Module Detail		
Subject Name	Physics	
Course Name	Physics 02(Physics Parts ,Class XI)	
Module Name/Title	Unit 9, Module 3, Speed of molecules of a gas	
	Chapter 13, Kinetic theory of gases	
Module Id	keph_201303_eContent	
Pre-requisites	Pressure, pressure exerted by gases, gaseous state of matter,	
	intermolecular separation of molecules of gases.	
Objectives	 After going through this module, the learners will be able to: Visualize the Distribution of velocity of molecules of gas Calculate with analysis the root mean square velocity of gas Understand variation of root mean square velocity of gas with temperature. Calculate average kinetic energy of a gas as a function of root mean square velocity of gas and temperature. Derive ideal gas laws from the kinetic theory 	
Keywords	Root mean square velocity, Maxwell Boltzmann distribution, Average kinetic energy. Ideal gas laws,	

1. Details of Module and its structure

2. Development Team

3. Role	Name	Affiliation
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1. UNIT SYLLABUS

UNIT 9: Behaviour of Perfect Gases and Kinetic Theory of Gases

SYLLABUS

Chapter13: Kinetic Theory:

Equation of state of a perfect gas, work done in compressing a gas. Kinetic theory of gasesassumptions, concept of pressure. Kinetic interpretation of temperature; rms speed of gas molecules; degrees of freedom, law of equi-partition of energy (statement only) and application to specific heat capacities of gases; concept of mean free path, Avogadro's number.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS

4 Modules

Module 1	
	 Microscopic and macroscopic view of interaction of heat with matter Kinetic theory Equation of state for a perfect gas PV= nRT
	• Statement of gas laws – Boyles law, Charles' law, pressure law, Dalton law of partial pressure
Module 2	
	Kinetic theory of gases
	 Assumptions made regarding molecules in a gas
	Concepts of pressure
	• Derivation for pressure exerted by a gas using the kinetic theory model
Module 3	
	Kinetic energy of gas molecules

	 rms speed of gas molecules Kinetic interpretation of temperature Derive gas laws from kinetic theory
Module 4	 Degrees of freedom Law of equipartition of energy Specific heat capacities of gases depends upon the atomicity of its molecules monoatomic molecule diatomic molecules polyatomic molecules

MODULE 3

3. WORDS YOU MUST KNOW

States of matter: the three states solid, liquid and gas in which matter can exist depending upon external conditions of temperature and pressure.

Interatomic / intermolecular forces a force exists between atoms and molecules of matter. The origin of force is electrostatic; it depends upon nature of material, State in which the material is in, external conditions of temperature and pressure.

Kinetic Theory: a theory that the particles of a gas move in straight lines with high average velocity, continually encounter one another and thus change their individual velocities and directions, and cause pressure by their impact against the walls of a container called also kinetic theory of gases.

Microscopic variables: Microscopic variables deal with the state of each molecule in terms of the mass, position, velocity etc. of each molecule.

Macroscopic variables: Macroscopic variables are the physical quantities which describe the state of a system as a whole e.g. Pressure, Volume, Temperature etc. They can be easily measured with laboratory instruments.

4. INTRODUCTION

Boyle discovered the law named after him in 1661. Boyle, Newton and several others tried to explain the behaviour of gases by considering that gases are made up of tiny atomic particles. The actual atomic theory got established more than 150 years later.

Kinetic theory explains the behaviour of gases based on the idea that the gas consists of rapidly moving atoms or molecules.

This is possible as the inter-atomic forces, which are short range forces that are important for solids and liquids, can be neglected for gases.

The kinetic theory was developed in the nineteenth century by Maxwell, Boltzmann and others. It has been remarkably successful.

It gives a molecular interpretation of pressure and temperature of a gas, and is consistent with gas laws and Avogadro's hypothesis.

It correctly explains specific heat capacities of many gases.

