1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part-1, Class XI)
Module Name/Title	Unit 6, Module 1, Kepler's laws ofplanetary Motion Chapter 8,Gravitation
Module Id	Keph_10801_eContent
Pre-requisites	Knowledge of solar system, vectors and vector addition, linear momentum, angular momentum and torque
Objectives	After going through this lesson, the learners will be able to: • **Know** Historical perspective of the planetary motion • **Understand** the relation between the period (or speed) of planets and their distance from the sun. • **State**Kepler's laws of planetary motion and **generalise** these laws to the motion of the satellite of planets. • **Apply** Newton's Universal law of gravitation and use it to gauge the magnitude of the gravitational force between smaller and larger masses • **Apply** law of gravitation to find the resultant gravitational force due to various distribution of masses using the principle of superposition involving vector addition
Keywords	Gravitational force, celestial bodies, ellipse, eccentricity, Areal velocity, universal gravitation constant

2. Development Team

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1. UNIT SYLLABUS

Unit VI: Gravitation

Chapter 8: Gravitation

Kepler's laws of planetary motion; universal law of gravitation; Acceleration due to gravity and its variation with altitude and depth; Gravitational potential energy and gravitational potential; escape velocity; orbital velocity of a satellite; Geo-stationary satellites.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 5 Modules

The above unit is divided into five modules for better understanding.

Module 1	• Gravitation • Laws of gravitation • Early studies • Kepler's laws

Module 2	• Acceleration due to gravity • Variation of g with altitude • Variation of g due to depth • Other factors that change g
Module 3	• Gravitational field • Gravitational energy • Gravitational potential energy • Need to describe these values
Module 4	• Satellites • India's satellite programme and target applications • Geo stationary satellites and Polar satellites • Escape velocity • India's space program
Module 5	• Numerical problems based on Gravitation

MODULE 1

3. WORDS YOU MUST KNOW

- Solar system: The sun, its eight planets with their satellites and the asteroid belt constitutes the solar system.

- Vectors: Physical quantities which have both magnitude and direction. They are added according to the vector laws of addition.

- Linear momentum: (P= m v) A physical quantity which is the product of mass and velocity. The rate of change of linear momentum is equal to the external force

  ($F = \frac{dP}{dt}$)

- Law of conservation of linear momentum: According to this law, the linear momentum of the system remains constant if the external force is zero.

- Angular momentum: (L = m v r when v is perpendicular to r).It is the analogue of linear momentum in rotational motion. The rate of change of angular momentum is torque $\tau = \frac{dL}{dt}$

- Law of conservation of angular momentum: According to this law, the angular momentum of the system remains constant if the external torque is zero.

4. INTRODUCTION

Human beings from time immemorial have gazed at the skies to enjoy, observe and analyze the movement of the stars, planets and other celestial bodies of the universe.

After centuries of work by many scientists, it was found that Gravitational force is the force responsible for holding the celestial objects together and for their observed motion in the sky.

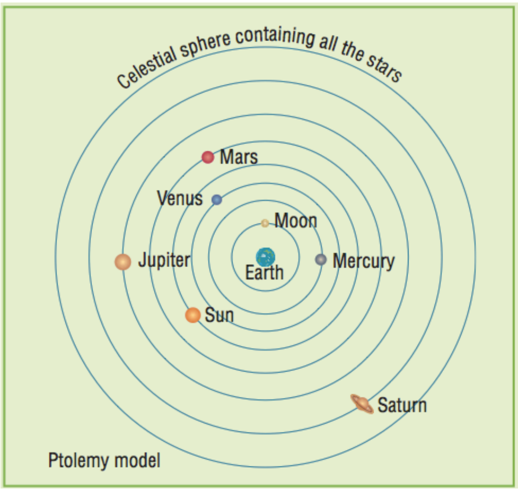
Gravitational forces are very weak forces of nature as compared to the electromagnetic forces and nuclear forces. The gravitational force between two objects in our surrounding is hardly noticeable. On the other hand, the same gravitational force can be so strong that it can hold a galaxy of stars together.

In the following sections we will try to understand more about the nature of the gravitational force and the law governing it.

