

## 1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 02 (Physics Part-2, Class XI)
Module Name/Title	Unit 7, Module 6, Pressure Exerted by Fluids Chapter 10, Mechanical Properties of Fluids
Module Id	Keph_201001_econtent
Pre-requisites	Students should have knowledge of force, thrust, pressure exerted by solids, definition and unit of pressure, scalar and vector quantities.
Objectives	After going through this lesson, the learners will be able to: <ul style="list-style-type: none"> <li>• Distinguish between solids and fluids</li> <li>• Define Pressure</li> <li>• Describe Pascal's law</li> <li>• Explain variation of Pressure with depth</li> <li>• Understand Atmospheric pressure and gauge pressure</li> <li>• Apply Pascal's law to explain hydraulic lift and hydraulic brakes</li> </ul>
Keywords	Pressure, Pascal's law, Atmospheric pressure, gauge pressure, hydraulic lift, hydraulic press, hydraulic brakes

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## 1. UNIT SYLLABUS

### UNIT 7: PROPERTIES OF BULK MATTER:

#### Syllabus

#### Chapter–9: Mechanical Properties of Solids:

Elastic behaviour, Stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, shear, modulus of rigidity, Poisson's ratio, elastic energy.

#### Chapter–10: Mechanical Properties of Fluids:

Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes). Effect of gravity on fluid pressure. Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, critical velocity, Bernoulli's theorem and its applications. Surface energy and surface tension, angle of contact, excess of pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise

#### Chapter–11: Thermal Properties of Matter:

Heat, temperature, thermal expansion; thermal expansion of solids, liquids and gases, anomalous expansion of water; specific heat capacity;  $C_p$ ,  $C_v$  - calorimetry; change of state - latent heat capacity. Heat transfer-conduction, convection and radiation, thermal conductivity, qualitative ideas of Blackbody radiation, Wien's displacement Law, Stefan's law, Greenhouse effect.

**2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS****17 Modules**

<b>Module 1</b>	<ul style="list-style-type: none"> <li>● Forces between atoms and molecules making up the bulk matter</li> <li>● Reasons to believe that intermolecular and interatomic forces exist</li> <li>● Overview of unit</li> <li>● State of matter</li> <li>● Study of a few selected properties of matter</li> <li>● Study of elastic behaviour of solids</li> <li>● Stationary fluid property: pressure and viscosity</li> <li>● Stationary liquid property: surface tension</li> <li>● Properties of Flowing fluids</li> <li>● Effect of heat on matter</li> </ul>
<b>Module 2</b>	<ul style="list-style-type: none"> <li>● Idea of deformation by external force</li> <li>● Elastic nature of materials</li> <li>● Elastic behaviour</li> <li>● Plastic behaviour</li> <li>● Tensile stress</li> <li>● Longitudinal Stress and longitudinal strain</li> <li>● Relation between stress and strain</li> <li>● Hooke's law</li> <li>● Young's modulus of elasticity 'Y'</li> </ul>
<b>Module 3</b>	<ul style="list-style-type: none"> <li>● Searle's apparatus</li> <li>● Experiment to determine Young's modulus of the material of a wire in the laboratory</li> <li>● What do we learn from the experiment?</li> </ul>
<b>Module 4</b>	<ul style="list-style-type: none"> <li>● Volumetric strain</li> <li>● Volumetric stress</li> <li>● Hydraulic stress</li> <li>● Bulk modulus K</li> <li>● Fish, aquatic life on seabed, deep sea diver suits and submarines</li> </ul>
<b>Module 5</b>	<ul style="list-style-type: none"> <li>● Shear strain</li> <li>● Shear stress</li> </ul>

	<ul style="list-style-type: none"> <li>● Modulus of Rigidity G</li> <li>● Poisson's ratio</li> <li>● Elastic energy</li> <li>● To study the effect of load on depression of a suitably clamped meter scale loaded at i)its ends ii)in the middle</li> <li>● Height of sand heaps , height of mountains</li> </ul>
Module 6	<ul style="list-style-type: none"> <li>● Fluids-liquids and gases</li> <li>● Stationary and flowing fluids</li> <li>● Pressure due to a fluid column</li> <li>● Pressure exerted by solid , liquids and gases</li> <li>● Direction of Pressure exerted by solids, liquids and gases</li> </ul>
Module 7	<ul style="list-style-type: none"> <li>● Viscosity- coefficient of viscosity</li> <li>● Stokes' Law</li> <li>● Terminal velocity</li> <li>● Examples</li> <li>● Determine the coefficient of viscosity of a given viscous liquid by measuring terminal velocity of a given spherical body in the laboratory</li> </ul>
Module 8	<ul style="list-style-type: none"> <li>● Streamline and turbulent flow</li> <li>● Critical velocity</li> <li>● Reynolds number</li> <li>● Obtaining the Reynolds number formula using method of dimensions</li> <li>● Need for Reynolds number and factors effecting its value</li> <li>● Equation of continuity for fluid flow</li> <li>● Examples</li> </ul>
Module 9	<ul style="list-style-type: none"> <li>● Bernoulli's theorem</li> <li>● To observe the decrease in pressure with increase in velocity of a fluid</li> <li>● Magnus effect</li> <li>● Applications of Bernoulli's theorem</li> <li>● Examples</li> <li>● Doppler test for blockage in arteries</li> </ul>
Module 10	<ul style="list-style-type: none"> <li>● Liquid surface</li> <li>● Surface energy</li> <li>● Surface tension defined through force and through energy</li> </ul>

	<ul style="list-style-type: none"> <li>• Angle of contact</li> <li>• Measuring surface tension</li> </ul>
Module 11	<ul style="list-style-type: none"> <li>• Effects of surface tension in daily life</li> <li>• Excess pressure across a curved liquid surface</li> <li>• Application of surface tension to drops, bubbles</li> <li>• Capillarity</li> <li>• Determination of surface tension of water by capillary rise method in the laboratory</li> <li>• To study the effect of detergent on surface tension of water through observations on capillary rise.</li> </ul>
Module 12	<ul style="list-style-type: none"> <li>• Thermal properties of matter</li> <li>• Heat</li> <li>• Temperature</li> <li>• Thermometers</li> </ul>
Module 13	<ul style="list-style-type: none"> <li>• Thermal expansion</li> <li>• To observe and explain the effect of heating on a bi-metallic strip</li> <li>• Practical applications of bimetallic strips</li> <li>• Expansion of solids, liquids and gases</li> <li>• To note the change in the level of liquid in a container on heating and to interpret the results</li> <li>• Anomalous expansion of water</li> </ul>
Module 14	<ul style="list-style-type: none"> <li>• Rise in temperature</li> <li>• Heat capacity of a body</li> <li>• Specific heat capacity of a material</li> <li>• Calorimetry</li> <li>• To determine specific heat capacity of a given solid material by the method of mixtures</li> <li>• Heat capacities of a gas have a large range</li> <li>• Specific heat at constant volume <math>C_V</math></li> <li>• Specific heat capacity at constant pressure <math>C_P</math></li> </ul>
Module 15	<ul style="list-style-type: none"> <li>• Change of state</li> <li>• To observe change of state and plot a cooling curve for molten wax.</li> <li>• Melting point, Regelation, Evaporation, boiling point,</li> </ul>

