

## 1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 02 (Physics Part-2, Class XI)
Module Name/Title	Unit 7, Module 8, Fluid Flow Chapter 10, Mechanical Properties of Fluids
Module Id	Keph_201003_econtent
Pre-requisites	Students should have knowledge of fluid at rest.
Objectives	<p>After going through this module, the learners will be able to:</p> <ul style="list-style-type: none"> <li>• Differentiate between Stream line and turbulent flow</li> <li>• Describe Critical velocity, and Reynold number</li> <li>• Derive Reynolds number formula using method of dimensions</li> <li>• Understand equation of continuity for fluid flow.</li> </ul>
Keywords	Stream line and turbulent flow, critical velocity, Reynolds number, Equation of continuity

## 2. Development Team

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**1. UNIT SYLLABUS**

**UNIT: 7**

**PROPERTIES OF BULK MATTER:**

**24 periods**

**Chapter–9: Mechanical Properties of Solids:** Elastic behaviour, Stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, shear modulus of rigidity, Poisson's ratio; elastic energy.

**Chapter–10: Mechanical Properties of Fluids:** Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes), effect of gravity on fluid pressure. Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, critical velocity, Bernoulli's theorem and its applications. Surface energy and surface tension, angle of contact, excess of pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise

**Chapter–11: Thermal Properties of Matter:** Heat, temperature, thermal expansion; thermal expansion of solids, liquids and gases, anomalous expansion of water; specific heat capacity;  $C_p$ ,  $C_v$  - calorimetry; change of state - latent heat capacity. Heat transfer-conduction, convection and radiation, thermal conductivity, qualitative ideas of Blackbody radiation, Wein's displacement Law, Stefan's law, Greenhouse effect.

**2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS**

**17 MODULES**

<b>Module 1</b>	<ul style="list-style-type: none"> <li>• <b>Forces between atoms and molecules making up the bulk matter</b></li> <li>• <b>Reasons to believe that intermolecular and interatomic</b></li> </ul>
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	<p>forces exist</p> <ul style="list-style-type: none"> <li>● Overview of unit</li> <li>● State of matter</li> <li>● Study of a few selected properties of matter</li> <li>● Study of elastic behaviour of solids</li> <li>● Stationary fluid property: pressure and viscosity</li> <li>● Stationary liquid property: surface tension</li> <li>● Properties of Flowing fluids</li> <li>● Effect of heat on matter</li> </ul>
<b>Module 2</b>	<ul style="list-style-type: none"> <li>● Idea of deformation by external force</li> <li>● Elastic nature of materials</li> <li>● Elastic behaviour</li> <li>● Plastic behaviour</li> <li>● Tensile stress</li> <li>● Longitudinal Stress and longitudinal strain</li> <li>● Relation between stress and strain</li> <li>● Hooke's law</li> <li>● Young's modulus of elasticity 'Y'</li> </ul>
<b>Module 3</b>	<ul style="list-style-type: none"> <li>● Searle's apparatus</li> <li>● Experiment to determine Young's modulus of the material of a wire in the laboratory</li> <li>● What do we learn from the experiment?</li> </ul>
<b>Module 4</b>	<ul style="list-style-type: none"> <li>● Volumetric strain</li> <li>● Volumetric stress</li> <li>● Hydraulic stress</li> <li>● Bulk modulus K</li> <li>● Fish, aquatic life on seabed, deep sea diver suits and submarines</li> </ul>
<b>Module 5</b>	<ul style="list-style-type: none"> <li>● Shear strain</li> <li>● Shear stress</li> <li>● Modulus of Rigidity G</li> <li>● Poisson's ratio</li> <li>● Elastic energy</li> <li>● To study the effect of load on depression of a suitably clamped meter scale loaded at i) its ends ii) in the middle</li> <li>● Height of sand heaps, height of mountains</li> </ul>
<b>Module 6</b>	<ul style="list-style-type: none"> <li>● Fluids-liquids and gases</li> <li>● Stationary and flowing fluids</li> <li>● Pressure due to a fluid column</li> <li>● Pressure exerted by solid, liquids and gases</li> </ul>