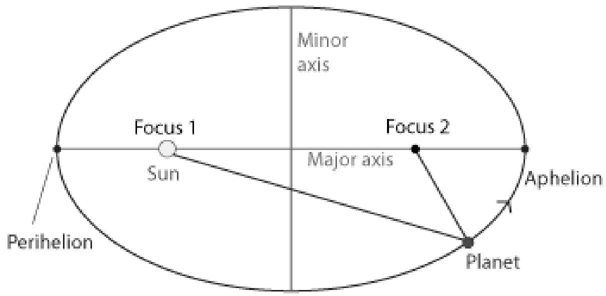
5. EARLY STUDIES

Models of the universe, mainly involving the planetary motion have been proposed by scientists from very early twice.

These models are described below:-



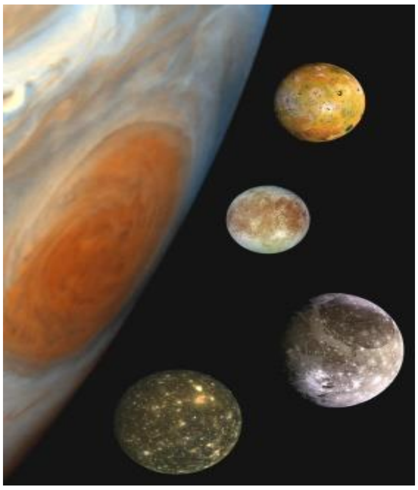
5.1 GEOCENTRIC MODEL:



We always see the sun and the stars rising in the sky every day and then setting again. This observation led scientists conclude that the **earth was stationary and was the centre of the universe**. According to this model all celestial objects, stars, the sun and the planets revolved around the earth in circular paths and hence it was called the **geocentric model**. To explain the observed motion of the planets against the background of stars, complicated schemes of motion were put forward by Ptolemy (around 2000 years ago). The planets were described as moving in circles with the center of the circles themselves moving in larger circles.

5.2 HELIOCENTRIC MODEL:

In the sixteen century a Polish monk, Nicolas Copernicus, proposed a definitive model in which the planets moved in circles **around a fixed central sun.** This was the **heliocentric model.**



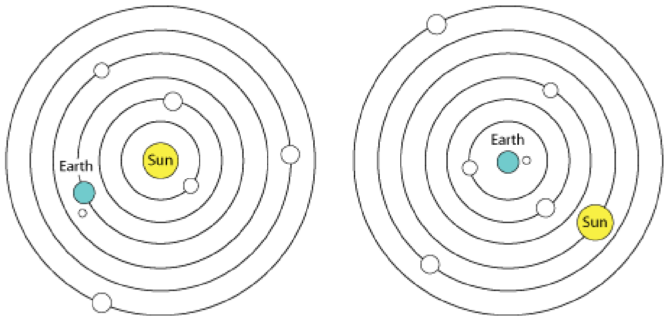
After the invention of telescope in 1609, the first scientific observations in astronomy were made by Galileo Galilei. With his telescope Galileo saw near Jupiter what he first thought to

be stars but then he observed that they disappeared for some time and reappeared again. This is when he realized that they were going around Jupiter. So he concluded that they were the moons of Jupiter and they had disappeared for some time when they were hidden behind Jupiter. From his observations Galileo discovered the four largest moons of Jupiter—Io, Europa, Ganymede, and Callisto. They were the first group of objects found to orbit another planet (as shown in figure).When Jupiter moved these moons of the Jupiter also moved along with it. These observations were in direct contradiction to the geocentric model in which all the celestial objects should move around the earth. Galileo had to face persecution from the state for his beliefs.

Tycho Brahe, a nobleman from Denmark in the late sixteenth century, was another scientist who spent his entire lifetime recording observations of the planetary motion with his naked eye. Meticulously compiled data based on these observations formed the basis of the Kepler's Laws of Planetary motion. Johannes Kepler was assistant of Tycho Brahe and he could analyze the data which was so carefully compiled by Brahe. This analysis led to the formulations of the three laws of planetary motion.