	<p>sublimation</p> <ul style="list-style-type: none"> <li>• Triple point of water</li> <li>• Latent heat of fusion</li> <li>• Latent heat of vaporisation</li> <li>• Calorimetry and determination of specific latent heat capacity</li> </ul>
Module 16	<ul style="list-style-type: none"> <li>• Heat Transfer</li> <li>• Conduction, convection, radiation</li> <li>• Coefficient of thermal conductivity</li> <li>• Convection</li> </ul>
Module 17	<ul style="list-style-type: none"> <li>• Black body</li> <li>• Black body radiation</li> <li>• Wien's displacement law</li> <li>• Stefan's law</li> <li>• Newton's law of cooling,</li> <li>• To study the temperature, time relation for a hot body by plotting its cooling curve</li> <li>• To study the factors affecting the rate of loss of heat of a liquid</li> <li>• Greenhouse effect</li> </ul>

**MODULE 6**

**3. WORDS YOU MUST KNOW**

**Matter** -anything that occupies space and has mass

**States of matter**- matter exists in three states solid, liquid and gas . a fourth state called plasma describes any in between state.

**Crystals** -Arrangement of atoms and molecules in different states of matter-atoms and molecules can be arrangement in a patterned way which has distinct separation, bond length, bond energies, bond inclinations; solids with regular arrangement are called crystals. The solids with no patterned stacking are called amorphous.

**Bulk properties of matter**-matter as a whole exhibits certain properties due to arrangement of its molecules

**Mass Density**-mass per unit volume in a specimen of matter .it depends upon temperature.

**Elasticity** –the property of bulk matter by virtue of which it regains its original shape and size once the external deforming forces is removed.

**Plasticity**- the property of bulk matter by virtue of which it does not regain its original shape and size once the external deforming forces is removed.

**Pressure exerted by a solid** –Thrust per unit area at the points of contact is called pressure .SI un it is  $\text{Nm}^{-2}$ , or pascal (Pa). Its dimensional formula is  $\text{M L}^{-1} \text{T}^{-2}$

**Point of contact**-when matter is placed in contact with a container the points of contact refer to points where the surfaces meet

**Scalar quantity**- a physical quantity which is completely described by its magnitude

**Vector quantity** a physical quantity which is completely described by both its magnitude and direction

**Stress:** The restoring force per unit area is known as stress. If F is the force applied and A is the area of cross section of the body,  
Magnitude of the stress =  $F/A$  .The SI unit of stress is  $\text{N m}^{-2}$  or pascal (Pa) .the dimensional formula for stress is  $\text{M L}^{-1} \text{T}^{-2}$

**Strain**-measure of deformity in shape of matter when subjected to external deforming force .it has no unit as it is a ratio. eg longitudinal strain =  $\frac{\text{change in length}}{\text{original length}}$

#### 4. INTRODUCTION

In this module, we shall study some common physical properties of liquids and gases. **Liquids and gases can flow and are therefore, called fluids.** It is this property that distinguishes liquids and gases from solids in a basic way.



Fluids are everywhere around us. Earth has an envelope of air and two-thirds of its surface is covered with water.

Water is not only necessary for our existence; every mammalian body constitute mostly of water. All the processes occurring in living beings including plants are mediated by fluids.

Thus understanding the behaviour and properties of fluids is important.

<https://static.pexels.com/photos/2422/sky-earth-galaxy-universe.jpg>

**How are fluids different from solids? What is common in liquids and gases?**

Unlike a solid, a fluid has no definite shape of its own. Solids and liquids have a fixed volume, whereas a gas fills the entire volume of its container. We have learnt in the previous modules that the volume of solids can be changed by stress. The volume of solid, liquid or gas depends on the stress or pressure acting on it. When we talk about fixed volume of solid or liquid, we mean its volume under atmospheric pressure. The difference between gases and solids or liquids is that for solids or liquids the change in volume due to change of external pressure is rather small. In other words solids and liquids have much lower compressibility as compared to gases.

Shear stress can change the shape of a solid keeping its volume fixed. The key property of fluids is that they offer very little resistance to shear stress; their shape changes by application of very small shear stress. The shearing stress of fluids is about million times smaller than that of solids.

The fluid flow is due to shear forces generated between different layers. we will consider this in detail in later modules.

**5. PRESSURE**

A sharp injection needle when pressed against our skin pierces it. Notice the flat end where the medical practitioner pushes it





<https://www.google.com/search?site=imghp&tbm=isch&q=syringe&tbs=sur:fmc#imgrc=HZHWm03scsP1JM>:

Our skin, however, remains intact when a blunt object with a wider contact area (say the back of a spoon) is pressed against it with the same force.



[https://www.google.com/search?site=imghp&tbm=isch&q=blunt%20knife&tbs=sur:fmc#imgrc=3UBH7-R\\_MuRNNM](https://www.google.com/search?site=imghp&tbm=isch&q=blunt%20knife&tbs=sur:fmc#imgrc=3UBH7-R_MuRNNM):

If an elephant were to step on a man's chest, his ribs would crack. A circus performer across whose chest a large, light but strong wooden plank is placed first, is saved from this accident.

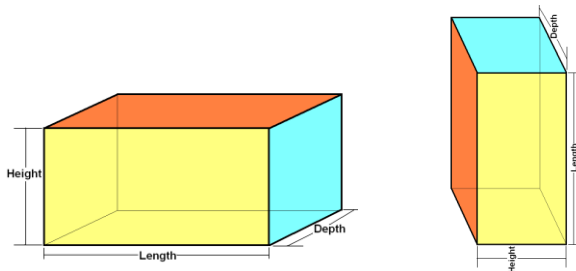
Such everyday experiences convince us that both the force and its coverage area are important. Smaller the area on which the thrust acts, greater is the pressure. This concept is known as pressure.

$$\text{pressure} = \frac{\text{Thrust}}{\text{area}}$$

**Consider these solids**

**Read the conditions and predict the pressure in the following cases**

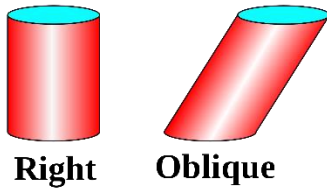
**a) A cuboid block of wood exerts different pressures, when it rests on different surfaces**



<https://upload.wikimedia.org/wikipedia/commons/d/dc/Cuboid.png>

- In which case would the pressure be the more and why?
- Is the force/thrust /normal force on the surface on which the cuboid rests the same in the two cases? Give reasons for your answer.

