1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 02 (Physics Part 2 ,Class XI)
Module Name/Title	Unit 9, Module 2, Pressure exerted by a gas Chapter 13, Kinetic Theory of Gases
Module Id	keph_201302_eContent
Pre-requisites	Gas, behaviour of gases, gas laws, gas equation, Dalton's law , law of partial pressures
Objectives	After going through this module, the learners will be able to; • Define the macroscopic and microscopic variables of a gas • Elaborate assumptions of kinetic theory • Understand the mechanism by which pressure is exerted by a gas. • Derive an expression for pressure exerted by gas in terms of microscopic variables from kinetic theory of gases.
Keywords	Macroscopic variables, Microscopic variables, Pressure, root mean square velocity. Average velocity

2. Development Team

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1. UNIT SYLLABUS

Unit 9: Behaviour of perfect gases and kinetic theory

Chapter13: Kinetic Theory:

Equation of state of a perfect gas, work done in compressing a gas. Kinetic theory of gases - assumptions, concept of pressure. Kinetic interpretation of temperature; rms speed of gas molecules; degrees of freedom, law of equi-partition of energy (statement only) and application to specific heat capacities of gases; concept of mean free path, Avogadro's number.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 4 Modules

Module 1	• **Microscopic and macroscopic view of interaction of heat with matter** • **Kinetic theory** • **Equation of state for a perfect gas** • **PV= nRT** • **Statement of gas laws – Boyles law, Charles' law, pressure law, Dalton law of partial pressure**
Module 2	• **Kinetic theory of gases** • **Assumptions made regarding molecules in a gas** • **Concepts of pressure** • **Derivation for pressure exerted by a gas using the kinetic theory model**
Module 3	• **Kinetic energy of gas molecules**

	• **rms speed of gas molecules** • **Kinetic interpretation of temperature** • **Derive gas laws from kinetic theory**
Module 4	• **Degrees of freedom** • **Law of equipartition of energy** • **Specific heat capacities of gases depend upon the atomicity of its molecules** • **monoatomic molecule** • **diatomic molecules** • **polyatomic molecules**

MODULE 2

3. WORDS YOU MUST KNOW

Elastic collision: a collision in which the total kinetic energy of the colliding bodies or particles is the same after the collision as it was before.

Momentum: the property or tendency of a moving object to continue moving. It may also be visualised as the impact capacity of a moving body Both mass of the body and its velocity are important to ascertain the magnitude of momentum. For an object moving in a line, the momentum is the mass of the object multiplied by its velocity (linear momentum); thus, a slowly moving, very massive body and a rapidly moving, light body can have the same momentum.

Atmospheric pressure: The pressure of the atmosphere at any point is equal to the weight of a column of air of unit cross sectional area extending from that point to the top of the atmosphere.

4. INTRODUCTION

Boyle discovered the law named after him in 1661. Boyle, Newton and several others tried to explain the behaviour of gases by considering that gases are made up of tiny atomic particles. The actual atomic theory got established more than 150 years later.

Kinetic theory explains the behaviour of gases based on the idea that the gas consists of rapidly moving atoms or molecules.

This is possible as the inter-atomic forces, which are short range forces that are important for solids and liquids, can be neglected for gases.

The kinetic theory was developed in the nineteenth century by **Maxwell, Boltzmann** and others. It has been remarkably successful. It gives a molecular interpretation of pressure and temperature of a gas, and is consistent with gas laws and Avogadro’s hypothesis.

It correctly explains specific heat capacities of many gases. It also relates measurable properties of gases such as viscosity, conduction and diffusion with molecular parameters, yielding estimates of molecular sizes and masses.

All things are made of atoms - little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another.

The scientific 'Atomic Theory' is usually credited to John Dalton. He proposed the atomic theory to explain the **laws of definite and multiple proportions** obeyed by elements when they combine into compounds.

The first law says that any given compound has a fixed proportion by mass of its constituents.

The second law says that when two elements form more than one compound, for a fixed mass of one element, the masses of the other elements are in ratio of small integers.