It also relates measurable properties of gases such as viscosity, conduction and diffusion with molecular parameters, yielding estimates of molecular sizes and masses.

The molecules in a gas are continuously moving in random directions with all possible speeds. It is almost impossible to find the speed of one individual molecule of the gas. But at normal temperature and pressure, the number of molecules in a gas is huge, so we can imagine a lot of activity inside a container with gas.

The volume of a gas depends upon the volume of the container; the pressure is the average pressure the gas molecules exert on the walls of a container enclosing the gas.

What about the situation when the gas is not enclosed say atmospheric air?

We will try and understand the activity of gas molecules in an enclosure.

5. DISTRIBUTION OF SPEED OF MOLECULES OF A GAS:

The number density of air molecules at standard temperature and pressure is nearly 2.5×10^{19} per centimetre cube. These numbers are really high and statistical probabilities can give the average distribution of the speeds of the gas molecules. Maxwell and Boltzmann could use statistical methods to correctly arrive at the distribution of speeds of these molecules at a given temperature.

Features of the Maxwell Boltzmann speed distribution:

• The graph has been plotted between the number of molecules and the speed of molecules.

- From the distribution it is evident that some molecules of the gas move very fast and some move very slowly, but most of the molecules have moderate speeds.
- Very few molecules of the gas move very fast or very slow.
- The area under the entire curve gives the total number of molecules of the gas.
- Collision between the molecules will change the individual molecular speed but their distribution will remain the same.



https://i.pinimg.com/originals/70/19/9a/70199ad461569b75d371bdfd51756f5e.gif

6. ROOT MEAN SQUARE SPEED AND ITS SIGNIFICANCE

Molecules of the gas are moving in all different directions. And since the number of molecules is very large, the number of molecules going in one direction is nearly equal to the number of molecules going in the opposite direction.

If we consider the velocity of the particles which is a vector quantity, almost half of them will have positive velocity and the other half negative velocity, so its average will come out to be zero.

To overcome this problem,

We take the square of velocities making them all positive.

Now the average of the squared velocities will not be zero.

The square root of the mean squared velocities is called **root mean square speed** (or **velocity**) of the molecules of the gas.

The root mean square speed is mathematically written as,

$$\mathbf{v}_{rms} = \sqrt{\frac{(\mathbf{v}_1^2 + \mathbf{v}_2^2 + \mathbf{v}_3^2 + \cdots + \mathbf{v}_N^2)}{N}}$$

• The peak of the graph showing the number of molecules versus their speed gives the maximum number of molecules which possess that particular speed. Hence it is called the **most probable speed** v_p



https://i.stack.imgur.com/K9UOH.png

- The graph of distribution of molecular speeds is **not symmetrical**.
- Due to the asymmetric nature of the graph, the **average speed** v_{avg} of the molecules is located on the right of the peak of the graph.
- The most probable speed $\mathbf{v_p}$ and the average speed of a gas v_{avg} are related to the root mean square speed $\mathbf{v_{rms}}$ of a gas.
- Mathematically it can be proved that

$$v_{avg} = 0.92 v_{rms}$$

$$v_p = 0.82 v_{rms}$$

7. AVERAGE KINETIC ENERGY OF A GAS:

All the molecules of the gas are moving. So all the molecules will have a kinetic energy associated with their motion. But again to find the kinetic energy associated with each molecule is nearly an impossible task, so we will consider the average kinetic energy of the molecules.

How do we find this average kinetic energy of the body? The root mean square value of the molecular speeds is a measure of this kinetic energy of the body at a certain temperature. If m

is the mass of any molecule and v_{rms} is the root mean square velocity, the average kinetic energy of molecules is given by

Average kinetic energy
$$=\frac{1}{2}mv_{rms}^2$$

But if the temperature of the gas is increased, more number of molecules will start moving faster. Similarly, if the temperature of the gas is decreased, more molecules will become slow. This change in the individual molecular speeds will change the root mean square value of the molecular speed which will change the average kinetic energy of the molecules of the gas.



