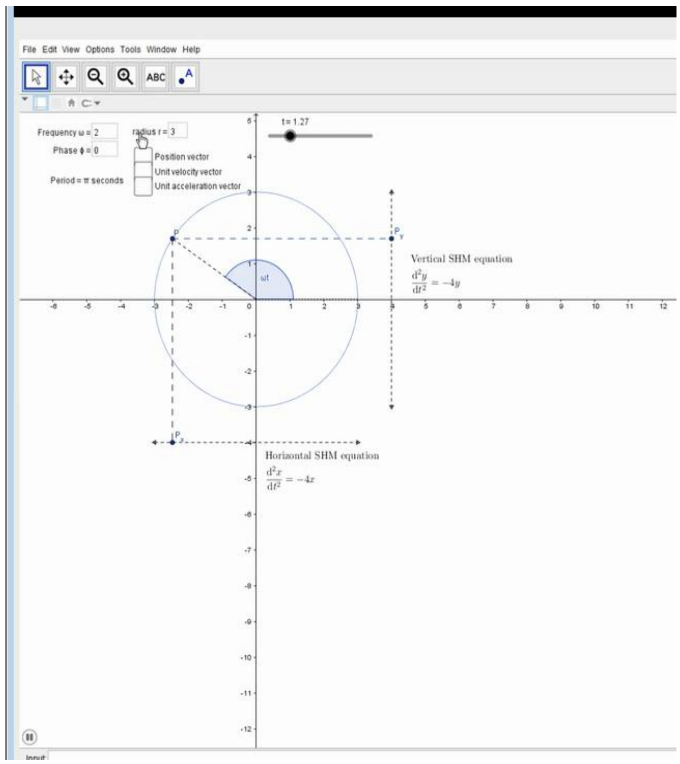
You would also recall that we derived the equations on the basis of the particle moving in a straight line, with total disregard to the cause of the motion.

In this module, we will do the same for simple harmonic motion.

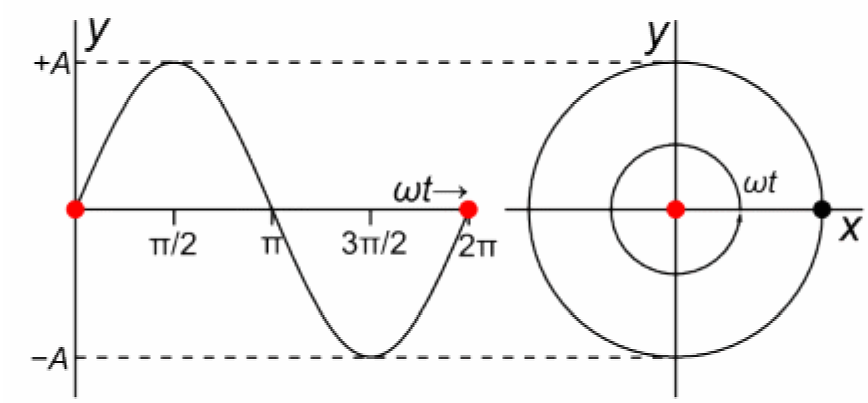
5. A PARTICLE EXECUTING SIMPLE HARMONIC MOTION WITH A CERTAIN PERIODIC TIME AND A PARTICLE MOVING IN A CIRCLE WITH THE SAME PERIODIC TIME

Is it possible to relate the two motions?
One is an SHM and the other is a circular motion?

But the condition is that they both take the same time to complete one oscillation or one rotation.



Observe the animation carefully



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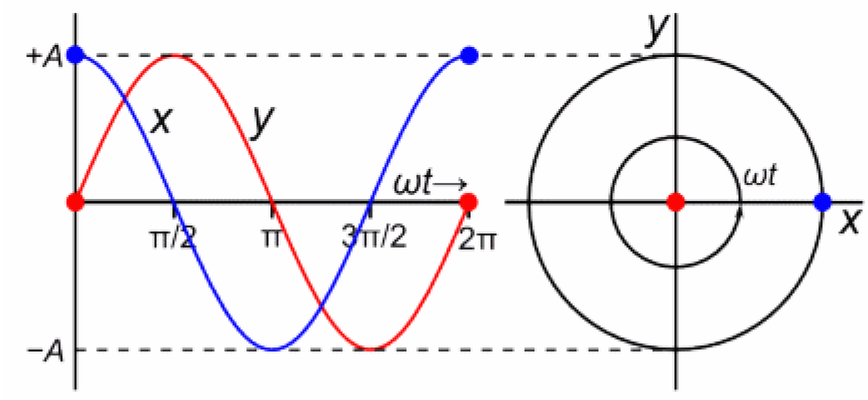