Dalton suggested, about 200 years ago, that the smallest constituents of an element are **atoms.** Atoms of one element are identical but differ from those of other elements.

A small number of atoms of each element combine to form a molecule of the compound.

Gay Lussac's law, also given in early 19th century, states: **When gases combine chemically to yield another gas, their volumes are in the ratios of small integers.**

Avogadro's law (or hypothesis) says: Equal volumes of all gases at equal temperature and pressure have the same number of molecules. Avogadro's law, when combined with Dalton's theory explains Gay Lussac's law. Since the elements are often in the form of molecules, Dalton's atomic theory can also be referred to as the molecular theory.

From many observations, in recent times we now know that molecules (made up of one or more atoms) constitute matter. The size of an atom is about an angstrom (10^{-10} m). In solids, which are tightly packed, atoms are spaced about a few angstroms (2 Å) apart. In liquids the separation between atoms is also about the same. In liquids the atoms are not as rigidly fixed as in solids, and can move around. This enables a liquid to flow. In gases the interatomic distances are in tens of angstroms.

We now know that atoms are not indivisible or elementary. They consist of a nucleus and electrons. The nucleus itself is made up of protons and neutrons. The protons and neutrons are again made up of quarks. Even quarks may not be the end of the story. There may be string like elementary entities. Nature always has surprises for us, but the search for truth is often enjoyable and the discoveries beautiful. In this chapter, we shall limit ourselves to understanding the behaviour of gases (and a little bit of solids), as a collection of moving molecules in incessant motion.

Let us now consider macroscopic variables and use them to understand changes in microscopic variables

Macroscopic and Microscopic variables:

Macroscopic variables are the physical quantities which describe the state of a system as a whole.

They can be easily measured from outside.

The laws of thermodynamics which have been developed relate these macroscopic variables.

For a gas, the state of the system is described by the macroscopic variables like

- **pressure,**
- **temperature,**
- **volume,**
- **internal energy,**
- **Number of moles of a gas.**

A system is said to be in thermodynamic equilibrium when these variables do not change with time.

The equation of state for an ideal gas relates these macroscopic variables,

$$PV = nRT$$

Here the symbols have their usual meaning.

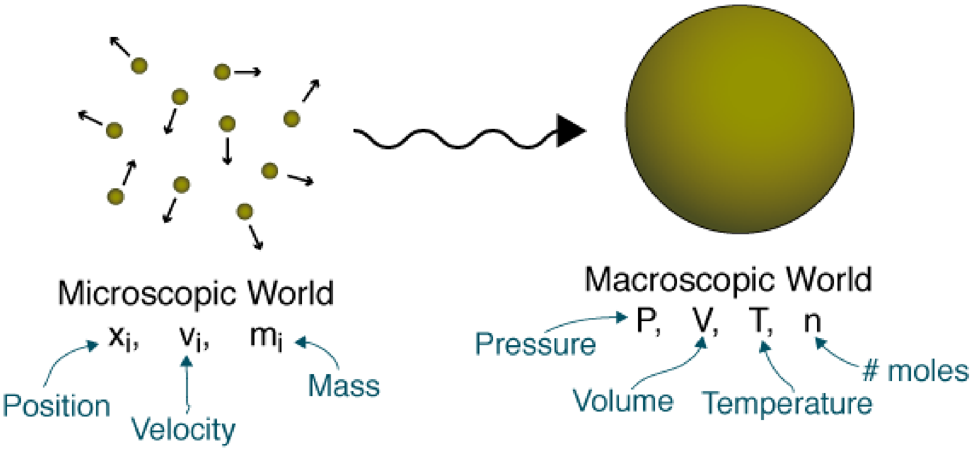
But if we consider a gas, say in a box, there are about 10^{20} molecules present in every centimetre cube of the box.

All these molecules are moving with different velocities, in different directions, colliding with each other and with the walls of the box. In other words, when we look inside the constituents of the box, we find a complete chaos. It is difficult to imagine such a system of the molecules in equilibrium.