TEST YOUR UNDERSTANDING:

1. **Spot the similarity in the two models given below**



The Copernicus Model The Ptolemy Model

Answer: All orbits in the above two models are circular.

https://cosmoquest.org/x/365daysofastronomy/2009/04/19/april-19th-ancient-indian-astronomy/

This may interest you it is about Indian astronomy

podcast script

Description: According to the Vedas, the holy scriptures of Ancient India, the birth of Indian astronomy dates to around second millennium BCE. India, who made the first mark in astronomy, had significant influence on other civilizations' development in the field. Since then, the world witnessed gigantic leaps in Indian astronomy which brought India to her current position in space research. This podcast attempts to explain the Who, What, When, Where and How of Ancient Indian Astronomy.

Bio: We are a group of friends with a common interest in astronomy. We wish to take this interest to the next level by participating in activities like this and also help others appreciate our universe better.

Today's Sponsor: This episode of "365 Days of Astronomy" is sponsored by Astrocamp Summer Mission of Idyllwild, California. Help introduce a child to the world of Astronomy. Learn more at www.astrocamp.org.

Transcript:

Hello and welcome to the three sixty five Days Of Astronomy programme's hundred and ninth podcast. This is Harith, Rahul and Rohith. We are a group of friends from Hyderabad, India with an interest in Astronomy. 34 years ago on this very day, India launched its first satellite Aryabhatta. It was named after one of the pioneers of Ancient Indian Astronomy which will be our topic of discussion.

Indian heritage and culture are influenced by the Vedas.

'What are the Vedas?'

The Vedas are among the oldest scared texts in the world. The Vedas were born around five thousand years ago during the transition of the world from the Neolithic Age to Bronze Age. In those days, knowledge was imparted to the next generation through recitations because they lacked a persistent storage medium like paper. So, though you may find some sources citing the oldest Vedic manuscripts dating back to 1500BC, you should keep in mind that they were in existence well before that.

The oldest mention of astronomy in India dates back to second millennium BC and has been found in the Vedas. In Indian languages, the science of astronomy is called "Khagola-Shastra". The word Khagola refers to the "cosmos" and Shastra means "Science".

'So, what motivated the people of those times to study the cosmos?'

The Bronze Age marked the beginning of agrarian civilizations. The first farmers needed to keep track of the seasons but had no formal way to do it. Indians, one of the oldest civilizations in our world, looked up towards the sky and noticed that a clock was staring at their faces.

The Ancient Indian astronomical works are generally divided into two classes. One class were alleged as revelations. The authors hid their names with the definite motive of making their astronomical theories and calculations acceptable to the common man. By doing so, they made them look like direct transmissions from the Gods. In Indian Mythology, seven Celestial bodies were believed to be around our Earth. They were the Sun, Mercury, Venus, Mars, Jupiter and Saturn. Only these bodies were considered because only these objects appeared to be in motion relative to the other "dots" in the sky. There were two other "evil" bodies, Rahu and Kethu, which were said to be "invisible" and would make the Sun disappear once in a while.

'Yes, you guessed it right. These are references to solar eclipses. In fact, prehistoric Indians were able to predict these occurrences precisely!'

A few excerpts from the Vedas can give us an idea of how advanced the Ancient Indian Astronomy was. For example, an old Sanskrit verse goes like this – "SarvaDishanaam, Suryaha, Suryaha, Suryaha". It translates to, "There are Suns in every direction". We now know that the Sun is a star. So the verse can be interpreted as their realization that all stars in the night sky and the Sun were similar. Also, the Sanskrit term for gravity is "Gurutvaakarshan", the roots of which are "Guru" and "Akarshan". Guru means "Master" and Akarshan "Attraction". The term Gurutvaakarshan means "to be attracted by the Master". The usage of Gurutvaakarshan in this context can be interpreted as a support to the recognition of the Heliocentric and Gravitation theories. Since, the term Guru corresponds to the male gender, it could be attributed to the Sun which was considered as a male, and not the Earth which was always referred to as a female. Unlike Newton who questioned why the apple fell down and hypothesized that it may be that the Earth attracted everything, ancient Indian astronomers realized that the Sun attracted the planets. They realised that, things not only fall towards the Earth but also towards the Sun.

The second class of these works were relatively new. The major contributors were distinguished astronomers like Lagadha, Aryabhatta, Bhaskara, Varahamihira, Brahmagupta and others. Surya Siddhanta, which was written by the Indian astronomers in 5th century, has explicit mentions of the discovery of the Heliocentric and Gravitation theories.

‘And what did these astronomers do?’