We can see from the graph that the peak in the blue curve representing a lower temperature has shifted towards right in the red curve representing a higher temperature.

http://www.entropy-book.com/wp-content/uploads/2013/10/Fig.-3-graph-hot-coldcorrect.jpg

8. KINETIC INTERPRETATION OF TEMPERATURE

From kinetic theory of gases, it can be mathematically proved that the temperature of a gas is a measure of the average kinetic energy of the gas.

We know that the pressure exerted by a gas is given by

$$\mathbf{P} = \frac{mNv_{rms}^2}{3V}$$

Where P = pressure,

m = mass of a molecule of gas,

N = total number of molecules of the gas,

V= volume of the gas,

 v_{rms} = root mean square velocity

Rearranging the equation, we can write

Hence.

$$PV = \frac{mNv_{rms}^2}{3}$$

For one mole of a gas, number of molecules (N) of the gas will be equal to the Avogadro's number and m N will be equal to the molar mass (M).

mN = M (Molar mass of gas)

The gas equation for one mole of a gas is given by

$$PV = RT$$

(Where the symbols have their usual meaning)

$$PV = \frac{M(v^2)_{\rm rms}}{3} = RT$$
$$v^2_{\rm rms} = \frac{3RT}{M}$$
$$v^2_{\rm rms} \propto T$$

This relates the macroscopic variable of temperature (T) with the microscopic variable of mean square velocity (v_{rms}^2). Thus the mean square velocity of the molecules of a gas is a measure of the temperature of the gas.

The average kinetic energy of the molecules in one mole of a gas is given by

$$\frac{1}{2}Mv^2_{\rm rms} = \frac{3\rm RT}{2}$$

The average kinetic energy per molecule of the gas is obtained by dividing the above equation by Avogadro's number (N).

$$\frac{1}{2}mv^2_{\rm rms} = \frac{3kT}{2}$$

Here m is the mass of a molecule and k = Boltzmann constant = $\frac{R}{N}$

This means that the kinetic energy of a gas will increase with increase in temperature. If the temperature of the gas is reduced the molecules will slow down.

Another interesting point to be noted is that the kinetic energy of the molecule is independent of the mass of the molecule. It depends only on temperature.

This means that hydrogen, oxygen and nitrogen molecules which are the main constituents of air will all have the same average kinetic energy per molecule at a certain temperature of air.

This is the kinetic interpretation of temperature.



http://web.mit.edu/16.unified/www/FALL/thermodynamics/chapter_4_files/image003.gif

The root mean square velocity of the gas is given by

$$\mathbf{v_{rms}} = \sqrt{\frac{3RT}{M}}$$

The above relation shows that the mean square velocity of the molecules of a gas will increase with temperature and decrease as the molecular mass increases.

THINK ABOUT THIS:

If the temperature of the gas is absolute zero, will the molecules of the gas come to a standstill?

EXAMPLE

The escape velocity on Mars is 5.0 km/s and the surface temperature is typically O $^{\circ}$ C. Calculate the rms speeds for (a) H₂, (b) O₂, and (c) CO₂ at this temperature. Which gas is likely to be found in the atmosphere of Mars?

SOLUTION



http://images.slideplayer.com/39/10839248/slides/slide_32.jpg

Conclusion:

None of the gases have their rms value of velocity comparable to the escape velocity of mars. Hence these gases are found in the Martian atmosphere. But there is an abundance of carbon dioxide in the atmosphere of mars which is justified from its low rms value of speed.

On the other hand, there is an extremely thin atmosphere on the moon because the escape velocity of moon is only 2380 m/s. The rms velocity of the gases in daytime when the temperature of moon is 100 degree Celsius becomes comparable to this escape velocity. Hence, most of the gases have escaped from the gravitational field of the moon.