	<ul style="list-style-type: none"> <li>• <b>Direction of Pressure exerted by solids, liquids and gases</b></li> </ul>
<b>Module 7</b>	<ul style="list-style-type: none"> <li>• <b>Viscosity- coefficient of viscosity</b></li> <li>• <b>Stokes' Law</b></li> <li>• <b>Terminal velocity</b></li> <li>• <b>Examples</b></li> <li>• <b>Determine the coefficient of viscosity of a given viscous liquid by measuring terminal velocity of a given spherical body in the laboratory</b></li> </ul>
<b>Module 8</b>	<ul style="list-style-type: none"> <li>• <b>Streamline and turbulent flow</b></li> <li>• <b>Critical velocity</b></li> <li>• <b>Reynolds number</b></li> <li>• <b>Obtaining the Reynolds number formula using method of dimensions</b></li> <li>• <b>Need for Reynolds number and factors effecting its value</b></li> <li>• <b>Equation of continuity for fluid flow</b></li> <li>• <b>Examples</b></li> </ul>
<b>Module 9</b>	<ul style="list-style-type: none"> <li>• <b>Bernoulli's theorem</b></li> <li>• <b>To observe the decrease in pressure with increase in velocity of a fluid</b></li> <li>• <b>Magnus effect</b></li> <li>• <b>Applications of Bernoulli's theorem</b></li> <li>• <b>Examples</b></li> <li>• <b>Doppler test for blockage in arteries</b></li> </ul>
<b>Module 10</b>	<ul style="list-style-type: none"> <li>• <b>Liquid surface</b></li> <li>• <b>Surface energy</b></li> <li>• <b>Surface tension defined through force and through energy</b></li> <li>• <b>Angle of contact</b></li> <li>• <b>Measuring surface tension</b></li> </ul>
<b>Module 11</b>	<ul style="list-style-type: none"> <li>• <b>Effects of surface tension in daily life</b></li> <li>• <b>Excess pressure across a curved liquid surface</b></li> <li>• <b>Application of surface tension to drops, bubbles</b></li> <li>• <b>Capillarity</b></li> <li>• <b>Determination of surface tension of water by capillary rise method in the laboratory</b></li> <li>• <b>To study the effect of detergent on surface tension of water through observations on capillary rise.</b></li> </ul>
<b>Module 12</b>	<ul style="list-style-type: none"> <li>• <b>Thermal properties of matter</b></li> <li>• <b>Heat</b></li> </ul>

	<ul style="list-style-type: none"> <li>• Temperature</li> <li>• Thermometers</li> </ul>
<b>Module 13</b>	<ul style="list-style-type: none"> <li>• Thermal expansion</li> <li>• To observe and explain the effect of heating on a bi-metallic strip</li> <li>• Practical applications of bimetallic strips</li> <li>• Expansion of solids, liquids and gases</li> <li>• To note the change in the level of liquid in a container on heating and to interpret the results</li> <li>• Anomalous expansion of water</li> </ul>
<b>Module 14</b>	<ul style="list-style-type: none"> <li>• Rise in temperature</li> <li>• Heat capacity of a body</li> <li>• Specific heat capacity of a material</li> <li>• Calorimetry</li> <li>• To determine specific heat capacity of a given solid material by the method of mixtures</li> <li>• Heat capacities of a gas have a large range</li> <li>• Specific heat at constant volume <math>C_V</math></li> <li>• Specific heat capacity at constant pressure <math>C_P</math></li> </ul>
<b>Module 15</b>	<ul style="list-style-type: none"> <li>• Change of state</li> <li>• To observe change of state and plot a cooling curve for molten wax.</li> <li>• Melting point, Regelation, Evaporation, boiling point, sublimation</li> <li>• Triple point of water</li> <li>• Latent heat of fusion</li> <li>• Latent heat of vaporisation</li> <li>• Calorimetry and determination of specific latent heat capacity</li> </ul>
<b>Module 16</b>	<ul style="list-style-type: none"> <li>• Heat Transfer</li> <li>• Conduction, convection, radiation</li> <li>• Coefficient of thermal conductivity</li> <li>• Convection</li> </ul>
<b>Module 17</b>	<ul style="list-style-type: none"> <li>• Black body</li> <li>• Black body radiation</li> <li>• Wien's displacement law</li> <li>• Stefan's law</li> <li>• Newton's law of cooling,</li> <li>• To study the temperature, time relation for a hot body by</li> </ul>

	<p>plotting its cooling curve</p> <ul style="list-style-type: none"> <li>• To study the factors affecting the rate of loss of heat of a liquid</li> <li>• Greenhouse effect</li> </ul>
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## MODULE 8

### 3. WORDS YOU MUST KNOW

**Fluids:** A fluid is a substance that can flow. This includes both gases and liquids.

**Hydrostatics:** Fluid statics or hydrostatics is the branch of fluid mechanics that studies fluids at rest.

**Stress:** The restoring force per unit area is known as stress. The restoring force is equal to the applied force. If  $F$  is the force applied and  $A$  is the area of cross section of the body, Magnitude of the stress =  $F/A$ . The SI unit of stress is  $\text{N m}^{-2}$  or pascal (Pa)

**Shearing strain:** shearing strain is defined as the ratio of relative displacement of the faces  $\Delta x$  of the cylinder to the length  $L$ .