Around 1200 BC, Lagadha, in his work VedƒÅngaJyotishya, listed several important aspects of the time and seasons, including lunar months, solar months, and their adjustment by a lunar leap month called Adikamasa; Adika means “Extra” and Masa means “Month”. Basically, after every 6 years during which a month spanned about thirty days, an extra month would be added to the calendar. Later Aryabhatta, in his book Aryabhatiya, mentioned that the Earth rotates about its axis, thereby causing what appears to be an apparent westward motion of the stars. He also mentioned that moonshine is reflected sunlight. Aryabhata wrote that 1,582,237,500 rotations of the Earth equal 57,753,336 lunar orbits. This is an extremely accurate fundamental astronomical ratio (1,582,237,500/57,753,336 = 27.3964693572), which is the lunar cycle, and is perhaps the oldest astronomical constant calculated to such precision.

‘And here is the icing on the cake. Aryabhata discovered these facts 1,500 years ago, that is, 10 ten centuries years before Copernicus and Galileo, the pioneers of European astronomy.

But they surely could not have made such precise calculations and observations without the help of instruments. Instruments like Clepsydra, Star clock and Gnomons were prominently used for determining time and seasons. It is generally accepted that gnomics is the oldest device to measure time and was invented by Indians. Golayantra, also known as the armillary sphere, was used for observation in India since early times, and finds mention in the works of ƒÄryabhata. Another fellow, by the name Bhaskaracharya, calculated the heights of terrestrial objects with the help of a long stick placed along the diameter of a semicircular disk. This disk had angular graduations and a pivoted chain at its center. This is basically the description of a sextant. In 12th century, Bhaskara II invented a device called ‘phalakayantra’. It consisted of a rectangular board with a pin and an index arm and was used to determine time from the sun’s altitude. A device named ‘KapƒÅlayantra’ was an equatorial sundial instrument used to determine the sun‚Äôs azimuth.

To sum it all up, India who made the first mark in astronomy, had significant influence on other civilizations’ early development in the field. Due to lack of telescopes, Indians could not make any significant progress. Nevertheless, even after centuries of stagnant research, India is catching up with the leaders of space technology.

End of podcast:

365 Days of Astronomy
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6. KEPLER'S LAWS OF PLANETARY MOTION

6.1 LAW OF ORBITS:

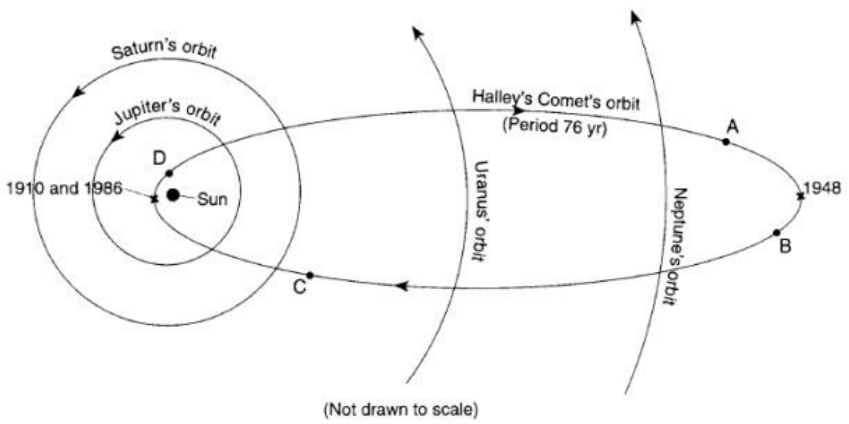
All planets move in elliptical orbits with the Sun situated at one of the foci of the ellipse.

At every point in its elliptical orbit:

Distance of the planet from Focus1 + distance of the planets from Focus 2 = constant

Assumption: The mass of the sun (M) is much larger than the mass of the planet(m).

Significance of the assumption: The centre of mass of the sun and the planet lies almost at the centre of the sun when m<<M. So the planet moves around the sun.



This shows a planet of mass m moving in an elliptical orbit around the sun.

a. THE ELLIPTICAL ORBIT

Figure2 shows the semi major axis (a) of the orbit of the planet with eccentricity (e). The distance of the closest approach of the planet to the sun is called the Perihelion (R_p) and the farthest distance of the planet from the sun is termed as the Aphelion (R_a).