9. GRAPH SHOWING THE DISTRIBUTION OF SPEED FOR NITROGEN GAS AT DIFFERENT TEMPERATURES

Salient features of the above distribution:

- The average speed and the root mean square speed of the molecules of Nitrogen increases with increase in temperature.
- The average speed of molecules is least at 300K and largest at 1200K.
- The relative number of molecules having average and rms speed decreases with increase in temperature. This happens because



the speed of molecules is spread out in a bigger range at temperatures.

https://upload.wikimedia.org/wikipedia/commons/1/1f/Maxwell-Boltzmann speed distributions.png

EXAMPLE

- a. At what temperature the molecule of nitrogen will have the same rms velocity as the molecule of oxygen at 400K?
- b. What is the root mean square velocity of the oxygen at 400K?
- c. What is the ratio of average kinetic energy of the molecules of oxygen and nitrogen?

SOLUTION

a. The root mean square velocity is given by, $v_{\rm rms} = \sqrt{\frac{3RT}{M}}$

Given that,

Root mean square velocity of nitrogen = root mean square velocity of oxygen

$$v_{\text{Oxygen}} = v_{\text{nitrogen}}$$

$$\sqrt{\frac{3\text{RT}_1}{M_{O_2}}} = \sqrt{\frac{3\text{RT}_2}{M_{N_2}}}$$

Here, M_{O_2} = molecular mass of oxygen = 32u &

 M_{N_2} = molecular mass of nitrogen = 28u

- T_1 = temperature of oxygen = 400K &
- T_2 is the temperature of nitrogen which is to be evaluated.

$$T_{2} = \frac{T_{1} \times M_{N_{2}}}{M_{O_{2}}}$$
$$T_{2} = \frac{400K \times 28}{32} = 350K$$

So, at a temperature of 350K the molecule of nitrogen will have the same rms velocity as the molecule of oxygen at 400K.

b. Root mean square velocity of oxygen molecule:

$$v_{\rm rms of oxygen at 400K} = \sqrt{\frac{3RT}{M}}$$
$$= \sqrt{\frac{3 \times 8.314 \times 400}{0.032}}$$

$$= 560 \text{m/s}$$

= $v_{
m rms}$ of nitrogen at 350K

c. Average kinetic energy of molecules = $\frac{3kT}{2}$

$$\frac{\text{average kinetic energy of oxygen}}{\text{average kinetic energy of nitrogen}} = \frac{\text{Temperature of oxygen}}{\text{Temperature of nitrogen}}$$
$$= \frac{400}{350}$$
$$= 1.14$$

Point to be noted: Although the root mean square velocity of the molecules of nitrogen is equal to that of the molecules of oxygen, the average kinetic energy of oxygen and nitrogen depend only on the temperature of the gas.

10. GRAPH SHOWING THE DISTRIBUTION OF SPEED FOR VARIOUS GASES

Salient features of the above distribution:

- Average speed of the molecules of the gas increases with decrease in the molecular mass of the gas
- Hydrogen has the least molecular mass has the maximum average speed
- Krypton has the largest molecular mass has the minimum average speed.



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• The speed of molecules is spread out in a bigger range for gases which have lower molecular mass.

EXAMPLE

Given below is the average speed of some gases at 25°. Refer to the bar graph below to answer the following.

- a. Which gas has the largest average speed of the molecules and which one has the least speed?
- b. What is the ratio of speed of oxygen molecules and the speed of hydrogen molecules?
- c. What is ratio of the molecular mass of oxygen and hydrogen?



https://slideplayer.com/slide/4137426/13/images/13/5.8+Average+Speed+of+Som e+Molecules.jpg

d. Is there a relation between (b) and (c)?