Shearing strain,  $\frac{\Delta x}{L} = \tan \theta$

**Viscosity:** Viscosity is a measure of a fluid's resistance to flow. It describes the internal friction of a moving fluid. A fluid with large viscosity resists motion because its molecular makeup gives it a lot of internal friction.

### 4. INTRODUCTION



Incense stick burning

[https://c1.staticflickr.com/6/5636/21165622395\\_339e2d6b3c\\_b.jpg](https://c1.staticflickr.com/6/5636/21165622395_339e2d6b3c_b.jpg)



Smoke rises smoothly for a while and then begins to form swirls and eddies.



When you light an incense stick or agarbatti, you must have noticed that the smoke rises in a smooth plume for some distance and then moves randomly in all directions as it continues its rise. Similarly, when a water-tap is turned on slowly, the water flow is smooth initially, but loses its smoothness when the speed of the outflow is increased.

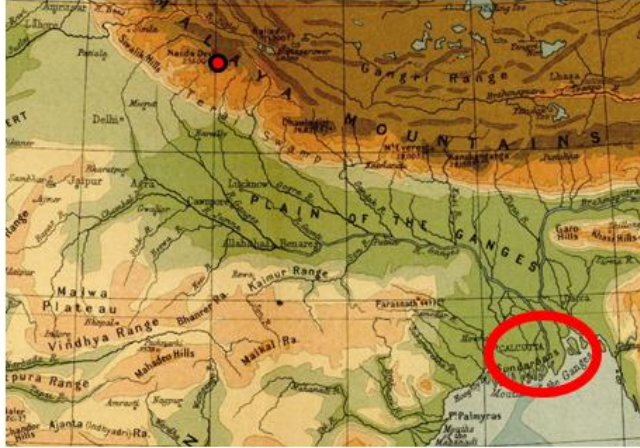
[https://c1.staticflickr.com/3/2878/10660352314\\_972c0dd560\\_b.jpg](https://c1.staticflickr.com/3/2878/10660352314_972c0dd560_b.jpg)



A stream or a river flows slowly when it runs through open country and on ground with less gradient and faster through waterfalls or in constricted areas like in a mountain valley . This is because water is an incompressible fluid, the pressure changes on it do not result in changes in its density. Consider the flow of ganga from Gangotri to Bay of Bengal.

<https://encrypted-tbn0.gstatic.com/images?q=tbn:ANd9GcRsuBP7wxENmZ-ygpHzAN2eEBfTq8ZOmMDapWeWqpvAZao6zC7seA>





<https://upload.wikimedia.org/wikipedia/commons/1/19/GangesValley%26Plain.jpg>

**Why does this happen?**

**What does flow of fluid has to do with its speed?**

This module will deal with answers to these questions.

In previous modules we have dealt with hydrostatics i.e. study of fluids at rest. In this module we are going to learn about fluids in motion. The study of fluids in motion is known as **fluid dynamics**.

## 5. CRITICAL VELOCITY

Whenever fluid is in motion, depending on its velocity of flow, it can either be smooth or irregular. Steady or smooth flow is achieved at low flow speeds. Beyond a limiting value, called critical velocity, this flow loses steadiness and becomes **irregular**, that is why when a water-tap is turned on slowly, the water flow is smooth initially, but loses its smoothness when either the speed of the outflow is increased or the water has fallen some distance from the spout under gravity





[https://upload.wikimedia.org/wikipedia/commons/thumb/7/77/Boy\\_drinks\\_from\\_a\\_tap\\_at\\_a NE WAH WASH water project %2810677817915%29.jpg/800px-Boy drinks from a tap at a NEWAH WASH water project %2810677817915%29.jpg](https://upload.wikimedia.org/wikipedia/commons/thumb/7/77/Boy_drinks_from_a_tap_at_a_NE_WAH_WASH_water_project_%2810677817915%29.jpg/800px-Boy_drinks_from_a_tap_at_a_NE_WAH_WASH_water_project_%2810677817915%29.jpg)



<https://static.pexels.com/photos/81902/dove-drinking-water-81902.jpeg>

Now let us understand the factors on which critical velocity of a liquid depends

### **CRITICAL VELOCITY OF LIQUID**

Critical velocity of a liquid is that limiting value of its velocity of flow up to which the flow is smooth or steady called streamlined and above which it becomes irregular called turbulent.

In the next section,

we will learn about **streamline and turbulent flow** in detail.