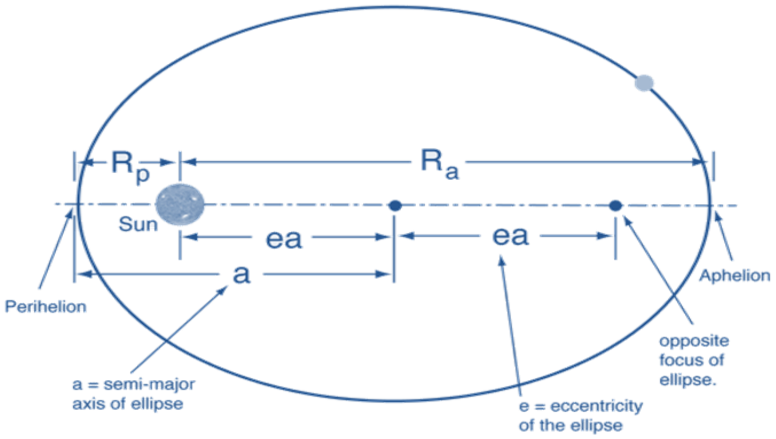


Figure 2

b. ECCENTRICITY OF ELLIPSE

Eccentricity of the ellipse 'e' is the ratio of distance from the centre of ellipse to any one of the foci and the semi major axis.

$e = (a - R_p)/a$ (symbols have the usual meaning)

Eccentricity of an ellipse tells about its deviation from a circle.

An eccentricity of zero corresponds to a circular orbit. Eccentricities of the planetary orbits are not large. The eccentricity of the earth's orbit is only 0.0167 which means that the earth's orbit around the sun is nearly circular. The eccentricity of orbit of comet Halley on the other hand is 0.97 (figure 3). This eccentricity is almost approaching unity so this comet's orbit is a long thin ellipse.

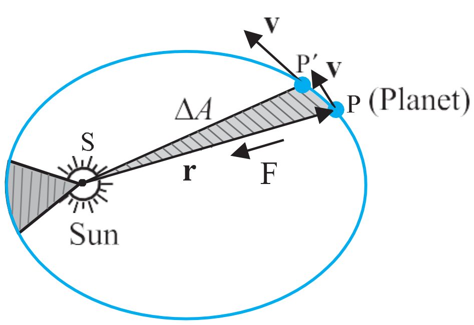


Figure 3

Video of how to make an ellipse on a paper with pins, string and pencil:

https://www.youtube.com/watch?v=PgP7eDXAOjQ

TEST YOUR UNDERSTANDING:

EXAMPLE:

If the eccentricity of the planetary orbit is zero, the distance between the two foci of the orbit will be

(a)__________ and the shape of the orbit will be

(b)___________

SOLUTION

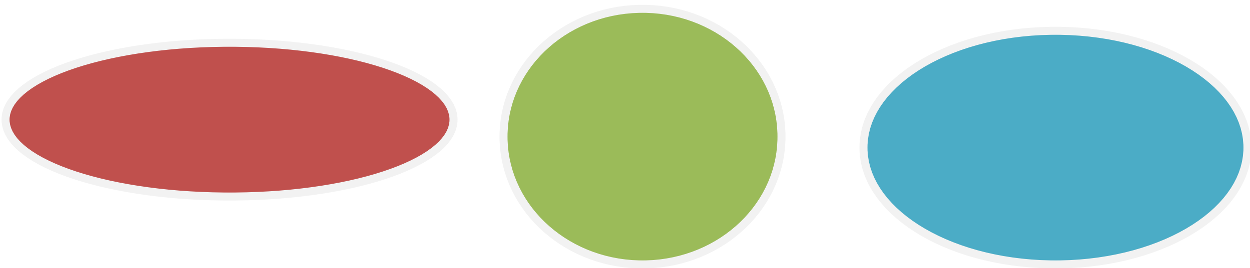
Since $e = (a- R_p)/a$

(a) Zero

(b) Circular

EXAMPLE:

Rank the given figures according to eccentricity of the ellipse greatest first:



(a) (b) (c)

SOLUTION:

Since $e = (a- R_p)/a$

a, c, b

EXAMPLE:

A satellite moving in an elliptical orbit is 360 km above the earth's surface at Aphelion and 180 km above the earth's surface at Perihelion. Take the radius of the earth to be 6400 km.