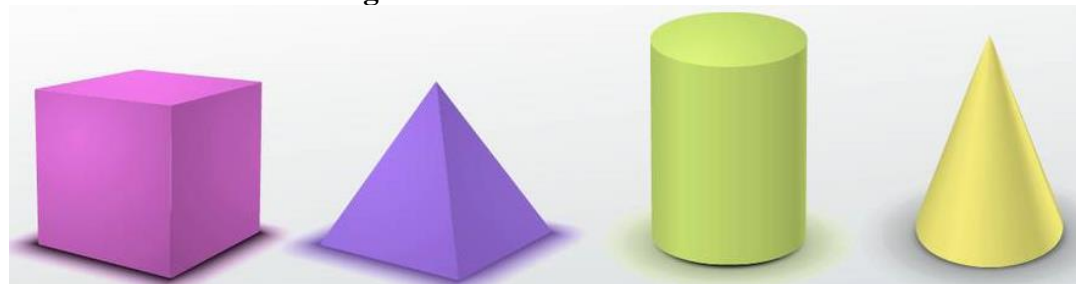
b) The mass of right and oblique blocks is the same. what about pressure?



c) The three cylinders are different in height, their base areas are the same. Predict the pressure exerted on the base, Think of a condition when the pressure will be the same in each case.



d) State the condition when all the differently shaped blocks would exert the same pressure. What if the cylinder is hollow, then, under what condition will it exert the same pressure AS OTHER SOLIDS in the figure



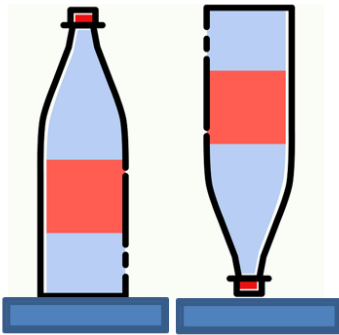
<https://study.com/cimages/videopreview/rirubt9efn.jpg>

e) What about the hour glass timer?



Do you think the pressure will change when sand is dropping from the top container to the lower container?

f) Would the pressure be different due to a water bottle placed in the two different ways, as shown

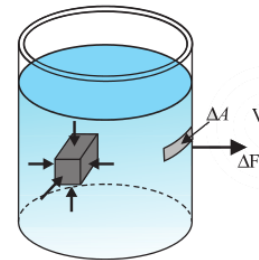


### FLUID PRESSURE

When we put our hand inside a bucket of water, we feel the force by the water on our hand. What is its direction? What is the pressure due to this force?

When an object is submerged in a **fluid at rest**, the fluid exerts a force on its surface. **This force is always normal to the object's surface.**

This is so because if there were a component of force parallel to the surface, the object will also exert a force on the fluid parallel to it; as a consequence of Newton's third law. This force will cause the fluid to flow parallel to the surface.



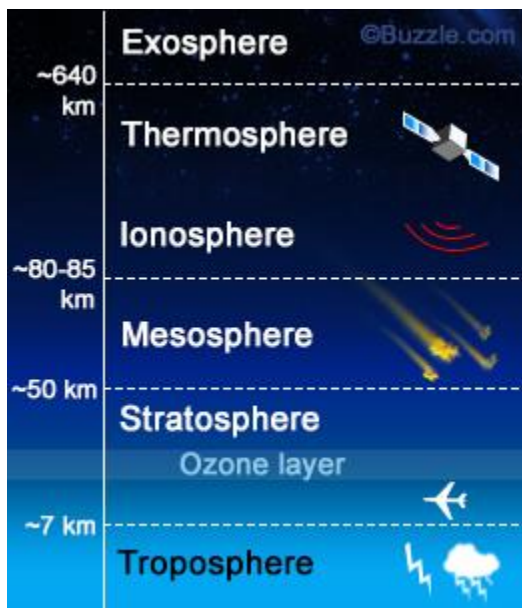
Since the fluid is at rest, this cannot happen.

Hence, the force exerted by the fluid at rest has to be perpendicular to the surface in contact with it. This is shown in Figure.

If a plastic ball is pushed inside the bucket of water and released, it rises to the surface again, the water (liquid) exerts this pressure in the upward direction

**A normal force acts on any object immersed in a container of liquid /fluid. Normal force is also exerted at every point on the surface of contact between the fluid and the container.**

Study the extent of atmosphere around the earth



<https://i.pinimg.com/originals/b9/87/a7/b987a7b604591c818eec51e9d6b197cd.jpg>

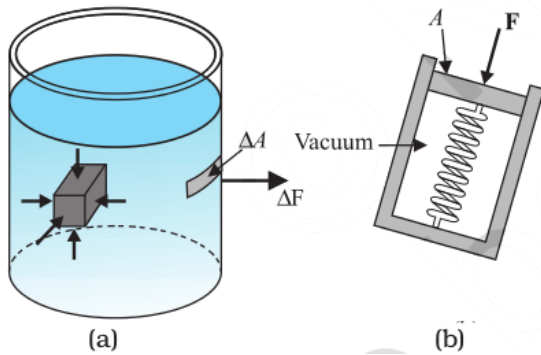
The atmospheric pressure is due to the blanket of air around the earth.

### MEASUREMENT OF FLUID PRESSURE

The normal force exerted by the fluid at a point may be measured.

An idealized form of one such pressure-measuring device is shown in Fig.

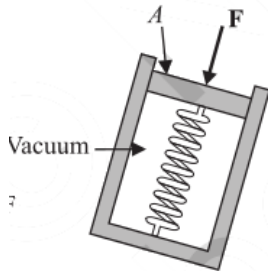
It consists of an evacuated chamber with a spring that is calibrated to measure the force acting on the piston. This device is placed at a point inside the fluid.



- (a) The force exerted by the liquid in the beaker on the submerged object or on the walls is normal (perpendicular) to the surface at all points
- (b) An idealized device for measuring pressure.

The inward force exerted by the fluid on the piston is balanced by the outward spring force

and is thereby measured.



If  $F$  is the magnitude of this normal force on the piston of area  $A$  then the average pressure  $P_{av}$  is defined as the normal force acting per unit area.

$$P_{av} = \frac{F}{A}$$

In principle, the piston area can be made arbitrarily small. The pressure is then defined in a limiting sense as

$$P = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A}$$

**Pressure is a scalar quantity?**

**Remember that it is the component of the force normal to the area under consideration and not the (vector) force that appears in the numerator in Equations**

**Its dimensions are  $[ML^{-1}T^{-2}]$ .**

**The SI unit of pressure is  $N\ m^{-2}$ .**

It has been named as pascal (Pa) in honour of the **French scientist Blaise Pascal (1623-1662)** who carried out pioneering studies on fluid pressure.

A common unit of pressure is the atmosphere (atm), i.e. the pressure exerted by the atmosphere at sea level ( $1\ atm = 1.013 \times 10^5\ Pa$ ).

**DENSITY**

Another quantity, that is indispensable in describing fluids, is the density  $\rho$ . For a fluid of mass  $m$  occupying volume  $V$ , so mass per unit volume

$$\rho = \frac{M}{V}$$

**Units and dimensions of density**

The dimensions of density are  $[ML^{-3}]$ .

Its SI unit is  $\text{kg m}^{-3}$ .

**It is a scalar quantity.**

A liquid is largely incompressible and its density is therefore, nearly constant at all pressures. Gases, on the other hand exhibit a large variation in densities with pressure.

**RELATIVE DENSITY**

The density of water at  $4^\circ\text{C}$  (277 K) is  $1.0 \times 10^3 \text{ kg m}^{-3}$ .

The relative density of a substance is the ratio of its density to the density of water at  $4^\circ\text{C}$ .

It is a dimensionless positive scalar quantity. For example the relative density of aluminium is 2.7.

Its density is  $2.7 \times 10^3 \text{ kg m}^{-3}$ .