The critical velocity  $V_c$  of a liquid flowing through a tube depends upon

- a) **coefficient of viscosity of the liquid,  $\eta$**
- b) **density of liquid,  $\rho$**
- c) **diameter of the tube,  $D$**

Let  $V_c = k \eta^a \rho^b D^c$

where k is constant of proportionality

**Using the methods of dimensions,**

$$[M^0 L T^{-1}] = [M L^{-1} T^{-1}]^a [M L^{-3}]^b [L]^c$$

Equating powers of M, L and T, We get

$$\begin{aligned} a + b &= 0 \\ -a - 3b + c &= 1 \\ -a &= -1 \end{aligned}$$

Solving these equations, we get

$$\begin{aligned} a &= 1 \\ b &= -1 \\ c &= -1 \end{aligned}$$

So  $V_c = \frac{k \eta}{\rho D}$

This shows critical velocity will be greater if  $\eta$  is large and  $\rho$  and  $D$  are small.

## 6. STREAMLINE AND TURBULENT FLOW

In **Fluid dynamics**, i.e. **study of fluid in motion** we will discuss the following types of flow

- **Streamline flow**
- **Turbulent flow**

### STREAMLINE FLOW

At **slow speeds**, the flow is smooth or steady. Such flow is called **streamlined**.

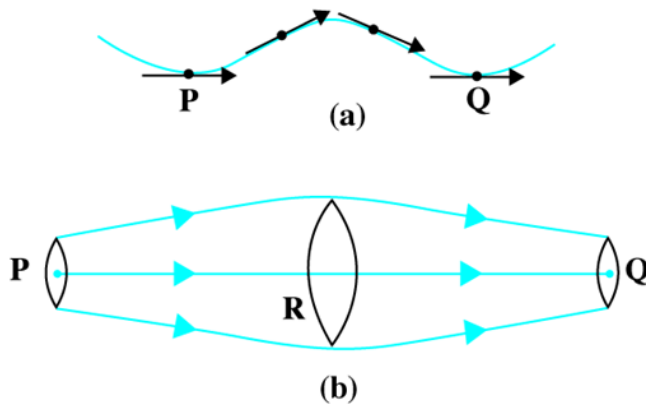
**The basic condition for steady flow is**

**The flow of the fluid is said to be steady if at any given point, the velocity of each passing fluid particle remains constant in time.**

This does not mean that the velocity at different points in space is same. The velocity of a particular particle may change as it moves from one point to another.

That is, at some other point the particle may have a different velocity, but every other particle which passes the second point behaves exactly as the previous particle that has just passed that point.

**Each particle follows a smooth path, and the paths of the particles do not cross each other. It is as though fluid particles were moving along ‘lines’ and as there is a ‘stream of them’ hence the word stream line**



**The meaning of streamlines**

**(a) A typical trajectory of a fluid particle.**

**(b) A region of streamline flow.**

Thus, when a liquid flow is such that each particle of the liquid is passing through a given point moves along the same path and has the same velocity as its predecessor had at that point, the flow is called **streamlined or steady flow**.

**The path taken by a fluid particle under a steady flow is a **streamline**.**

It is defined as a curve whose tangent at any point is in the direction of the fluid velocity at that point.

**PROPERTIES OF STREAMLINES:**

- No two streamlines can cross, for if they do, an oncoming fluid particle can have two possible direction of motion at that point. It can go either one way or the other and

the flow would not be steady. Hence, in steady flow, the map of flow is stationary in time.

- **The tangent at any point on the streamline gives the direction of the fluid velocity at that point.**
- **Greater the number of streamlines passing through a section of the fluid, larger is the fluid velocity at that point.**
- **Fluid velocity remains constant at any point of a streamline, but it may be different at different points of the same streamline.**

Have you observed streamline flow in open drains, kitchen sink or river?

Match the features highlighted above.



[https://upload.wikimedia.org/wikipedia/commons/5/55/Drain\\_-\\_Mayflower\\_Woods\\_-\\_geograph.org.uk\\_-\\_663722.jpg](https://upload.wikimedia.org/wikipedia/commons/5/55/Drain_-_Mayflower_Woods_-_geograph.org.uk_-_663722.jpg)

## **TURBULENT FLOW**

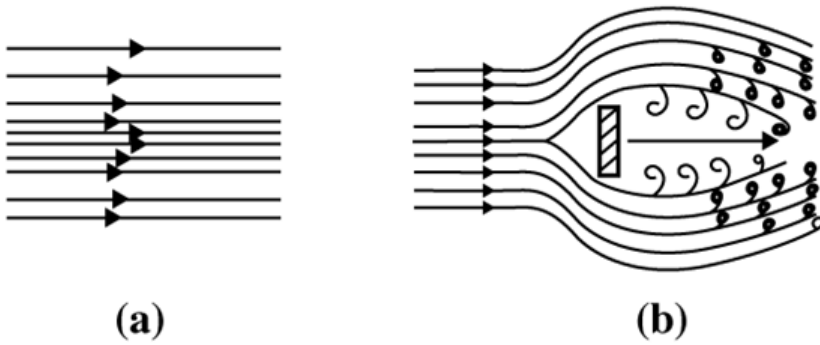
Liquid flows in layers. Each layer of the liquid slides over the other layer. It behaves as if different lamina are sliding over one another. Such a flow is **called laminar flow**. Layers flow without mixing when flow is laminar.