The microscopic description of such a system requires us to specify the state of each molecule in terms of the mass, position, velocity etc. of the each molecule. Again an impossible task!

Clearly there has to be some link between the macroscopic and microscopic variables. For example if we want to measure the temperature of this gas, we would just insert a thermometer in the box and note the reading on the thermometer.

We cannot follow the individual motion of all the molecules of the gas. But the state of all these molecules taken together averages out and we are able to measure the macroscopic state of the whole system.



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In kinetic theory of gases, we work with a simplistic model which describes the constituent particles of the gas and the interactions between the particles of the gas. It attempts to make a bridge between the microscopic and the macroscopic states of the gas. So the macroscopic states like pressure, temperature can be expressed in terms of the microscopic states like the velocity, mass, etc. of the individual molecules.

5. ASSUMPTIONS OF THE KINETIC MODEL OF GAS

Kinetic theory of gases is a statistical model which works when the number of particles is very large. This is justified for gases because there are about 10^{20} molecules per cm^3.

The theory is developed for the classical monoatomic ideal gas. So the assumptions for the ideal gas hold true in this theory namely

- **The molecules of the gas are identical spherical particles having negligible size as compared to the intermolecular distances.**
- **The force of interaction between the molecules is negligible.**
- **The collision between the molecules and between the molecules and wall of the container is elastic and hence involves no loss in the kinetic energy.**
- **Time spent during a collision is negligibly small as compared to the time between two successive collisions.**
- **The molecules obey the Newton's laws of motion.**

The statistical model also assumes that,

Even though the molecules are in random motion all the time, the average number of molecules in any given volume is constant.

The molecules of the gas do not move preferably in some direction. This means that there is an equal probability of the molecules going in the x, y and z directions.

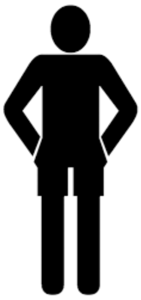
6. PRESSURE

(i) THE CONCEPT

Pressure is defined as the force applied perpendicular to a surface of unit area.

$$\text{Pressure} = \frac{\text{Force applied perpendicular to the surface}}{\text{Area of the surface}}$$

Pressure is measured in the SI unit of N/m^2 or Pascal.



When the person is standing on the ground, the perpendicular force on the ground is equal to the weight mg of the person. The area on the ground where the force is acting is nearly equal to the area of his two feet in contact with the ground. So the pressure on the ground is the ratio of the weight of the man and this area.

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When the person is lying down on the ground, the perpendicular force on the ground is again equal to the weight mg of the person. But now, the area on

https://i.pinimg.com/originals/a4/60/c0/a460c0bbd1eb9f7eaf75f8e64ce7e133.jpg

the ground where the force is acting is the area of his whole body which is in contact with the ground. So the pressure on the ground is the ratio of the weight of the man and the greater area.

In which case is the pressure on the ground greater?

Answer is obviously the first case when the person is standing on the ground.

(ii)EVIDENCES OF PRESSURE EXERTED BY A GAS

Everyone must have had an experience with a tyre of a vehicle while driving on the roads requires a certain air pressure. We need an air-pump, which can fill air into the flat tyre. The air which goes into the tyre increases the pressure and the tyre gets road ready

http://www.palmdaletow.com/uploads/5/5/7/6/55762443/4427145_orig.jpg

An interesting event demonstrated the power of the air pressure. The men closed the shutters of rail road car in the evening and went home. In the morning to their surprise, they found the rail road car had completely collapsed.

The air inside it was hot due to the steam engines which used to run the car. At night the temperatures dropped and the pressure of the air inside the car became much less than the atmospheric pressure. So the rail road car collapsed due to the atmospheric pressure.

Original status of the rail road car

http://ridingmode.com/wp-content/uploads/2016/05/11.jpg

http://geekologie.com/2016/01/22/imploding-tanker.jpg

So there are ample evidences of the fact that gases exert pressure.