The densities of some common fluids are displayed in Table densities of some common fluids at stp\*

**DENSITIES OF SOME COMMON FLUIDS AT STP\***

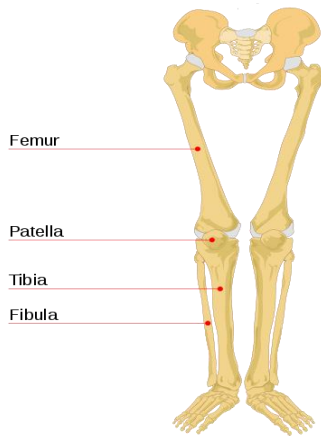
Fluid	$\rho (\text{kg m}^{-3})$
Water	$1.00 \times 10^3$
Sea water	$1.03 \times 10^3$
Mercury	$13.6 \times 10^3$
Ethyl alcohol	$0.806 \times 10^3$
Whole blood	$1.06 \times 10^3$
Air	1.29
Oxygen	1.43
Hydrogen	$9.0 \times 10^{-2}$
Interstellar space	$\approx 10^{-20}$

*STP means standard temperature (0 °C) and 1 atmospheric pressure.*

**Do you think** density and relative density would play any role in pressure exerted by a fluid?

**EXAMPLE**

The two thigh bones (femurs), each of cross-sectional area  $10 \text{ cm}^2$  support the upper part of a human body of mass  $40 \text{ kg}$ . Estimate the average pressure sustained by the femurs.



[https://commons.wikimedia.org/wiki/File:Human\\_leg\\_bones\\_labeled.svg](https://commons.wikimedia.org/wiki/File:Human_leg_bones_labeled.svg)

**SOLUTION**

Total cross-sectional area of the femurs is  $A = 2 \times 10 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$ . The force acting on them is  $F = 40 \text{ kg wt} = 400 \text{ N}$  (taking  $g = 10 \text{ m s}^{-2}$ ). This force is acting vertically down and hence, normally on the femurs. Thus, the average pressure is

$$P_{\text{av}} = \frac{F}{A} = 2 \times 10^5 \text{ N m}^{-2}$$

**EXAMPLE**

A  $50 \text{ kg}$  girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter  $1.0 \text{ cm}$ . What is the pressure exerted by the heel on the horizontal floor?

**SOLUTION**

Weight of the girl ( $W$ ) = mass  $\times$  acc due to gravity

$$w = 50 \text{ kg} \times 9.8 \text{ m s}^{-2} = 490 \text{ N}$$

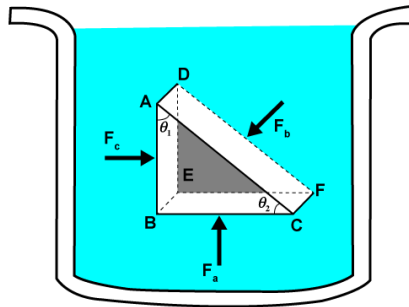
$$\text{Area of the heel (A)} = \pi r^2 = 3.14 \times (0.5 \times 0.5) \text{cm}^2 = 3.14 \times 0.5 \times 10^{-4} \text{m}^2$$

$$\text{Pressure} = \frac{W}{A} = \frac{490 \text{N}}{3.14 \times 0.5 \times 0.5} \times 10^4 = 624.2 \times 10^4 \text{Nm}^2 = \mathbf{6.24 \times 10^6 \text{Nm}^2}$$

## 6. PASCAL'S LAW

**The French scientist Blaise Pascal observed that the pressure in a liquid at rest is the same at all points if they are at the same depth below the free surface.**

This fact may be demonstrated in a simple way.



**The figure shows an element in the interior of a fluid at rest. This element ABC-DEF is in the form of a right-angled prism.**

### Proof of Pascal's law:

ABC-DEF is an element of the interior of a fluid at rest. This element is in the form of a right-angled prism. The element is small so that the effect of gravity can be ignored, but it has been enlarged for the sake of clarity.

In principle, this prismatic element is very small so that every part of it can be considered at the same depth from the liquid surface and therefore, the effect of the gravity is the same at all these points. But for clarity we have enlarged this element.

The forces on this element are those exerted by the rest of the fluid and they must be normal to the surfaces of the element as discussed above.

Thus, the fluid exerts pressures  $P_a$ ,  $P_b$  and  $P_c$  on this element of area corresponding to the normal forces  $F_a$ ,  $F_b$  and  $F_c$  as shown in Figure, on the faces BEFC, ADFC and ADEB denoted by  $A_a$ ,  $A_b$  and  $A_c$  respectively. Then

$$F_b \sin \theta = F_c, F_b \cos \theta = F_a \text{ (by equilibrium)}$$



$A_b \sin \theta = A_c$ ,  $A_b \cos \theta = A_a$  (by geometry)

Thus,

$$\frac{F_b}{A_b} = \frac{F_c}{A_c} = \frac{F_a}{A_a}; \quad P_b = P_c = P_a$$

Hence, **pressure exerted at a level is same in all directions in a fluid at rest.**

It again reminds us that like other types of stress, pressure is not a vector quantity.

No direction can be assigned to it. The force against any area within (or bounding) a fluid at rest and under pressure is normal to the area, regardless of the orientation of the area.

**Now consider a fluid element in the form of a horizontal bar of uniform cross-section. The bar is in equilibrium. The horizontal forces exerted at its two ends must be balanced or the pressure at the two ends should be equal.**

**This proves that for a liquid in equilibrium the pressure is same at all points in a horizontal plane.**

Suppose the pressure were not equal in different parts of the fluid, then there would be a flow as the fluid will have some net force acting on it.

Hence in the absence of flow or in stationary fluid, the pressure in the fluid must be same everywhere. Wind is flow of air due to pressure differences.

**Pour some water in tumbler, tilt it and see that the surface always remains horizontal.**

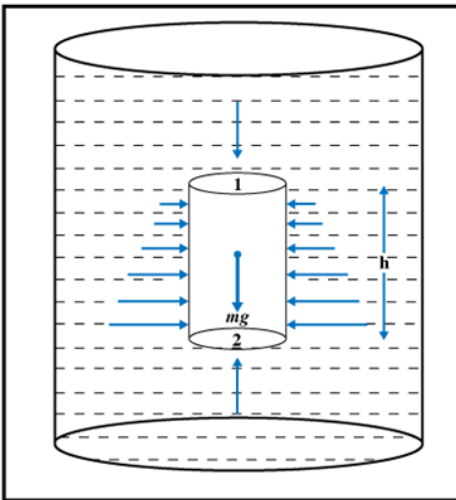


[https://encrypted-tbn0.gstatic.com/images?q=tbn:ANd9GcT7qGc8CSnQQ4i5F\\_1-ULvP2\\_Yu9FJ3uTZt3TFdhds9AOcMHZQx](https://encrypted-tbn0.gstatic.com/images?q=tbn:ANd9GcT7qGc8CSnQQ4i5F_1-ULvP2_Yu9FJ3uTZt3TFdhds9AOcMHZQx)

Try the same with a transparent water bottle. Cap it and place it horizontally on any surface, the water surface will always be horizontal.

## 7. VARIATION OF PRESSURE WITH DEPTH

Consider a fluid at rest in a container. In Figure



**Point 1 is at height 'h' above a point 2.**

The pressures at points 1 and 2 are  $P_1$  and  $P_2$  respectively. Consider a cylindrical element of fluid having area of base  $A$  and height  $h$ . As the fluid is at rest the resultant horizontal forces should be zero and the resultant vertical forces should balance the weight of the element.