SOLUTION

- (a) Molecules of hydrogen gas has the largest average speed and carbon dioxide has the least speed.
- **(b)** $\frac{\text{Speed of oxygen molecules}}{\text{Speed of hydrogen molecules}} = \frac{490}{1960} = \frac{1}{4}$
- (c) $\frac{\text{Molecular mass of oxygen molecules}}{\text{Molecular mass of hydrogen molecules}} = \frac{32}{2} = 16$
- (d) Speed of hydrogen molecules Speed of oxygen molecules = $\sqrt{\frac{\text{Molecular mass of oxygen molecules}}{\text{Molecular mass of hydrogen molecules}}}$

Inference: It is clearly evident from the above chart that the molecular speed decreases with the increases in the molecular mass. The heavier the molecule, lesser is its agility. So as compared to oxygen molecules, the hydrogen molecules move four times faster.

11. THE IDEAL GAS LAWS FROM THE KINETIC THEORY

The expression for pressure in terms of the microscopic variables and the kinetic interpretation of temperature can be used to arrive at the various gas laws. This also establishes the validity of kinetic theory of gases.

Boyles law:

Or

According to this law

"At a given temperature, the pressure of a given mass of an ideal gas is inversely proportional to the volume occupied by it".

The expression for pressure can be written as,

The root mean square velocity v_{rms} depends on the temperature of the given gas. But the temperature is constant, so, v_{rms} is also constant. Also, for a given mass of gas, total number of molecules N and mass of individual molecule m is constant.

So, the right hand side of the expression (a) is a constant and we have,

$$PV = constant$$

 $\mathbf{P} \propto \frac{1}{v}$

Hence, we have

So we have arrived at the Boyle's law.

EXAMPLE

A 100-g sample of CO₂ occupies a volume of 55 L at a pressure of 1 atm. If the volume is increased to 80 L and the temperature is kept constant, what is the new pressure?

SOLUTION:

Using Boyle's law:

$$P_1V_1 = P_2V_2$$

Given that $P_1 = 1$ atm, $V_1 = 55$ L and $V_2 = 80$ L

$$P_2 = \frac{P_1 V_1}{V_2} = \frac{1 \text{ atm} \times 55 \text{ L}}{80 L} = 0.688 \text{ atm}$$

The answer shows that the pressure decreases with increase in volume.

Charles law:

According to this law

"At a given volume, the pressure of a given mass of a gas is proportional to its absolute temperature"

Analysing the expression for pressure,

$$\mathbf{P} = \frac{\mathrm{N}m \, v_{rms}^2}{3V}$$
$$\mathbf{V} = \frac{\mathrm{N}m \, v_{rms}^2}{3P}$$

We know that for an ideal gas, the root mean square velocity v_{rms} depends on the temperature of the given gas.

$$v_{rms}^2 \propto T$$

The rest of the terms on the right hand side of the expression (b): total number of molecules (N), mass of each molecule (m) and the pressure (P) are constant

$\mathbf{V} \propto \mathbf{T}$

So we have arrived at the Charles' law.

EXAMPLE

A cylinder of helium gas is used to fill helium in balloons. As the balloons get filled, the number of helium molecules in the cylinder reduces. How does it affect the rms speed of the remaining molecules of helium in the cylinder?



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SOLUTION:

The root mean square speed of any gas depends on the temperature of the gas and its molecular mass. If the temperature of the gas in the helium gas cylinder does not change, the decrease in the number of molecules of the gas in the cylinder will not affect the rms speed of the molecules.

Avogadro's law

According to this law:

"Equal volume of all gases has equal number of molecules when they are at the same temperature and pressure"

The expression of pressure is,

$$PV = \frac{Nm v_{rms}^2}{3}$$

We consider two gases in two containers having the same volume and are at same temperature and pressure.