When there is turbulence, the layers mix, and there are significant velocities in directions other than the overall direction of flow.

For example,

Fig. (a) describes a laminar flow where the velocities at different points in the fluid may have different magnitudes but their directions are parallel.

And fig(b) shows turbulent flow.



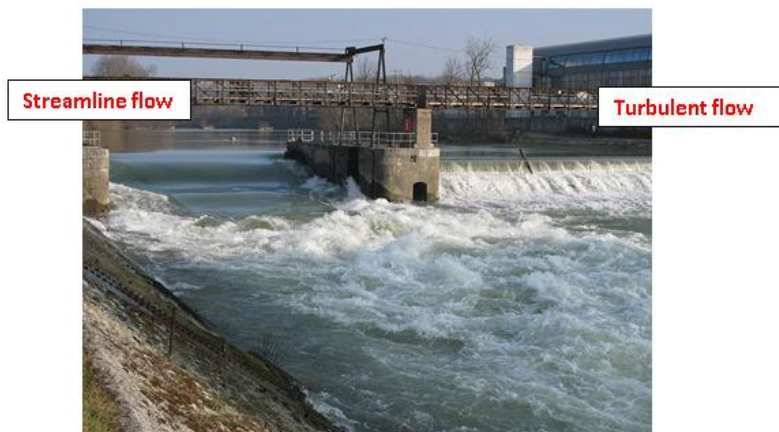
(a) Some streamlines for fluid flow.

(b) A jet of air striking a flat plate placed perpendicular to it. This is an example of turbulent flow.

We can say that,

When the rate of flow of a fluid is large, the flow no longer remains laminar, but becomes turbulent.

In a turbulent flow the velocity of the fluids at any point in space varies rapidly and randomly with time. Some circular motions called eddies are also generated. **An obstacle placed in the path of a fast moving fluid causes turbulence**



<https://upload.wikimedia.org/wikipedia/commons/thumb/4/4b/Laminar and turbulent flows at the Noisiel dam.jpg/800p>

### Laminar and turbulent flows at a dam

Examples of turbulent flow;

- The smoke rising from a burning stack of wood is turbulent.
- Oceanic currents are turbulent.
- Twinkling of stars is the result of atmospheric turbulence.



- The wakes in the water and in the air left by cars, aeroplanes and boats are also turbulent.

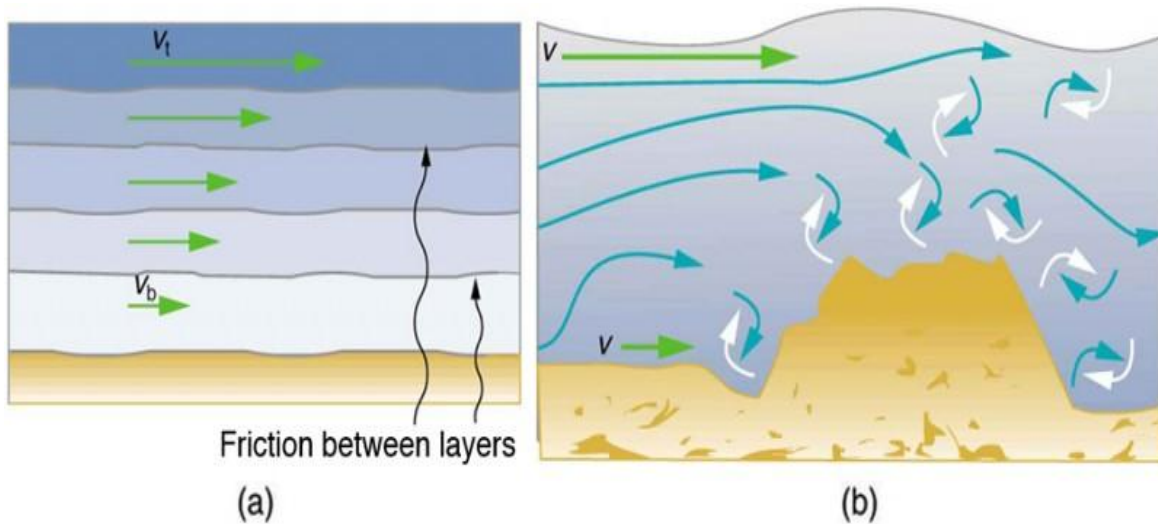
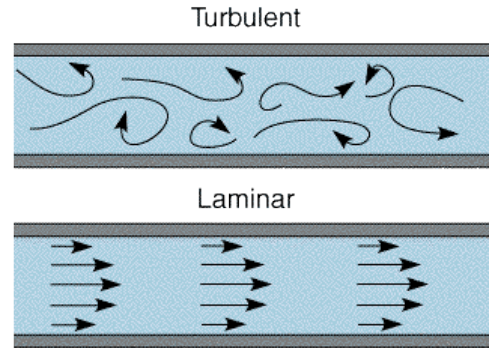
### CAUSE OF TURBULENCE

Turbulence has **two main causes**.