**Here though we are using the term fluid, we are explaining using a liquid only.**

**Fluid under gravity. The effect of gravity is illustrated through pressure on a vertical cylindrical column.**

The **forces acting in the vertical direction** are due to the fluid pressure at the top ( $P_1A$ ) acting downward, at the bottom ( $P_2A$ ) acting upward.

If  $mg$  is weight of the fluid in the cylinder we have

$$(P_2 - P_1) A = mg$$

Now, if  $\rho$  is the mass density of the fluid, we have the mass of fluid to be

$$m = \rho V = \rho hA$$

so that

$$P_2 - P_1 = \rho gh$$

**Pressure difference depends on the vertical distance  $h$  between the points (1 and 2), mass density of the fluid  $\rho$  and acceleration due to gravity  $g$ .**

If the point 1 under discussion is shifted to the top of the fluid (say water), which is open to the atmosphere,  $P_1$  may be replaced by atmospheric pressure ( $P_a$ ) and we replace  $P_2$  by  $P$ . gives

$$P = P_a + \rho gh$$

Thus, the pressure  $P$ , at depth below the surface of a liquid open to the atmosphere is greater than atmospheric pressure by an amount  $\rho gh$ .

The excess of pressure,  $P - P_a$ , at depth  $h$  is called a **gauge pressure** at that point.

### HYDROSTATIC PARADOX

The area of the cylinder is not appearing in the expression of absolute pressure in Eq. (10.7). Thus, the height of the fluid column is important and not cross sectional or base area or the shape of the container. **The liquid pressure is the same at all points at the same horizontal level (same depth below the free surface).**

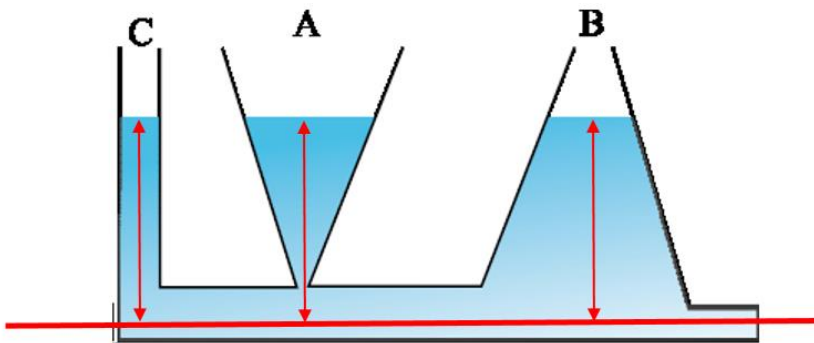
The result is appreciated through the example of **hydrostatic paradox**.

Consider three vessels A, B and C as shown in the figure of different shapes.

They are connected at the bottom by a horizontal pipe.

On filling with water the level in the three vessels is the same though they hold different amounts of water.

This is so, because water at the bottom has the same pressure below each section of the vessel.



**Illustration of hydrostatic paradox. The three vessels A, B and C contain different amounts of liquids, all upto the same height.**

**NOTE** The amount of water in each vessel is different

**EXAMPLE**

What is the pressure on a swimmer 10 m below the surface of a lake?

**SOLUTION**

Here

$h = 10 \text{ m}$  and  $\rho = 1000 \text{ kg m}^{-3}$ . Take  $g = 10 \text{ m s}^{-2}$

$$P = P_a + \rho gh$$

$$= 1.01 \times 10^5 \text{ Pa} + 1000 \text{ kg m}^{-3} \times 10 \text{ m s}^{-2} \times 10 \text{ m}$$

$$= 2.01 \times 10^5 \text{ Pa}$$

$$\approx 2 \text{ atm}$$

**This is a 100% increase in pressure from surface level.**

**At a depth of 1 km the increase in pressure is 100 atm!**

Submarines are designed to withstand such enormous pressures.

**EXAMPLE**

Torricelli's barometer used mercury. Pascal duplicated it using French wine of density  $984 \text{ kg m}^{-3}$ .

Determine the height of the wine column for normal atmospheric pressure.

**SOLUTION**

Atmospheric pressure  $= 1.01 \times 10^5 \text{ Pa}$

So we need to calculate **what length of tube filled with wine will exert the same pressure!**

$$P = h d g$$

$$h = \frac{p}{dg} = \frac{1.01 \times 10^5 \text{ Nm}^2}{984 \text{ kgm}^{-3} \times 9.8 \text{ ms}^{-2}} = 10.5 \text{ m}$$

**Also calculate the length of a tube required to give the same pressure as wine at the base of the tube. Density of water is  $1000 \text{ kg m}^{-3}$**

**EXAMPLE**

A vertical off-shore structure is built to withstand a maximum stress of  $10^9$  Pa. Is the structure suitable for putting up on top of an oil well in the ocean?

Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.

**SOLUTION**

Pressure due to 3 km water column = 3 000 x density of sea water x  $9.8 \text{ ms}^{-2}$

$$3000m \times 1.03 \times 10^3 \text{kgm}^3 \times 9.8\text{ms}^{-2} = 30.282 \times 10^6 \text{Pa}$$

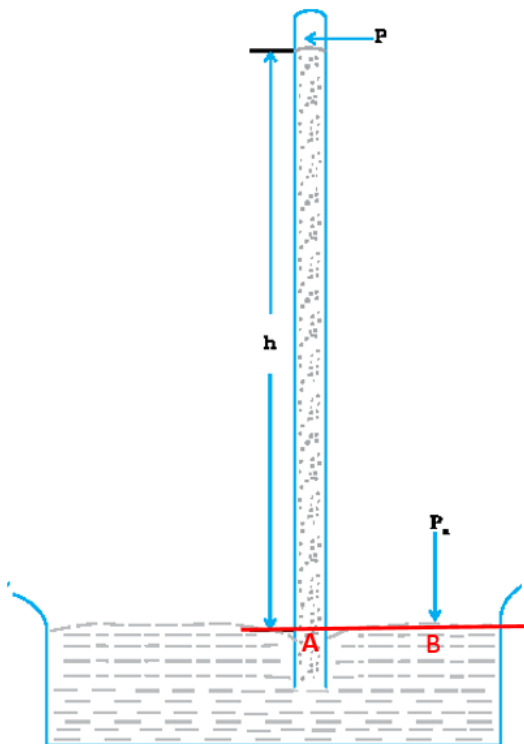
The structure is safe as it can withstand far greater pressure or stress

**8. ATMOSPHERIC PRESSURE AND GAUGE PRESSURE**

The pressure of the atmosphere at any point is equal to the weight of a column of air of unit cross sectional area extending from that point to the top of the atmosphere.

At sea level it is  $1.013 \times 10^5$  Pa (1 atm).

**MERCURY BAROMETER**



The mercury barometer.

Italian scientist **Evangelista Torricelli (1608-1647)** devised for the first time, a method for measuring atmospheric pressure.