GAUGING THE MAGNITUDE OF ATMOSPHERIC PRESSURE

Pressure of 1 Pascal means a force of 1N is acting perpendicularly on an area of $1m^2$. 1N force is almost equivalent to the weight of 100g mass.

$$F = mg$$

If m = 100g = 0.1kg and g = $10m/s^2$ (approximately)

$$F = 0.1 \times 10 = 1N$$

Atmospheric pressure is equal to $1.013 \times 10^5 = 101.3 \times 10^3$ Pascal.

10^5 Newton is equivalent to the weight of 10,000kg.

If m = 10,000kg   and g = $10 m/s^2$ (approximately)

F = 10,000 x 10 = 10^5N

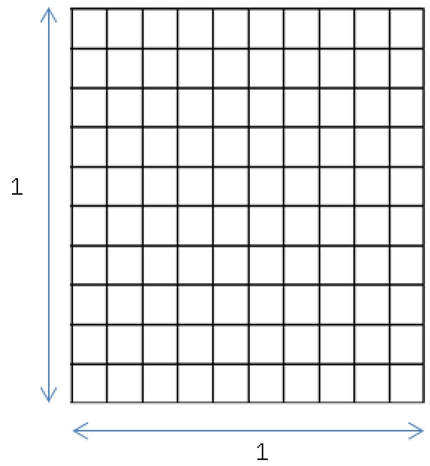
Let us imagine this value

Approximate weight of an elephant =   5000kg x 10 N

Pressure exerted by an elephant per $m^2 = 50 \times 10^3$ Nm^{-2}

Hence atmospherics pressure is two times the value!!

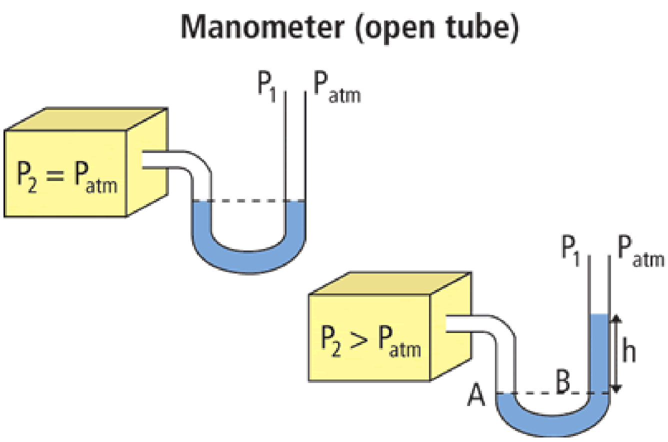
Or in terms of a 100 kg person it will be 100 times the weight!!



The figure shows a grid of area 1m x 1m. The grid is divided into 100 squares and a 100kg mass kept on each square so that the total mass on the 1 square meter area is 10,000 kg

The pressure on the $1m^2$ area will then be nearly equal to the atmospheric pressure.

The magnitude of atmospheric pressure can be understood and appreciated by considering a mass of 10,000kg distributed over a surface area of $1m^2$. The pressure on the surface would be 10^5 Pascal.



(iii)MEASUREMENT OF PRESSURE OF A GAS

The pressure exerted by the gas is measurable with instruments called **manometer.**

A mercury manometer is shown in the figure. The instrument has a U-tube which is open to the atmosphere at one end. It is filled with mercury and graduated.

https://www.isa.org/uploadedImages/Content/Standards_and_Publications/ISA_Publications/InTech_Magazine/2004/March/040328.gif

At the other end is the container having the gas whose pressure is to be measured. If the pressure of the gas is more than the atmospheric pressure, mercury in the other longer arm of U-tube rises (and falls if the pressure of the gas is less than the atmospheric pressure).

Difference in the height (h) the mercury in the two arms gives the excess pressure exerted by the gas in comparison to atmospheric pressure.