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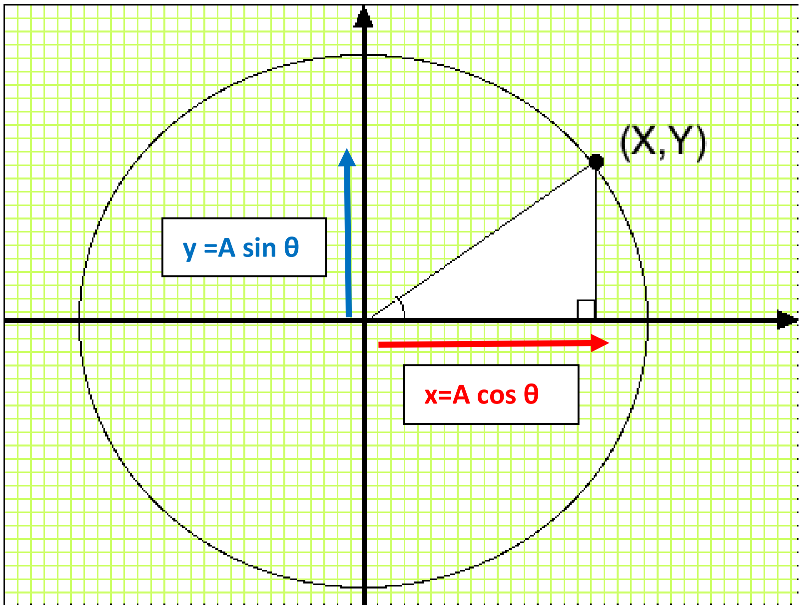
Notice the dot moving in a circle. At the same time observe the dot moving along the vertical and the horizontal lines- the time has been synchronized.

Now consider the figure the point (x, y) and the right angled triangle



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6. USING SINE AND COSINE FUNCTION

As the particle moves along the circle in the anticlockwise direction, its position described by (x,y) changes continuously.

The motion of the feet of perpendiculars dropped from the particle on the diameters along x-axis or y-axis within the limit of radius (A) of the circle is given by

$x = A\cos\theta$, y = A sinθ

These indicate the particle position along x-axis and y-axis.

So, when θ = 0, cos θ =1 and x =A, or the particle is at its extreme position. Its displacement from the center, or the mean position, and A is the amplitude.

What is θ?

As we said, it is the angular displacement of the particle with respect to the x-axis.

If **ω is the angular velocity of the particle moving in the circle,**

θ = ω t, where t is the time taken by the particle to have an angular displacement θ.

$\boldsymbol{\omega = \frac{2\pi}{T}}$, 2π is the angle in radians covered by the radius at the center of the circle for one complete revolution. T is the time taken to complete one revolution.

The graph drawn alongside is the sine or cosine curve.

The angle in question is the angle subtended on x or y-axis. The two motions are periodic and the red and blue dots move with the same periodic time.

This enables us to use a sine or a cosine function to describe the displacement of an oscillating particle at any instant.

If we take the case of Simple Harmonic Motion, $\omega = 2\pi/T$ can be expressed as $\omega=2\pi f$

ω is called angular frequency. It is 2π times the frequency.

Note - Angular frequency has no physical interpretation

We can therefore write the **displacement equation** as

$x = A \cos \theta = A \cos \omega t = A \cos 2\pi f\, t$

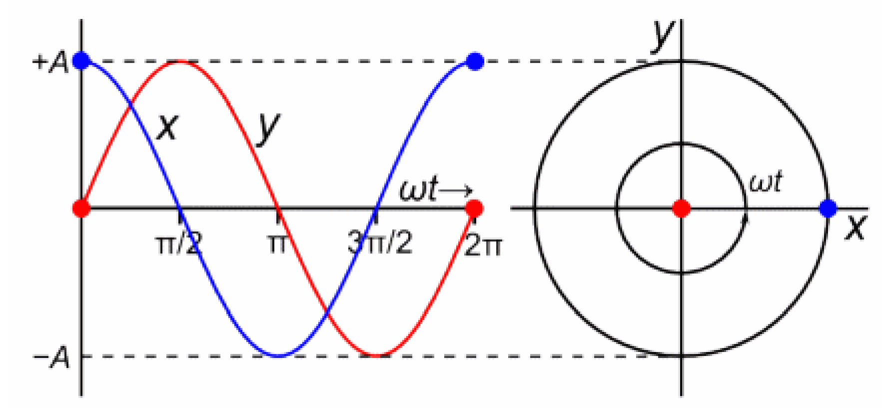
Here,

A is the amplitude of the oscillator,

f its frequency and

x the displacement from extreme position at instant t.