A long glass tube closed at one end and filled with **mercury** is inverted into a trough of mercury as shown in Figure. **This device is known as mercury barometer.**

**The space above the mercury column in the tube is nearly a vacuum, contains only mercury vapour whose pressure  $P$  is so small that it may be neglected.**

**The pressure inside the column at point A must equal the pressure at point B, which is at the same level. Pressure at B = atmospheric pressure =  $P_a$**

$$P_a = \rho gh$$

where  $\rho$  is the density of mercury and  
 **$h$  is the height of the mercury column inside the tube.**

In experiments, it is found that the mercury column in the barometer has a height of about **76 cm** at sea level equivalent to **one atmosphere (1 atm)**.

This can also be obtained using the value of  $\rho$  in equation  $P_a = \rho gh$

A **common way** of stating pressure is in terms of cm or mm of mercury (Hg).

**A pressure equivalent of 1 mm is called a torr (after Torricelli).**

$$1 \text{ torr} = 133 \text{ Pa.}$$

**The mm of Hg and torr are used in medicine and physiology.**

**In meteorology, a common unit is the bar and millibar.**

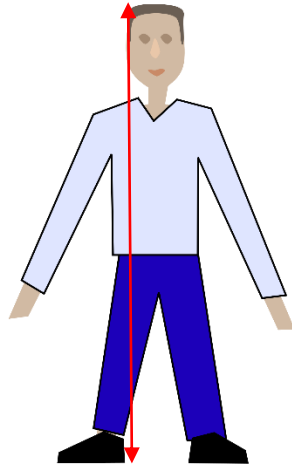
$$1 \text{ bar} = 10^5 \text{ Pa}$$

### **EXPLAIN WHY**

(a) We use mercury as barometric liquid, when water is cheaper and easily available

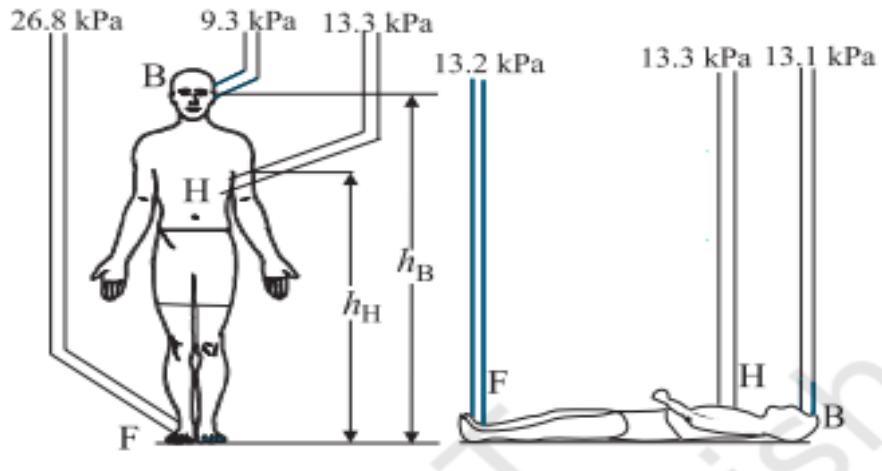
(b) The blood pressure in humans is greater at the feet than at the brain.

**Hint**



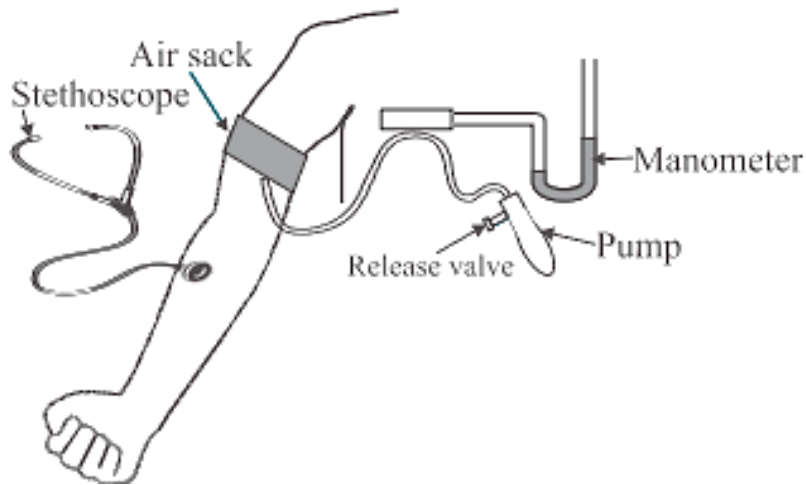
**WHAT IS BLOOD PRESSURE?**

In evolutionary history there occurred a time when animals started spending a significant amount of time in the upright position. This placed a number of demands on the circulatory system. The venous system that returns blood from the lower extremities to the heart underwent changes. You will recall that veins are blood vessels through which blood returns to the heart. Humans and animals such as the giraffe have adapted to the problem of moving blood upward against gravity. But animals such as snakes, rats and rabbits will die if held upwards, since the blood remains in the lower extremities and the venous system is unable to move it towards the heart.



Schematic view of the gauge pressures in the arteries in various parts of the human body while standing or lying down. The pressures shown are averaged over a heart cycle.

Figure shows the average pressures observed in the arteries at various points in the human body.



**There are two reasons why the upper arm is used.**

First, it is at the same level as the heart and measurements here give values close to that at the heart.

Secondly, the upper arm contains a single bone and makes the artery there (called the brachial artery) easy to compress.

We have all measured pulse rates by placing our fingers over the wrist. Each pulse takes a little less than a second. During each pulse the pressure in the heart and the circulatory system goes through a maximum as the blood is pumped by the heart (systolic pressure) and a minimum as the heart relaxes (diastolic pressure).

The **sphygmomanometer** (blood pressure instrument) is a device, which measures these extreme pressures.

The gauge pressure in an air sack wrapped around the upper arm is measured using a manometer or a dial pressure gauge.

**The blood pressure of a patient is presented as the ratio of systolic/diastolic pressures. For a resting healthy adult it is typically 120 / 80 mm of Hg (120 / 80 torr).**

Pressures above 140/90 require medical attention and advice. High blood pressures may seriously damage the heart, kidney and other organs and must be controlled.



- (c) **Atmospheric pressure at a height of about 6 km decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than 100 km**

**Hint:** density of air is not uniform, air gets rarefied with altitude

- (d) **Hydrostatic pressure is a scalar quantity even though pressure is force divided by area.**

**Hint** at a point hydrostatic pressure is the same in all directions

- (e) **Atmosphere pressure =  $1.013 \times 10^5 \text{ Pa}$  =  $1.013 \times 10^5 \text{ Nm}^{-2}$**

**this is equivalent to pressure exerted by 100 people of 100 kg weight ( equal to approximately 1000N ) each,**

**Imagine 100 persons standing in  $\text{m}^2$  area the pressure is very large. Why do we not feel this pressure?**

**Hint** pressure at a level is the same in all directions hence we do not feel the pressure.

- (e) **What can you say about human blood pressure as compared to atmospheric pressure?**

Blood pressure is slightly higher than atmospheric pressure, this keeps the body in shape.



<https://www.buckley.af.mil/News/Photos/igphoto/2000063823/>

blood oozes out slowly after pin prick , as the blood pressure is only slightly greater than atmospheric pressure

- (f) **Why do mountain climbers sometimes bleed through their nose and ears at high altitude?**

Atmospheric pressure decreases with altitude making the blood pressure much higher than the outside atmospheric pressure the human body is exposed to this results in bleeding.