(a) What will be the value of its semi major axis?

- **A. 6000 km**
- **B. 6500 km**
- **C. 6580 km**
- **D. 7500 km**

(b) What will be the eccentricity of the orbit?

- **A. 0.01**
- **B. 0.12**
- **C. 0.0135**
- **D. 0.55**

SOLUTION:

The distance from the centre of the earth to the satellite at Aphelion

R_a = 6400 km + 360 km = 6760 km

The distance from the centre of the earth to the satellite at Perihelion

R_p =6400 km + 180 km = 6580 km

(a) Semi major axis a = $(R_a + R_p)/2$ = (6760 + 6580)/2 = 13340/2 = 6670km

(b) Eccentricity of the orbit e = $(a – R_p)/a$ = 0.0135

6.2 LAW OF AREAS:

The line that joins any planet to the sun sweeps equal areas in equalintervals of time

CONSEQUENCE OF THIS LAW:

Planets move slowly when they are far away from the sun and move faster when they are closer to the sun.

CAUSE OF THIS LAW:

This law follows from the fact that gravitational forces are central forces meaning that the gravitational force between the planet and the sun acts along the line joining them.

THE AREAL VELOCITY:

The position of the planet is given by r with respect to the sun. The momentum of the planet at this position is p. In a small time Δt, the planet sweeps out a small triangle having base line r and height $v\Delta t$.

The area of this triangle is given by $\Delta A = \frac{1}{2}(r \times v\Delta t)$

$\Delta A / \Delta t = \frac{1}{2}(r \times v)$

Hence areal velocity $\Delta A / \Delta t = \frac{1}{2}(r \times p)/m$ (since $v = p/m$)

$= L / (2m)$ (L = angular momentum)

CONSERVATION OF ANGULAR MOMENTUM:

Angular momentum of a moving object about a certain point remains conserved if the object does not experience any external torque.

Here the gravitational force between the planet and the sun acts along the line joining them. So the torque experienced by the planet is zero.

Hence the angular momentum of the planet about the sun remains constant throughout the planetary orbit which makes the areal velocity also constant.

L = mvr = constant

So as r increases v decreases and vice versa. This shows that a planet slows down when it is far from the sun and speeds up when it comes near the sun.

THINK ABOUT THESE:

EXAMPLE:

A satellite is in an elliptic orbit around the earth with aphelion of 6R and perihelion of 2R where R is the radius of the earth. What will be the ratio of the satellite's velocity at apogee and perigee?

A. 1:3

B. 3:1

C. 3:2

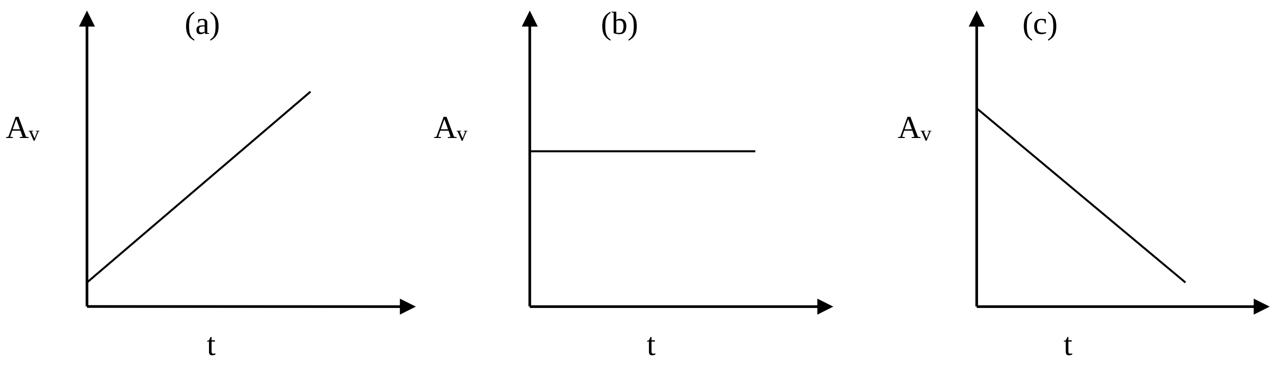
D. 2:3

SOLUTION

1:3 (using $m_1v_1r_1 = m_2v_2r_2$)

EXAMPLE:

Which among the following represents the correct graph of areal velocity versus time for the planet mars?