See the blue line trace in the graph



Alternately

We can similarly use the equation

$y = A \sin \theta$ or $y = A \sin \omega t$ or $y = A \sin 2\pi f\, t$

In this case, if $\theta = 0$, $y = 0$ or the particle will be at its mean position, or when $t = 0$, $\sin\omega t = 0$

What if t = T/4?

sin ω t = sin $\frac{2\pi}{T}\frac{T}{4} = sin\frac{\pi}{2} = sin\,90 = 1$

y =A sin ω t =A or the particle is at its extreme position

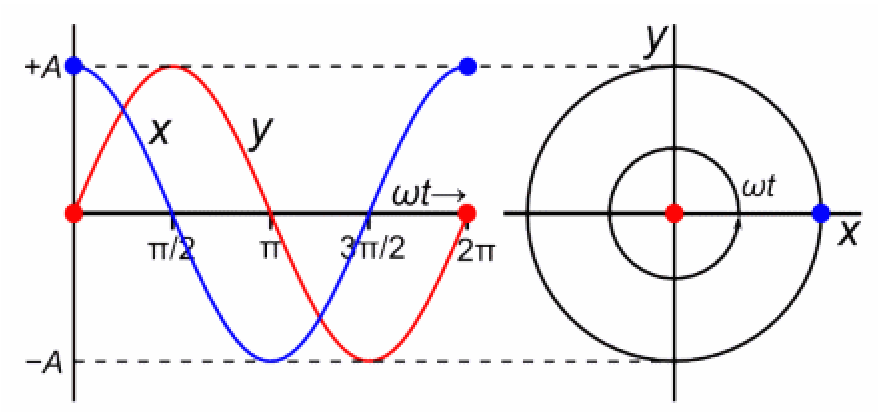
At t = 3T/4?

Here,

$$sin\,\omega\,t = sin\,\frac{2\pi}{T}\frac{3T}{4} = sin\frac{3\pi}{2} = sin\left(\pi + \frac{\pi}{2}\right) = -sin\frac{\pi}{2} = -1$$

$$y = -A$$

See the red line trace in the graph



What did we do here, in order to derive the equation of motion to express displacement of the oscillator at any instant of time?

We considered a periodic motion (a particle continuously moving in a circle with constant speed or constant angular velocity) coupled it with an SHM, which in our case was the motion of the foot of the perpendicular dropped from the particle on the x-axis or y-axis**. Common between the two situations is the time period, frequency and the amplitude of the oscillator which is equal to the radius of the circle.**

What is different is the fact that the **particle moves in a circle**, while the **foot of the perpendicular** on the diametric x-axis or y-axis executes **simple harmonic motion.**

EXAMPLE

What is meant by 'amplitude of an oscillator = 1.2mm'?

SOLUTION

The maximum displacement on either side of the equilibrium position is 1.2 mm.

EXAMPLE

A harmonic oscillator has a period of 0.2 s and amplitude of 10 cm, write its equation of motion for displacement.

SOLUTION

$T = 0.2s,\ \ \omega = \frac{2\pi}{0.2} = 10\pi$

$$y = A\sin\theta = A\sin\omega t = 10(\mathrm{cm})\sin 10\,\pi\, t$$

Or $$x = A\cos\theta = A\cos\omega t = 10\ (\mathrm{cm})\cos 10\,\pi\, t$$

EXAMPLE

In the above example, find the position of the particle at t = 6 s.

SOLUTION

$T = 0.2\ s\ \omega = \frac{2\pi}{0.2} = 10\pi$

$$y = A\sin\omega t = 10(\mathrm{cm})\sin 10\,\pi\, t$$

$$y = 10(\mathrm{cm})\sin 10\,\pi\, 6$$

$$y = 10\ sin\ 60\ \pi = 10\sin 30(2\pi) = 10\sin 2\pi = 0$$

Particle is at mean position at 6 sec.