- g) **Why do dams have broad bases?**



[https://commons.wikimedia.org/wiki/File:Sardar Sarovar Dam 2006, India.jpg](https://commons.wikimedia.org/wiki/File:Sardar_Sarovar_Dam_2006,India.jpg)

### OPEN-TUBE MANOMETER

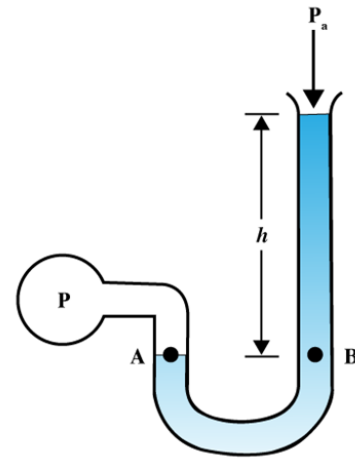
**An open-tube manometer is a useful instrument for measuring pressure differences.**

It consists of a U-tube containing a suitable liquid i.e. a low density liquid (such as oil) for measuring small pressure differences and a high density liquid (such as mercury) for large pressure differences. Why?

One end of the tube is open to the atmosphere and other end is connected to the system whose pressure we want to measure [see Fig].

The pressure  $P$  at A is equal to pressure at point B.  
What we normally measure is the gauge pressure, which is  $P - P_a$ , given by Eq.

$P - P_a = h d g$   
and is proportional to manometer height  $h$ .



The open tube manometer

**Pressure is same at the same level on both sides of the U-tube containing a fluid.**

For liquids the density varies very little over wide ranges in pressure and temperature and we can treat it safely as a constant for our present purposes.

Gases on the other hand, exhibits large variations of densities with changes in pressure and temperature. **Unlike gases, liquids are therefore, largely treated as incompressible.**

**EXAMPLE**

The density of the atmosphere at sea level is  $1.29 \text{ kg/m}^3$ . Assume that it does not change with altitude. Then how high would the atmosphere extend?

**SOLUTION**

We use  $\rho gh = 1.29 \text{ kg m}^{-3} \times 9.8 \text{ m s}^{-2} \times h \text{ m} = 1.01 \times 10^5 \text{ Pa}$   
 $\therefore h = 7989 \text{ m} \approx 8 \text{ km}$

In reality the density of air decreases with height.

So does the value of  $g$ .

The atmospheric cover extends with decreasing pressure over 100 km.

We should also note that the sea level atmospheric pressure is not always 760 mm of Hg. A drop in the Hg level by 10 mm or more is a sign of an approaching storm.

**EXAMPLE**

At a depth of 1000 m in an ocean

- What is the absolute pressure?
- What is the gauge pressure?
- Find the force acting on the window of area  $20 \text{ cm} \times 20 \text{ cm}$  of a submarine at this depth, the interior of which is maintained at sea-level atmospheric pressure.

(The density of sea water is  $1.03 \times 10^3 \text{ kg m}^{-3}$ ,  $g = 10 \text{ m s}^{-2}$ .)

**SOLUTION**

Here  $h = 1000 \text{ m}$  and  $\rho = 1.03 \times 10^3 \text{ kg m}^{-3}$ .

- From Equation for absolute pressure

$$\begin{aligned} P &= P_a + \rho gh \\ &= 1.01 \times 10^5 \text{ Pa} \end{aligned}$$

$$\begin{aligned}
 &+ 1.03 \times 10^3 \text{ kg m}^{-3} \times 10 \text{ m s}^{-2} \times 1000 \text{ m} \\
 &= 104.01 \times 10^5 \text{ Pa} \\
 &\approx \mathbf{104 \text{ atm}}
 \end{aligned}$$

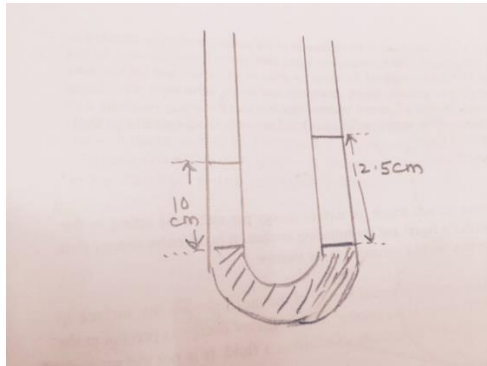
(b) Gauge pressure is  $P - P_a = \rho gh = P_g$   
 $P_g = 1.03 \times 10^3 \text{ kg m}^{-3} \times 10 \text{ ms}^2 \times 1000 \text{ m}$   
 $= 103 \times 10^5 \text{ Pa}$   
 $\approx \mathbf{103 \text{ atm}}$

(c) The pressure outside the submarine is  $P = P_a + \rho gh$  and the pressure inside it is  $P_a$ .  
Hence, the net pressure acting on the window is gauge pressure,  $P_g = \rho gh$ .  
Since the area of the window is  $A = 0.04 \text{ m}^2$ ,  
the force acting on it is  
 $F = P_g A = 103 \times 10^5 \text{ Pa} \times 0.04 \text{ m}^2 = \mathbf{4.12 \times 10^5 \text{ N}}$

**EXAMPLE**

**A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit?**

**SOLUTION**



Let the density of the liquid be  $d$   
The pressure on the top of mercury in both the tubes is the same  
 $h_w d_w g = h_l d_l g$   
 $\therefore d_l = h_w / h_l = 10 / 12.5 = \mathbf{0.8}$

**EXAMPLE**

In the previous problem, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms? (Specific gravity of mercury = 13.6)

**SOLUTION**

On pouring 15 cm of water and spirit each into the respective arms of the tube, the height of water and spirit in the two arms will become  $10 + 15 = 25$  cm and  $12.5 + 15 = 27.5$  cm, since the density of water is greater than that of spirit, mercury will rise in the arm containing spirit.

Let the difference in level of mercury be  $h$

Pressure exerted by 25 cm of water column = pressure exerted by 27.5 cm of spirit column + pressure exerted by  $h$  cm of mercury column

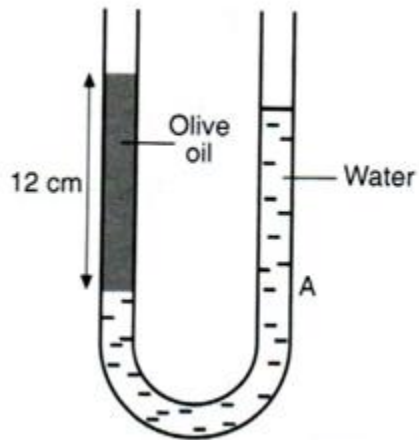
$$25 \times 1 \times g = 27.5 \times 0.8 \times g + h \times 13.6 \times g$$

$$25 = 22 + 13.6 h$$

$$h = \frac{25 - 22}{13.6} = 0.22 \text{ cm}$$

**EXAMPLE**

You could have oil and water in a U tube as shown



A U-tube with 30 cm of water is set up vertically and 12 cm of olive oil is poured carefully into the left-hand limb of the tube. It is observed that water will be pushed up in the other limb. The density of water and mustard oil are  $1000 \text{ kg m}^{-3}$  and  $920 \text{ kg m}^{-3}$  respectively. Given A is at same level as the boundary between olive oil and water.