The above expression for the two gases is then given by

PV =
$$\frac{Nm_1(v_1^2)_{rms}}{3}$$
, PV = $\frac{Nm_2(v_2^2)_{rms}}{3}$

Pressure and volume remaining constant, the left hand side of the expression is constant and same for both the gases

$$PV = \frac{Nm_1(v_1^2)_{rms}}{3} = \frac{Nm_2(v_2^2)_{rms}}{3}$$

$$N_{1}m_{1}v_{1_{\rm rms}}^{2} = N_{2}m_{2}v_{2_{\rm rms}}^{2}$$

$$\frac{1}{2}N_{1}m_{1}v_{1_{\rm rms}}^{2} = \frac{1}{2}N_{2}m_{2}v_{2_{\rm rms}}^{2}$$

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Since, the temperature of the two gases is same, the average kinetic energy of their molecules is also same.

$$\frac{1}{2}m_1v_{1\,\rm rms}^2 = \frac{1}{2}m_2v_{2\,\rm rms}^2$$

Hence,

 $N_1 = N_2$

We have arrived at the Avogadro's law.

EXAMPLE

A small room, at temperature T = 300K and one atmospheric pressure, measures 3.5m x 3m x 4m. Find

- a. The total number of molecules in the room.
- b. The total kinetic energy of molecules of the gas.

SOLUTION

a. Volume of the room = $3.5 \times 3 \times 4 = 42 \text{m}^3$

According to Avogadro's law:

In a volume of 22.4 litres or 0.0224 m³, there are 6.023 x 10^{23} molecules.

So, in a volume of 42 m³ there will be $\frac{6.023 \times 10^{23} \times 42}{0.0224} = 1.13 \times 10^{27}$ molecules

b. Average kinetic energy of a molecule $=\frac{3kT}{2}$ $=\frac{3 \times 1.38 \times 10^{-23} \times 300}{2}$

$$= 6.21 \times 10^{-21} \text{J}$$

Total kinetic energy of the molecules

= Number of molecules x Average kinetic energy of each molecule

 $= 1.13 \text{ x } 10^{27} \text{ x } 6.21 \times 10^{-21} \text{J}$ $= 7 \text{ x } 10^{6} \text{ J}$

Interesting observation: The number of molecules in a small room is a huge value and the total kinetic energy of the molecules is also very large although the kinetic energy per molecule is quite small.

Sound propagates through a gas by disturbing the motion of the molecules of the gas. This disturbance is passed from molecule to molecule through molecular collisions and is referred to as the sound wave. So, the speed of sound waves in a gas having higher root mean square velocity will be higher. For example, the speed of sound in hydrogen gas in 1350 m/s and the speed of sound in nitrogen gas is 350 m/s. The root mean square velocity of hydrogen is 1920m/s and for Nitrogen it is 510 m/s.

The speed of the sound wave is less than the root mean square speed of the molecules of gas.

12. SUMMARY

- Molecules in a gas move with varied speeds. Maxwell and Boltzmann gave a distribution of speed versus the number of molecules of the gas. From this distribution, the most probable speed, the average speed and the root mean square speed can be calculated.
- Most probable speed is the speed which is possessed by maximum number of molecules of the gas. So, it corresponds to the peak of Maxwell distribution.
- Average speed is the arithmetic mean of the speeds of all the molecules of the gas.
- Root mean square speed (rms value) is the square root of the mean of the squares of the velocity of the molecules of the gas.

$$v_{\rm rms} = \sqrt{\frac{3\rm RT}{M}}$$

- This rms value of speed depends only on the temperature and the mass of the molecules.
- The average kinetic energy depends on the mean square speed of the molecules which in turn depends on the temperature of the gas. This gives the kinetic interpretation of temperature.
- The equation below gives the total kinetic energy of one mole of a gas.

$$\frac{1}{2}Mv^2_{\rm rms} = \frac{3RT}{2}$$

• The Boyle's law, Charles law and Avogadro's law can be obtained from the kinetic theory and the macroscopic variables of Pressure, Volume, and temperature can be related to the microscopic variables like the root mean square velocity and mass of individual molecules of the gas.

• The speed of sound in gas is comparable to the root mean square speed of molecules but it is always less than the rms value of molecules of the gas.