EXAMPLE

Motion of an oscillator is given by $y = 7sin\left(0.5\pi t + \frac{\pi}{4}\right)$

Calculate

- **i) Amplitude**
- **ii) Frequency**
- **iii) Periodic time**
- **iv) Initial phase**
- **v) Displacement at t =2 s**
- **vi) Time taken by the oscillator to move from one extreme position to the other**

SOLUTION

i) 7 units

ii) $2\pi f = 0.5\pi$

$f = 0.25 Hz$

iii) Periodic time = 1/f = 1/0.25 = 4 s

iv) consider the argument $\left(\boldsymbol{0.5\pi t + \frac{\pi}{4}}\right)$

This time-dependent quantity, (ω t +ϕ) is called the **phase of the motion**. The value of phase at t = 0 is ϕ and is called the phase constant or phase angle; the second term $\frac{\pi}{4}$ gives the initial phase.

v) $y = 7sin\left(0.5\pi t + \frac{\pi}{4}\right)$ at t =2 s

$y = 7sin\left(0.5\pi(2) + \frac{\pi}{4}\right)$

$$y = 7sin\left(\pi + \frac{\pi}{4}\right) = -7sin\frac{\pi}{4}$$

sin (180° + θ) = - sin θ

$$y = -7\frac{1}{\sqrt{2}} = -7\ x\ 0.71 = -4.97\ \ units$$

vi) Half the periodic time = 2s

EXAMPLE

Which of the following functions of time represent (a) simple harmonic motion and (b) periodic but not simple harmonic? Give the period for each case.

(1) sin ω*t* – cos ω*t*

(2) sin² ω*t*

SOLUTION

a) $\sin\omega t - \cos\omega t = \sin\omega t - \sin\left(\frac{\pi}{2} - \omega t\right)$

$$= 2\cos\left(\frac{\pi}{4}\right)\sin\left(\omega t - \frac{\pi}{4}\right)$$

$$= \sqrt{2}\ sin\left(\omega t - \frac{\pi}{4}\right)$$

This function represents a simple harmonic motion having a period $T = 2\pi/\omega$ and a phase angle $(-\pi/4)$ or $(7\pi/4)$

b)

$$\sin^2\omega t = \frac{1}{2} - \frac{1}{2}\cos 2\ \omega t$$

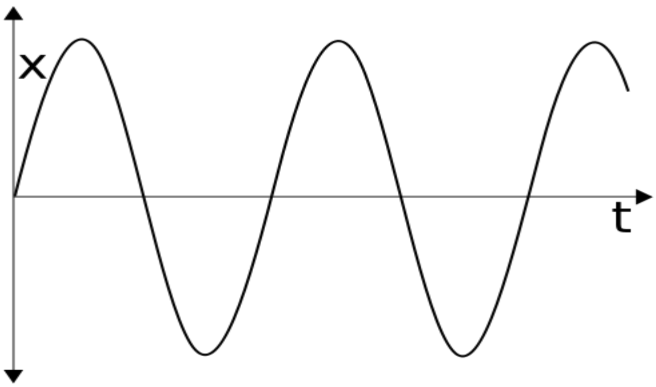
The function is periodic having a period $T = \frac{\pi}{\omega}$

It also represents a harmonic motion with the point of equilibrium occurring at ½ instead of zero.

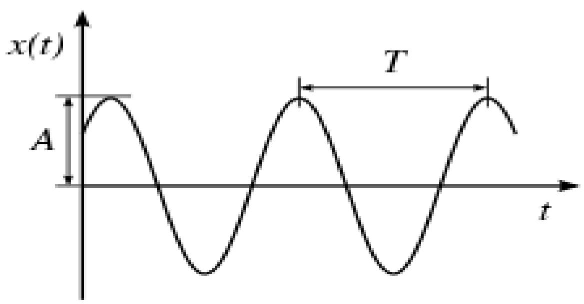
7. DISPLACEMENT TIME GRAPH

We can use these equations to draw a graph between the displacement of a particle executing simple harmonic motion and the time t.