**First**, any obstruction or sharp corner, such as in a faucet, creates turbulence by imparting velocities perpendicular to the flow,

**Second**, high speeds cause turbulence.

The drag both between adjacent layers of fluid and between the fluid and its surroundings forms swirls and eddies, if the speed is high enough.



The figure shows (a) Laminar flow occurs in layers without mixing. (b) An obstruction in the vessel produces turbulence. Turbulent flow mixes the fluid.

This video shows laminar flow.

[https://youtu.be/p08\\_KITKP50](https://youtu.be/p08_KITKP50)

This video shows types of flow of liquid

<https://youtu.be/s4LvI3rgttw>

**THINK ABOUT THESE**

- When the air flows in the atmosphere, does it go turbulent after some fixed speed limit?
- Can the critical velocity for the same liquid be changed by varying its viscosity?

### APPLICATION OF LAMINAR AND TURBULENT FLOW

In the human body normally, blood flow is laminar. However, under conditions of flow at high speed, particularly in the ascending aorta, laminar flow can be disrupted and become **turbulent**. When this occurs, blood does not flow linearly and smoothly in adjacent layers, but instead the flow can be described as being chaotic.

The turbulent flow also occurs in large arteries at branch points, in diseased and narrowed (stenotic) arteries. Turbulence increases the energy required to drive blood flow because turbulence increases the loss of energy in the form of friction, which generates heat. Turbulence does not begin to occur until the velocity of flow becomes high enough that the flow lamina breaks apart.

This video shows laminar flow and turbulent flow of blood in veins.

<https://youtu.be/ABAgil5kkI0>

### 7. REYNOLDS NUMBER

Osborne Reynolds (1842-1912) observed that turbulent flow is less likely for viscous fluid flowing at low rates. He defined a dimensionless number, whose value gives one an approximate idea whether the flow would be turbulent. This number is called the Reynolds number,  $R_e$ , which is given by

$$R_e = \rho v d / \eta$$

where  $\rho$  is the density of the fluid flowing with a speed  $v$ ,  
 $d$  stands for the diameter of the pipe,  
 and  $\eta$  is the viscosity of the fluid.

$R_e$  is a dimensionless number and therefore, it remains same in any system of units.

### 8. REYNOLDS NUMBER FORMULA USING METHOD OF DIMENSIONS

It has been found that

$R_e$  directly proportional to velocity  $v$



If  $R_e$  depends upon density of the fluid, diameter of the pipe and viscosity of the fluid .the dimensions of these may be written

$R_e$  is a number, so dimension less  $[M^0 L^0 T^0]$

$\rho$  is the density of the fluid flowing  $[M^1 L^{-3}]$

$v$  speed of fluid flow , $LT^{-1}$

$d$  diameter of the pipe  $L$

$\eta$  viscosity of the fluid  $[M^1 L^{-1}T^{-1}]$

substituting the dimensional formula, we get

$$[M^0 L^0 T^0] = [M^1 L^{-3}]^a [L T^{-1}]^b [L] [M^1 L^{-1}T^{-1}]^c$$

using **principle of homogeneity**,

$$-1 - c = 0$$

$$a + c = 0$$

$$-3a + b - c + 1 = 0$$

Which gives  $a=1$   $b=1$   $c=-1$

$$R_e = v \frac{\rho d}{\eta}$$

## 9. NEED FOR REYNOLDS NUMBER AND FACTORS AFFECTING ITS VALUE

It is found that

- flow is streamline or laminar for  $R_e$  less than 1000.
- The flow is turbulent for  $R_e > 2000$ .
- The flow becomes unsteady for  $R_e$  between 1000 and 2000.

The critical value of  $R_e$  (known as critical Reynolds number), at which turbulence sets, is found to be the same for the geometrically similar flows.

For example when oil and water with their different densities and viscosities, flow in pipes of same shapes and sizes, turbulence sets in at almost the same value of  $R_e$ . Using this fact a small

scale laboratory model can be set up to study the character of fluid flow. They are useful in designing of ships, submarines, racing cars and aeroplanes.

$R_e$  can also be written as

$$\begin{aligned} R_e &= \rho v^2 / (\eta v/d) \\ &= \rho A v^2 / (\eta A v/d) \\ &= \text{inertial force/force of viscosity.} \end{aligned}$$

Thus  $R_e$  represents the ratio of inertial force (force due to inertia i.e. mass of moving fluid or due to inertia of obstacle in its path) to viscous force.