If pressure of the gas is more than the atmospheric pressure, then

Pressure of the gas (P) = Atmospheric Pressure (P_o) + $h\rho g$

$$P = P_o + h\rho g$$

Excess pressure of the gas = $P - P_o$

EXAMPLE:

In a mercury manometer, the difference in the heights of the mercury column in the two arms of the U-tube is 2cm in manometer.

a. **Find the excess pressure of the gas in the enclosed volume as shown in figure.**
b. **If water is used instead of mercury in the manometer, what will be the corresponding difference in the heights of water column in the two arms of the U-tube?**

Given: Density of mercury (ρ_{Hg}) = 13.6 x 10^3 kg/m^3, Density of water (ρ_W) = 10^3 kg/m^3, g = 10m/s^2

SOLUTION:

a. Density of mercury (ρ_{Hg}) = 13.6 x 10^3 kg/m^3, difference in height (h) = 2cm = 0.02m and taking g = 10m/s^2

Excess pressure of the gas = $h\rho_{Hg}g$

$$= 0.02 \times 13.6 \times 10^3 \times 10 = 2720 \text{ Pascal}$$

b. Density of water (ρ_W) = 10^3 kg/m^3

Excess pressure of the gas = $h'\rho_W g$ = 2720 Pascal

Difference in height of the water column in the two arms of the U tube of the water manometer = h'

$$= \frac{2720}{1000 \times 10} = 0.27\text{m} = 27\text{cm}$$

Analysis: The excess pressure of the gas is independent of the choice of the type of manometer. But we use mercury manometer because the density of mercury is 13.6 times the density of water. So the difference in the height of the two liquids in the two arms of the U-tube increases 13.6 times if water is used instead of mercury.

7. PRESSURE EXERTED BY A GAS ACCORDING TO KINETIC THEORY OF GASES

What causes the gas to exert pressure?

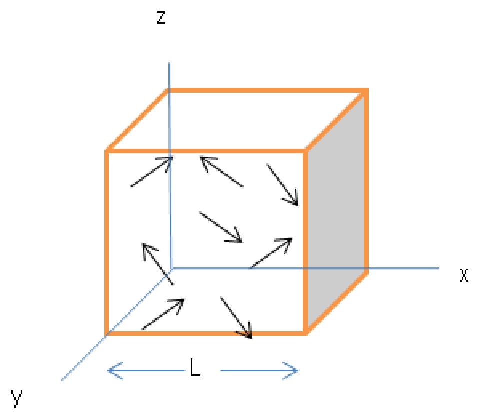
To understand this we must look inside the gas. Kinetic theory of gases enables us to analyse the motion of molecules of gas and we can explain the pressure of the gas in terms of the microscopic variables like the mass, velocity of the individual molecules of the gas.

If we could look inside a gas, we would see billions and billions of atoms moving randomly, colliding with each other and colliding with the walls of the container. Each time a molecule hits the wall, the wall experiences an impact or a force. We can calculate the force experienced by the wall due to an individual molecule using the Newton's laws of motion. According to Newton,

"The rate of change of momentum of an object is equal to the external force applied and the change is in the direction of the applied force"

The force on the wall due to one molecule can then be extended to all molecules and the total force on the wall can be calculated using statistical probabilities which can be applied when there is large number of molecules.

We consider **an ideal gas inside** a **cubical container of side L.** According to the assumptions of the kinetic theory, the collisions between the molecules and the collisions between the molecules and the walls of the container are elastic collisions. This means that there is no loss of kinetic energy in the collisions.



One of the vertices of the cube is at the origin of the co-ordinate axes **x**, **y** and **z** as shown in the figure.

We consider a molecule of mass m moving with a velocity $\vec{v}$.

The components of the velocity along the axes is given by $\vec{v}_x$, $\vec{v}_y$ and $\vec{v}_z$.