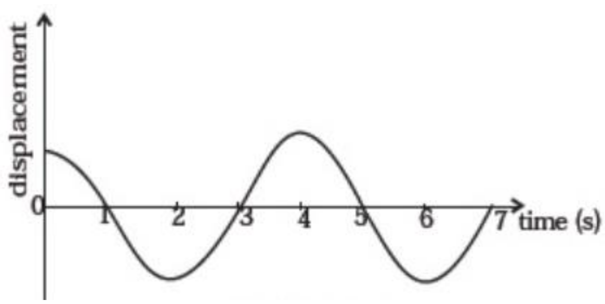
X =A sin ωt

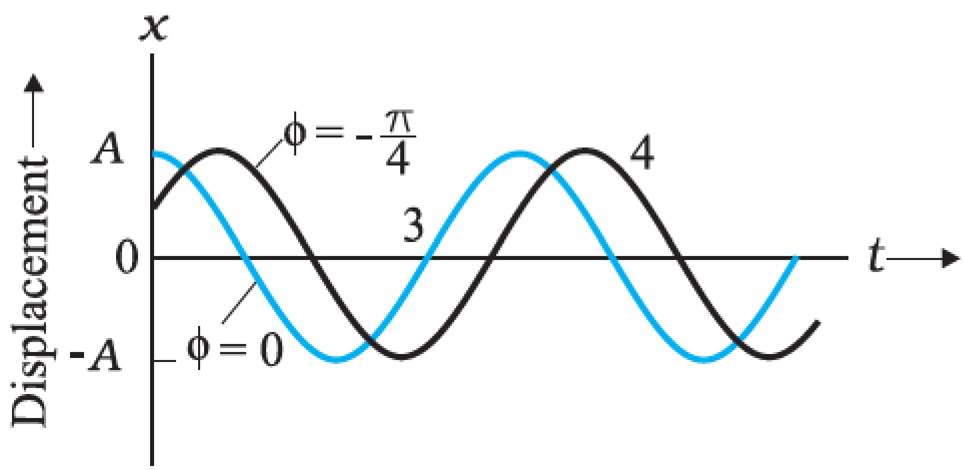


X =A sin (ωt +ϕ)



Y=A cos **ωt**





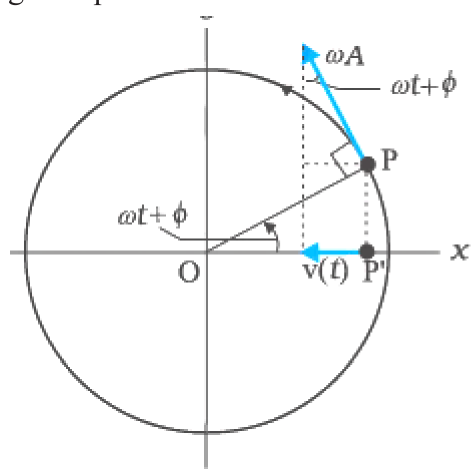
The curves 3 and 4 are for $\phi = 0$ and $-\pi/4$ respectively. The amplitude A is same for both the plots.

8. VELOCITY AND ACCELERATION OF THE PARTICLE IN SHM

The speed of a particle v in uniform circular motion is its angular speed ω times the radius of the circle A.

$$v = \omega A$$

The direction of velocity v at a time t is along the tangent to the circle at the point where the particle is located at that instant. From the geometry of Fig., it is clear that the velocity of the projection particle P′ at time t is



$$\mathbf{v(t) = -\omega A sin(\omega t + \phi)}$$

The velocity, v (t), of the particle P′ is the projection of the velocity v of the reference particle, P.

Where the negative sign shows that v (t) has a direction opposite to the positive direction of x-axis.

$v(t) = -\omega A \sin(\omega t + \phi)$ gives the **instantaneous velocity** of a particle executing SHM, where displacement is given by

$$\mathbf{x(t) = Acos(\omega t + \phi)}$$

We can, of course, obtain this equation without using geometrical argument, directly by differentiating

$$x(t) = A\cos(\omega t + \phi)$$

with respect of t:

As rate of change of position is velocity

$$v(t) = \frac{dx(t)}{dt} = -A\omega \sin(\omega t + \phi)$$

The differentiation of displacement with respect to time gives instantaneous velocity.