### THINK ABOUT THIS

- **If viscosity is the property of liquid then can we assume that the liquid will flow with the same speed in different medium, say if we pour water in air, or in a chamber of smoke , or say a chamber of hydrogen ?**
- **In a bent pipe when the liquid will flow the particles which are farther from the from the curve will move with the greater velocity then the particle which are inside, then at any instant how can we say they are behaving similarly and following a streamline flow?**
- **The velocity of the particle in the streamline flow is equal to its predecessor at that point or to its adjacent particle?**
- **Between milk and water motion of which will be more streamline?**
- **Between water and milk which one will have more velocity during streamline flow?**

Turbulence dissipates kinetic energy usually in the form of heat. Racing cars and planes are engineered to precision in order to minimize turbulence. The design of such vehicles involves experimentation and trial and error.

On the other hand, turbulence (like friction) is sometimes desirable. Turbulence promotes mixing and increases the rates of transfer of mass, momentum and energy. The blades of a kitchen mixer induce turbulent flow and provide thick milk shakes as well as beat eggs into a uniform texture.

**This video shows Reynolds number for laminar and turbulent flow.**

[https://youtu.be/\\_6UDB\\_TZKDK](https://youtu.be/_6UDB_TZKDK)

### EXAMPLE

**The flow rate of water from a tap of diameter 1.25 cm is 0.48 L/min. The coefficient of viscosity of water is  $10^{-3}$  Pa s. After sometime the flow rate is increased to 3 L/min. characterize the flow for both the flow rates**

**SOLUTION**

Let the speed of the flow be  $v$  and the diameter of the tap be  $d = 1.25 \text{ cm}$ .

The volume of the water flowing out per second is

$$Q = v \times \pi d^2 / 4$$

$$v = 4 Q / d^2 \pi$$

$$R_e = 4 \rho Q / \pi d \eta$$

$$= 4 \times 10^3 \text{ kg m}^{-3} \times Q / (3.14 \times 1.25 \times 10^{-2} \text{ m} \times 10^{-3} \text{ Pa s})$$

$$= 1.019 \times 10^8 \text{ m}^{-3} \text{ s Q}$$

Since initially

$$Q = 0.48 \text{ L / min} = 8 \text{ cm}^3 / \text{s} = 8 \times 10^{-6} \text{ m}^3 \text{ s}^{-1},$$

we obtain,

$$R_e = 815$$

Since this is below 1000, the flow is steady.

After some time when

$$Q = 3 \text{ L / min} = 50 \text{ cm}^3 / \text{s} = 5 \times 10^{-5} \text{ m}^3 \text{ s}^{-1},$$

$$R_e = 5095$$

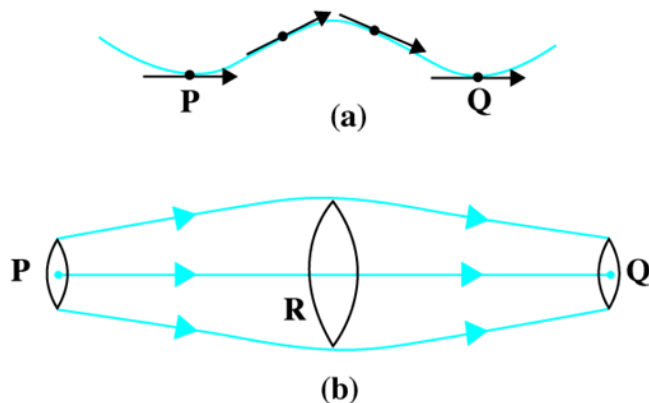
The flow will be turbulent.

You may carry out an experiment in your washbasin to determine the transition from laminar to turbulent flow.

**This video shows Reynolds experiment**

[https://youtu.be/\\_b1cd59Sm8U](https://youtu.be/_b1cd59Sm8U)

**10. EQUATION OF CONTINUITY**



Consider planes perpendicular to the direction of fluid flow e.g., at three points P, R and Q in Fig. (b). The plane pieces are so chosen that their boundaries be determined by the same set of streamlines. This means that number of fluid particles crossing the surfaces as indicated at P, R and Q is the same.

If area of cross-sections at these points are  $A_P, A_R$  and  $A_Q$  and speeds of fluid particles are  $v_P, v_R$  and  $v_Q$ , then the mass of fluid  $\Delta m_P$  crossing at  $A_P$  in a small interval of time  $\Delta t$  is  $\rho_P A_P v_P \Delta t$ .

Similarly, mass of fluid  $\Delta m_R$  flowing or crossing at  $A_R$  in a small interval of time  $\Delta t$  is  $\rho_R A_R v_R \Delta t$  and mass of fluid  $\Delta m_Q$  is  $\rho_Q A_Q v_Q \Delta t$  crossing at  $A_Q$ .