So the total momentum $\vec{P}$ of the molecule in its components is given by

$$\vec{P} = m\vec{v}_x + m\vec{v}_y + m\vec{v}_z$$

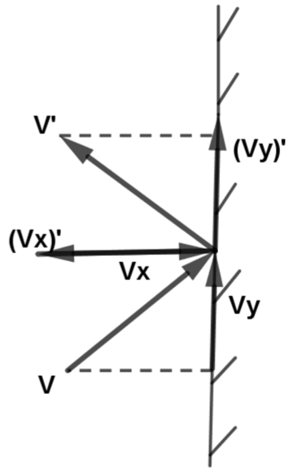
We consider a molecule to be striking the shaded wall of the container which is perpendicular to the x axis and is in the yz plane with velocity $\vec{v}$.

Since it is an elastic collision and the wall is much more massive as compared to the molecule, the molecule rebounds with the same speed after the collision with the wall.

The y and z component of the velocity do not change during the collision. The change occurs in the x component of the velocity which gets inverted or changes direction. As seen from the two dimensional vector diagram.

Initial y component of velocity **Vy = Vy' (F**inal y component of velocity)

Initial x component of velocity **Vx = - Vx'** (Final x component of velocity)



This can be extended to three dimensions to show that the z component of velocity also does not change during the collision.

The momentum of the molecule in the x direction also changes from m$\vec{v}_x$ to - m$\vec{v}_x$.

Hence the change in momentum of the molecule in the x direction = -2mv_x.

Since the momentum of the molecule and the wall system is conserved during collision,

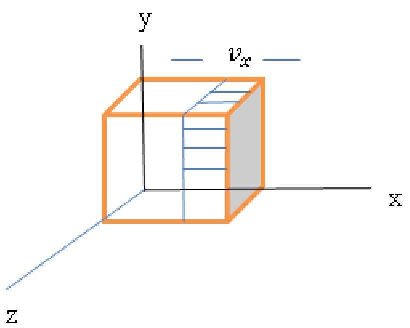
the change in momentum of wall = 2mv_x

The molecules hitting this wall in a time interval Δt are those that are within distance v_xΔt of the wall and are moving to the right.

The total number of molecules in this volume is the product of the number density of molecules n and the volume ΔV of this shaded region of the box and is equal to nΔV

Volume, ΔV = (v_xΔt) A where A is the area of the wall

So, the total number of molecules in the shaded region is **n(v_xΔt) A**



According to statistical probabilities and the assumptions of kinetic theory in which we do not consider any preferential directions of movement of molecules, **we consider only half of the number of molecules in the shaded region to be moving to the right.**

So the number of molecules colliding with the wall (shaded) in time Δt is given by,

$$N = ½\, n(v_x \Delta t)\, A$$

So the total change in the momentum of the wall is given by,

$$\Delta P = N\,(2mv_x) = ½\,(n(v_x \Delta t)\, A)\,(2mv_x) = nm{v_x}^2\, A\Delta t$$

Hence the force exerted by the molecules on the wall is,

$$\mathbf{F = \frac{\Delta P}{\Delta t} = nm{v_x}^2\, A}$$

And this force is acting perpendicular to the wall.

The magnitude of the pressure is the magnitude of this perpendicular force divided by the area,

$$\text{Pressure, } P = \frac{F}{A} = nm{v_x}^2$$

The above analysis has been done for one of the many molecules of the gas.

But this can be extended to the rest of the molecules also. Instead of taking the x component of velocity specifically for the molecule, we can take the average value of the x component of velocity of any molecule. For this we find the mean of the square of the velocities (all molecules).

Hence the expression for pressure becomes,

$$\text{Pressure, } P = nm({v_x}^2)_{avg}$$

Since there is no preferential direction, we have

$$({v_x}^2)_{avg} = ({v_y}^2)_{avg} = ({v_z}^2)_{avg}$$

And the average of mean square velocity of the molecules is given by

$$(v^2)_{avg} = ({v_x}^2)_{avg} + ({v_y}^2)_{avg} + ({v_z}^2)_{avg}$$

$$\therefore ({v_x}^2)_{avg} = (v^2)_{avg}/3$$

The square root of this average mean square velocity is known as the root mean square velocity v_{rms} .