We can also write the v in terms of A and x as

We know, $x(t) = A\cos(\omega t + \phi)$

Differentiate $x(t)$ w.r.t. t, we get

$$\frac{dx}{dt} = -A\omega \sin(\omega t + \phi) = v$$

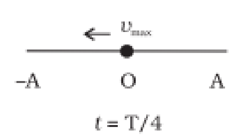
$$v^2 = A^2\omega^2 \sin^2(\omega t + \phi)$$

$$= A^2\omega^2 \sqrt{1 - \cos^2(\omega t + \phi)}$$

$$\mathbf{v = \omega\sqrt{A^2 - x^2}}$$

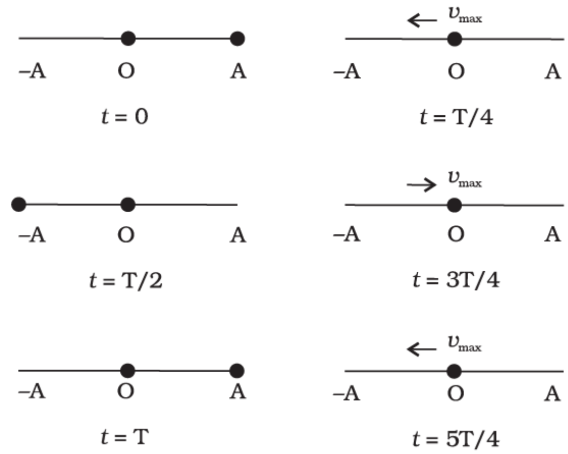
INTERPRETATION OF RESULT

- The maximum speed = $\pm A\omega$ occurs, when $\sin(\omega t + \phi) = 1, (\omega t + \phi) = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$
- The speed is zero when displacement is maximum $\pm A$ (the negative sign indicates that at t = 0 the displacement is maximum say +A and will be -A at T/2).



- At the mean position, the speed is maximum and the displacement at that instant is zero.
- The negative sign indicates the likely direction of velocity at any instant. If the particle reaches $\pm$ A its speed becomes zero and it would be ready to move towards the mean position.

The location of the particle in SHM at the discrete values t = 0, T/4, T/2, 3T/4, T, 5T/4. The time after which motion repeats itself is T. T will remain unchanged, no matter what location you choose as the initial (t = 0) location. The speed is maximum for zero displacement (at x = 0) and zero at the extremes of motion.

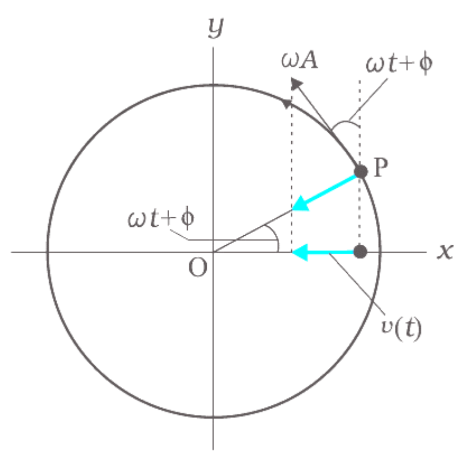


The method of reference circle can also be used for obtaining **instantaneous acceleration** of a particle undergoing SHM. We know that the centripetal acceleration of a particle P in uniform circular motion has a magnitude v^2/A or $\omega^2 A$, and it is directed towards the center i.e., the direction is along PO.

The **instantaneous acceleration** of the projection particle P′ is then

$$a(t) = -\omega^2 A\cos(\omega t + \phi)$$

$$= -\omega^2 x(t)$$



The acceleration, $a(t)$, of the particle P′ is the projection of the acceleration a of the reference particle P. It gives the acceleration of a particle in SHM.

The same equation can again be obtained directly by differentiating velocity v(t) given by

v(t)=- ω A sin (ω t+∅) **with respect to time**:

acceleration = rate of change of velocity

$$a(t) = \frac{d}{dt}v(t) = \frac{d}{dt}[-\omega A \sin(\omega t + \phi)]$$

We note from,

$$\mathbf{a(t) = -\omega^2 x(t)}$$

The important properties that acceleration of a particle in SHM has

- **Instantaneous Acceleration is proportional to displacement.**
- **Acceleration is maximum at extreme position**
- **Acceleration is zero in the mean position**
- **For x(t) > 0, $a(t)$< 0 and for x(t) < 0, $a(t)$> 0. Thus, whatever the value of x between –A and A, the acceleration a(t) is always directed towards the centre.**

Redefine SHM

A to and fro motion about a mean position where the acceleration is always directed towards the mean position and is proportional to the displacement from the mean position