- a) What is the length of water column above point A  
 b) An additional 6 cm of olive oil is added into the left-hand limb. How much further will the water level rise in the right limb?

**SOLUTION**

a)  $P_{\text{mustard oil}} = P_{\text{water}}$

$$h d g = 0.12 \text{ m} \times 10 \text{ ms}^{-2} \times 920 \text{ kg m}^{-3} = h \times 10 \text{ ms}^{-2} \times 1000 \text{ kg m}^{-3}$$

$$h = \frac{0.12 \text{ m} \times 920 \text{ kg m}^{-3}}{1000 \text{ kg m}^{-3}} = 0.1104 \text{ m} = \mathbf{11.04 \text{ cm}}$$

b) new height of water in the right limb of the U tube when 6 cm of olive oil is added to the left limb

$$h' = \frac{0.18 \text{ m} \times 920 \text{ kg m}^{-3}}{1000 \text{ kg m}^{-3}} = \mathbf{16.56 \text{ cm}}$$

Also watch

<https://www.youtube.com/watch?v=qG6cfwQBhHM>

**9. HYDRAULIC MACHINES**

Let us now consider **what happens when we change the pressure on a fluid contained in a vessel.**

Consider a horizontal cylinder with a piston and three vertical tubes at different points. The pressure in the horizontal cylinder is indicated by the height of liquid column in the vertical tubes. It is necessarily the same in all. If we push the piston, the fluid level rises in all the tubes, again reaching the same level in each one of them.

This indicates that when the pressure on the cylinder was increased, it was distributed uniformly throughout.

We can say whenever external pressure is applied on any part of a fluid contained in a vessel, it is transmitted undiminished and equally in all directions.

**This is the Pascal's law for transmission of fluid pressure and has many applications in daily life.**

A number of devices such as **hydraulic pressure, hydraulic lift and hydraulic brakes** are based on the Pascal's law. In these devices fluids are used for transmitting pressure.

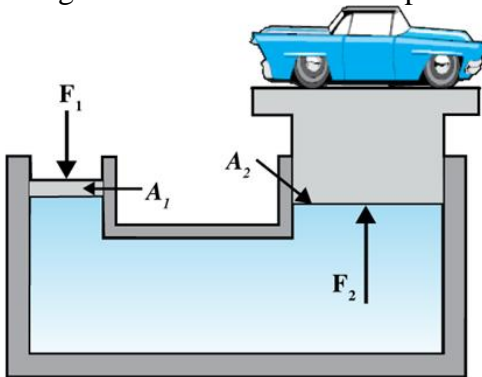
In a hydraulic lift as shown in Figure

two pistons are separated by the space filled with a liquid.

A piston of small cross section  $A_1$  is used to exert a force  $F_1$  directly on the liquid.

The pressure  $P = \frac{F_1}{A_1}$  is transmitted throughout the liquid to the larger cylinder attached with a larger piston of area  $A_2$ , which results in an upward force of  $P \times A_2$ . Therefore, the piston is capable of supporting a large force (large weight of, say a car, or a truck, placed on the platform)  $F_2 = PA_2 = \frac{F_1 A_2}{A_1}$ . By changing the force at  $A_1$ , the platform can be moved up or down.

Thus, the applied force has been increased by a factor of  $\frac{A_2}{A_1}$  and this factor is the mechanical advantage of the device. The example below clarifies it.



Schematic diagram illustrating the principle behind the hydraulic lift, a device used to lift heavy loads.

These videos show Pascal's Law and Hydraulic Brake System Working animation

<https://youtu.be/WSWHgXZqjD4>

<https://youtu.be/d66EiKwySt4>

### EXAMPLE

Two syringes of different cross sections (without needles) filled with water are connected with a tightly fitted rubber tube filled with water. Diameters of the smaller piston and larger piston are 1.0 cm and 3.0 cm respectively.

(a) Find the force exerted on the larger piston when a force of 10 N is applied to the smaller piston.

(b) If the smaller piston is pushed in through 6.0 cm, how much does the larger piston move out?

**SOLUTION**

(a) Since pressure is transmitted undiminished throughout the fluid,

$$F_2 = \frac{A_2}{A_1} F_1 = \frac{\pi(3/2 \times 10^{-2} m)^2}{\pi(1/2 \times 10^{-2} m)^2} \times 10 \text{ N}$$

$$= 90 \text{ N}$$

(b) Water is considered to be perfectly incompressible. Volume covered by the movement of smaller piston inwards is equal to volume moved outwards due to the larger piston.

$$L_2 = \frac{A_1}{A_2} L_1 = \frac{\pi(1/2 \times 10^{-2} m)^2}{\pi(3/2 \times 10^{-2} m)^2} \times 6 \times 10^{-2} m$$

$$\varphi 0.67 \times 10^{-2} m = 0.67 \text{ cm}$$

**Note, atmospheric pressure is common to both pistons and has been ignored.**

**EXAMPLE**

**In a car lift compressed air exerts a force  $F_1$  on a small piston having a radius of 5.0 cm. This pressure is transmitted to a second piston of radius 15 cm. If the mass of the car to be lifted is 1350 kg, calculate  $F_1$ .**

**What is the pressure necessary to accomplish this task?**

**( $g = 9.8 \text{ ms}^{-2}$ ).**

**SOLUTION**

Since pressure is transmitted undiminished throughout the fluid,

$$F_1 = \frac{A_1}{A_2} F_2 = \frac{\pi(5 \times 10^{-2} m)^2}{\pi(15 \times 10^{-2} m)^2} \times (1350 \text{ N} \times 9.8 \text{ m s}^{-2})$$

$$= 1470 \text{ N}$$

$$\approx 1.5 \times 10^3 \text{ N}$$

The air pressure that will produce this force is

$$P = \frac{F_1}{A_1} = \frac{1.5 \times 10^3 \text{ N}}{\pi(5 \times 10^{-2} m)^2} = 1.9 \times 10^5 \text{ Pa}$$

This is almost double the atmospheric pressure.



**10. SUMMARY**

- The basic property of a fluid is that it can flow. The fluid does not have any resistance to change of its shape. Thus, the shape of a fluid is governed by the shape of its container.
- A liquid is incompressible and has a free surface of its own. A gas is compressible and it expands to occupy all the space available to it.
- If  $F$  is the normal force exerted by a fluid on an area  $A$  then the average pressure  $P_{av}$  is defined as the ratio of the force to area

$$P_{av} = \frac{F}{A}$$

- The unit of the pressure is the pascal (Pa). It is the same as  $\text{N m}^{-2}$ . Other common units of pressure are  
 $1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$   
 $1 \text{ bar} = 10^5 \text{ Pa}$   
 $1 \text{ torr} = 133 \text{ Pa} = 0.133 \text{ kPa}$   
 $1 \text{ mm of Hg} = 1 \text{ torr} = 133 \text{ Pa}$
- *Pascal's law* states that: Pressure in a fluid at rest is same at all points which are at the same height. A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel.
- The pressure in a fluid varies with depth  $h$  according to the expression  
 $P = P_a + \rho gh$   
 where  $\rho$  is the density of the fluid, assumed uniform.