Hence, $$\sqrt{(v^2)_{avg}} = v_{rms}$$

So we will refer to $(v^2)_{avg}$ as (v_{rms}^2)

So the pressure of the gas can now be expressed as

$$\mathbf{P = \frac{nmv_{rms}^2}{3}}$$

(n is the number density of the molecules = total number of molecules N per unit volume V)

$$P = \frac{Nmv_{rms}^2}{3V} = \frac{Mv_{rms}^2}{3V}$$

(Product of total number of molecules and mass of an individual molecule (Nm)

= Total mass (M) of the molecules of the gas)

$$P = \frac{\rho v_{rms}^2}{3}$$

Total mass of the molecules per unit volume (of container) M/V= density of the gas ρ)

These expressions give the pressure of the gas in terms of the microscopic variables of the gas.

Points to be noted:

- **Pressure of a gas increases with increase in the number density of molecules**
- **Pressure exerted by a gas is more if the mass of the gas molecules is more**
- **Pressure increases with increase in temperature of the gas. Increase in temperature causes the velocity of the molecules to increase which in turn increases the pressure exerted by the gas.**

EXAMPLE

A cubic metal box with sides of 10 cm contains air at a pressure of 1 atm and a temperature of 300 K. The box is sealed so that the volume is constant.

a. **Find the net force on each wall of the box.**
b. **If the pressure of the air inside the box is 1.1 atm, what will be the net force on each wall of the box.**

SOLUTION:

a. Pressure in the box = 1 atm = 1.01×10^5 Pa

Area of each wall = 0.1m x 0.1m = $0.01 m^2$

Force on each wall due to air inside the box = Pressure x Area of each wall

$= 1.01 \times 10^5 \times 0.01 = 1.01 \times 10^3$ N

But outside the box also there is air which is at 1 atm.

So, the force on each wall due to the air outside the box is also = 1.01×10^3 N

Hence, the net force on each wall due to air pressure is zero.

b. If pressure in the box = 1.1 atm = 1.111×10^5 Pa

Force on each wall due to air inside the box = $1.111 \times 10^5 \times 0.01 = 1.111 \times 10^3$ N

Force on each wall due to the air outside the box = 1.01×10^3 N

Net force on each wall = $(1.111 \times 10^3 - 1.01 \times 10^3)$
$= 0.101 \times 10^3$N = 101 N

Analysis: When the pressure inside and outside the container is same there is no force on the walls of the container. But if there is even a slight pressure difference inside and outside the wall, there is a substantial force on the walls of the container.

In a flexible container like a balloon you push the air with a certain pressure and the inflated balloon extends. What can you say about the pressure inside and outside the balloon?

8. SUMMARY:

Macroscopic variables: Macroscopic variables are the physical quantities which describe the state of a system as a whole e.g. Pressure, Volume, Temperature etc. They can be easily measured with laboratory instruments.

Microscopic variables: Microscopic variables deal with the state of each molecule in terms of the mass, position, velocity etc. of the each molecule.

Kinetic theory of gases enables macroscopic variables to be expressed in terms of microscopic variables.

Assumptions of the kinetic theory

- The molecular size is negligible as compared to the intermolecular distances.
- The force of interaction between the molecules is negligible
- The collision between the molecules and between the molecules and wall of the container is elastic.
- Time spent during a collision is negligibly small
- The molecules obey the Newton's law of motion

Pressure $= \dfrac{\text{Force applied perpendicular to the surface}}{\text{Area of the surface}}$

Pressure is measured in Pascal in SI units.

Pressure of a gas can be measured using a manometer.

Pressure of a gas is due to the molecules of the gas continuously colliding with the walls of the container. With the help of kinetic theory of gases, pressure can be expressed in terms of the microscopic variables and is given as

$$P = \frac{N m v_{rms}^2}{3V} = \frac{\rho v_{rms}^2}{3}$$

Where P = pressure,

V = volume,

N = total number of molecules of gas,

m = mass of individual molecule,

ρ = density of gas and,

v_{rms} = root mean square velocity of the molecules.