9. EQUATION OF MOTION

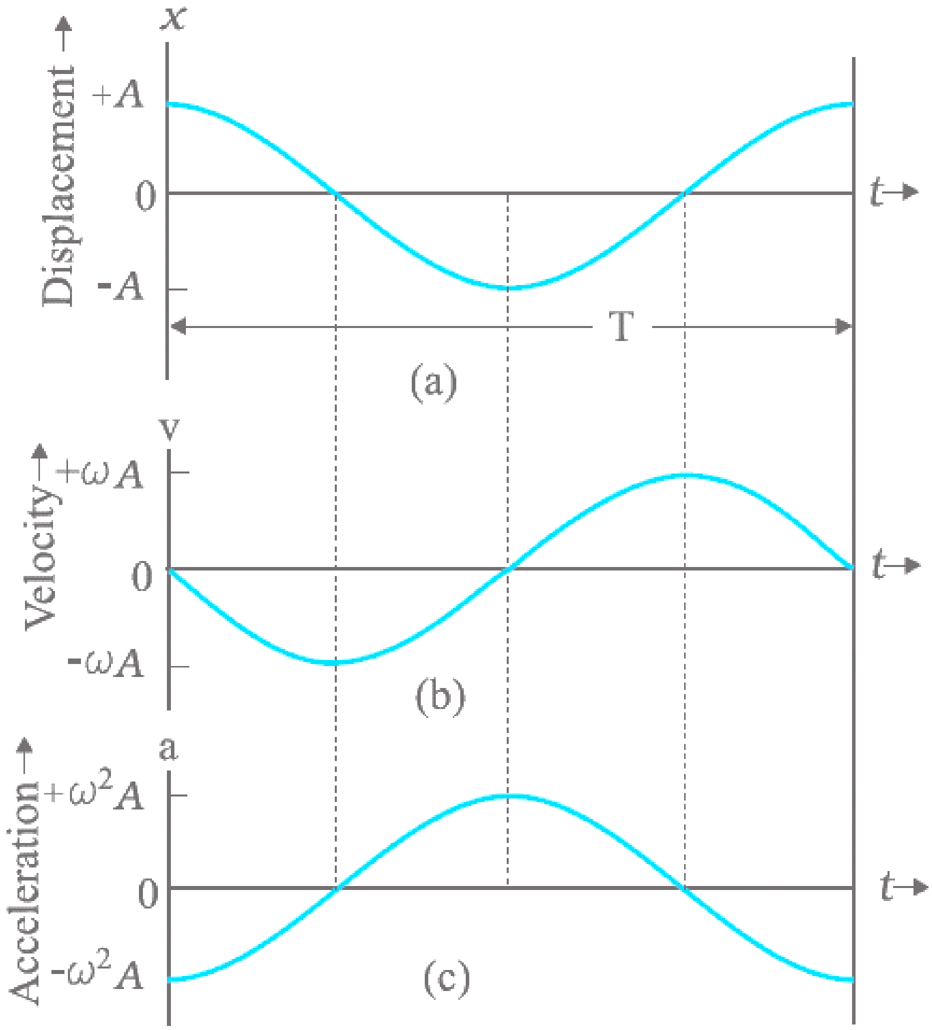
In fact, SHM may be defined as the motion in which the acceleration is directly proportional to the displacement and is in a direction opposite to the displacement.

For simplicity, let us put ϕ = 0 and write the expression for x (t), v (t) and $a(t)$

$$x(t) = A \cos \omega t,$$
$$v(t) = -\omega A \sin \omega t,$$
$$a(t) = -\omega^2 A \cos \omega t$$

These are the equations of motion for a harmonic oscillator. The term harmonic implies 'rhythmic'.

The corresponding plots for the equations are



Notice that these graphs are drawn to different scales.

Displacement, velocity and acceleration of a particle in simple harmonic motion have the same period T, but they differ in phase.

- All quantities vary sinusoidally with time;
- Only their maxima differ
- The different plots differ in phase.
- x varies between $-A$ to A;
- $v(t)$ varies from $-\omega A$ to $+\omega A$ and
- $a(t)$ from $-\omega^2 A$ to $\omega^2 A$.
- With respect to displacement plot, velocity plot has a phase difference of $\pi/2$ and acceleration plot has a phase difference of π.