**The mass of liquid flowing out equals the mass of liquid flowing in, holds in all cases. And there are no “traffic jams”**

Therefore,  
 $\rho_P A_P v_P \Delta t = \rho_R A_R v_R \Delta t = \rho_Q A_Q v_Q \Delta t$

**For flow of incompressible fluids**

$$\rho_P = \rho_R = \rho_Q$$

the equation reduces to

$$A_P v_P = A_R v_R = A_Q v_Q$$

which is called the **equation of continuity and it is a statement of conservation of mass in flow of incompressible fluids.**

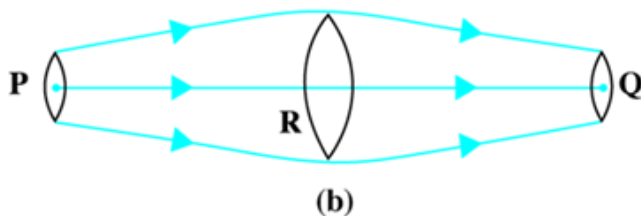
In general

$$Av = \text{constant}$$

**Av gives the volume flux or flow rate and remains constant throughout the pipe.**

Thus, at narrower portions where the streamlines are closely spaced, velocity increases and its vice versa.

From



It is clear that  $A_R > A_Q$  or  $v_R < v_Q$ , the fluid is accelerated while passing from R to Q.

**THINK ABOUT THIS**

- Would equation of continuity hold for pipes with uniform area of cross section?

- Would the equation be true for open pipes?
- Would the equation be true if the density of fluid can change as in gaseous flow?
- Would the equation of continuity hold for a pipe bend, or looped?
- Would the equation of continuity hold for rivers on the mountain terrain, in the plains?

**EXAMPLE**

Water tank has a hole in its wall at a distance of 10m below the free surface of water. The diameter of the hole is 2mm. compute the velocity of efflux of water from the hole and the rate of flow of water?

**SOLUTION-**

The velocity of efflux of water is

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \text{ m/s}^2 \times 10\text{m}}$$

$$v = 14 \text{ m/s}$$

The rate of flow of water =  $A \times v = \pi r^2 \times v$

$$= 3.14(1 \times 10^{-3}\text{m})^2 \times 14 \text{ m/s}$$

$$= 4.4 \times 10^{-5} \text{ m}^3/\text{s}$$

**EXAMPLE**

The cylindrical tube of spray pump has a cross section of  $0.8 \text{ cm}^2$ , one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is  $1.5 \text{ m min}^{-1}$ , what is speed of ejection of the liquid through the holes?

**SOLUTION:**

Here, cross section of the tube,

$$a_1 = 8.0 \text{ cm}^2 = 8.0 \times 10^{-4} \text{ m}^2;$$

The speed of the liquid in the tube,

$$v_1 = 1.5 \text{ m min}^{-1} = \frac{1.5}{60} \text{ m s}^{-1} = 0.025 \text{ m s}^{-1}$$

$$\text{Diameter of a hole, } D = 1.0 \text{ mm} = 10^{-4} \text{ m}$$

Therefore, cross section of a hole,

$$\frac{\pi D^2}{4} = \frac{\pi}{4} \times (10^{-4})^2 = \frac{\pi}{4} \times 10^{-8} \text{m}^2$$

Therefore, total cross-section of 40 holes,

$$a_2 = \frac{\pi}{4} \times 10^{-8} \times 40 \text{m}^2$$

If  $v_2$  is the speed of ejection of the liquid through holes,

$$a_1 v_1 = a_2 v_2$$

Or 
$$v_2 = \frac{a_1 v_1}{a_2} = \frac{8.0 \times 10^{-4} \times 0.025}{\frac{\pi}{4} \times 10^{-8} \times 40} = 0.637 \text{ m s}^{-1}$$

## 11. SUMMARY

- Critical velocity of a liquid is that limiting value of its velocity of flow upto which the flow is smooth or, steady called streamlined and above which it becomes irregular called turbulent.
- A streamline is a map of fluid flow. In a steady flow two streamlines do not intersect as it means that the fluid particle will have two possible velocities at the point.
- In a turbulent flow the velocity of the fluids at any point in space varies rapidly and randomly with time.
- The critical Reynolds number for the onset of turbulence is in the range 1000 to 10000, depending on the geometry of the flow. For most cases  $R_E < 1000$  signifies laminar flow;  $1000 < R_E < 2000$  is unsteady flow and  $R_e > 2000$  implies turbulent flow.
- In general  **$A\mathbf{v} = \text{constant}$**  which is called the **equation of continuity** and it is a statement of conservation of mass in flow of incompressible fluids.