EXAMPLE

A body oscillates with SHM according to the equation (in SI units),

x = 5 cos [2π t + π/4].
At t = 1.5 s,
Calculate the
(a) displacement,
(b) speed and
(c) acceleration of the body

SOLUTION

The angular frequency ω of the body = 2π s^{-1} and its time period T = 1 s.
At t = 1.5 s
(a) displacement = $(5.0m)cos[(2\pi s^{-1}) \times 1.5s + \pi/4]$
$= (5.0m)cos[(3\pi) + \pi/4]$
$= -5.0 \times 0.707$ m
$= -3.535$ m

(b) Using Eq., $v(t) = -\omega A sin(\omega t + \phi)$, the speed of the body
$= -(5.0m)(2\pi s^{-1})sin[(2\pi s^{-1}) \times 1.5s + \pi/4]$
$= -(5.0m)(2\pi s^{-1})sin[(3\pi) + \pi/4]$
$= 10\pi \times 0.707 ms^{-1}$
$= 22\ ms^{-1}$

(c) Using Eq. $a(t) = -\omega^2 A cos\omega t$, the acceleration of the body
$= -(2\pi s^{-1})^2 \times$ displacement
$= -(2\pi s^{-1})^2 \times (5 \times -0.707)$
$= 140\ m\ s^{-2}$

EXAMPLE
A particle executing SHM has velocities of 4 cm s^{-1} and 3 cm s^{-1} at distances of 3 cm and 4 cm from the mean position
Calculate
i) amplitude of oscillation
ii) periodic time
iii) velocity at mean position.

SOLUTION

$$\mathbf{x = A cos\omega t}$$

$$\mathbf{v = -A\omega sin\omega t}$$

Square both equations

$$\mathbf{x^2 = A^2 cos^2 \omega t}$$

Multiply both sides by ω^2

$$\omega^2 x^2 = \omega^2 A^2 \cos^2 \omega t$$

The velocity equation

$$v = -A\omega \sin\omega t$$

Squaring both sides

$$v^2 = A^2\, \omega^2 \sin^2\, \omega t$$

Adding

$\omega^2 x^2 = \omega^2 A^2 \cos^2 \omega t$ **and** $v^2 = A^2\, \omega^2 \sin^2\, \omega$

$\omega^2 x^2 + v^2 = \omega^2 A^2 (\cos^2 \omega t + \sin^2\, \omega t)$

$\omega^2 x^2 + v^2 = \omega^2 A^2$

$$v = \omega\sqrt{A^2 - x^2}$$

This equation is useful in the problem

SHM has velocities of 4 cm /s and 3 cm/s at distances of 3 cm and 4 cm from the mean position

4cm/s = $\omega\sqrt{A^2 - 3^2}$

3 cm /s = $\omega\sqrt{A^2 - 4^2}$

dividing $\frac{4}{3} = \frac{\sqrt{A^2-3^2}}{\sqrt{A^2-4^2}}$

$$\frac{16}{9} = \frac{A^2 - 9}{A^2 - 16}$$

$$16A^2 - 256 = 9A^2 - 81$$

$$7A^2 = 175$$

$$A^2 = 25$$

$$A = \pm 5\text{cm}$$

b) Substitute for A and find ω and then T

c) Find max velocity

10. SUMMARY

In this module you have learnt

- In simple harmonic motion (SHM), the displacement x (t) of a particle from its equilibrium position is given by,
  $x(t) = A\cos(\omega t + \varphi)$

- (displacement), in which A is the amplitude of the displacement, the quantity $(\omega t + \varphi)$ is the phase of the motion, and φ is the phase constant.

- The **angular frequency ω** is related to the period and frequency of the motion by,
  (Angular frequency = $2\pi f = \frac{2\pi}{T}$).
- Simple harmonic motion is the projection of uniform circular motion on the diameter of the circle in which the latter motion occurs.

- The particle velocity and acceleration during SHM as functions of time are given by,
  velocity $v(t) = -\omega A\sin(\omega t + \varphi)$,
  acceleration $a(t) = -\omega^2 A\cos(\omega t + \varphi) = -\omega^2 x(t)$,

  Thus we see that both velocity and acceleration of a body executing simple harmonic motion are periodic functions, having the
  velocity amplitude $v_{max} = \omega A$ and
  acceleration amplitude $a_{max} = \omega^2 A$,
- $v = \omega\sqrt{A^2 - x^2}$ is a useful result
- We can plot graphs for the three equations and see a phase relation between them.
- The velocity is maximum at mean position and zero at extreme position the acceleration is always directed towards the mean position.
- We can define simple harmonic motion in which **a particle executes to and fro motion about a mean position , for which the acceleration is always directed towards the mean position and is proportional to the displacement from the mean position**