ANSWER:

(b)

6.3LAW OF PERIODS:

The Square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet.

In equation form, this is given as:

$T^2 \propto r^3$

Hence, $T^2/r^3 = \text{constant}$

i .e.,

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

Where *T* is the period (time for one orbit) and *r* is the average radius.

CONSEQUENCE OF THIS LAW:

Time period is more for planets which have a greater orbital radius.

ASSUMPTION:

Planetary orbits were taken to be circular with the radius of the orbit equal to the semi major axis of the orbit which was the average distance between the sun and the planet.

CAUSE OF THE LAW:

The centripetal force for the motion of the planet in circular orbit is provided by the gravitational force between the sun and the planet.

Hence the law was proved with the help of Newtonian mechanics. We will take up the proof in the next section after the study of Newton's law of gravitation.

CHECKING THE VALIDITY OF THE LAW:

To check the validity of the law we will take the data of the time periods of the planets of the solar system and their average distances from the sun (semi-major axis)

Table-1

Planet	Average orbital radius of the planets (a)		Time period of planets in earth years (T)	T^2/a^3 (for T in years and a in AU)
	(x 10^{10}m)	(in AU)		
Mercury	5.79	0.387	0.241	1.002
Venus	10.8	0.723	0.615	1.001
Earth	15	1.000	1.000	1.000
Mars	22.8	1.524	1.881	1.000
Jupiter	77.8	5.203	11.86	0.999
Saturn	143	9.537	29.42	0.998
Uranus	287	19.19	83.75	0.993
Neptune	450	30.07	163.7	0.986

Interpretation of Table 1:

i. Eight planets of the solar system have been taken (Pluto excluded).
ii. Average orbital radius equal to the semi major axis of the orbit is given in metres and also in AU.
   a. For example from the data in the table above orbital radius of Mars = 22.8 x 10^{10} m = (22.8 x 10^{10}/ 15 x 10^{10})AU = 1. 524AU.
   b. Here, AU is Astronomical unit and 1AU is equal to the distance of the earth from the sun which is equal to 15 x 10^{10}m.
iii. The ratio $T^2 \approx a^3$ is taken for T in years and a in AU and interestingly we find that this value is nearly unity for all the planets.
iv. So we can say that $T^2 = a^3$ for all the planets (for T in years and a in AU).
v. This is true for even the satellites orbiting the planets.

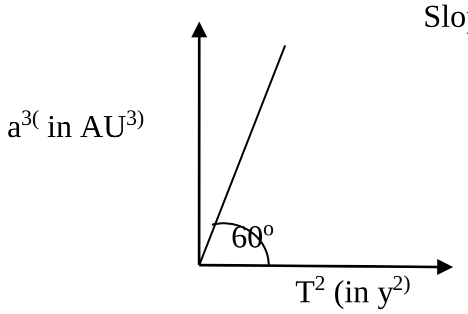
Video stating the Kepler's laws:
https://www.youtube.com/watch?v=a9N7ogaeg9g

CONSIDER THESE

EXAMPLE:

The following graph with slope m is shown for the cube of average orbital radius of planets (a^3 in AU^3) and the square of the time period (T^2 in y^2). Spot the mistake in the graph.

Slope m = 1.732



ANSWER:

Slope m should be nearly unity. So the line should make an angle of 45^o instead of 60^o.

EXAMPLE:

The average orbital radius of the moon about the sun is 384000 km with an orbital period of 27.3 days. Using this data the average orbital radius of an artificial satellite about the earth having an orbital period of 1hour was found to be 5.08 x 10^3km.

a. **What is unreasonable about the result?**
b. **From the above data find the time period of a satellite of earth orbiting just above the surface of the earth. Radius of the earth = 6400km**
c. **What is the significance of the result obtained in b?**

ANSWER:

a. Orbital radius of satellite is less than the radius of the earth which is unreasonable.
b. Time period = 84 minutes
c. This is the limiting value of the time period of a satellite about the earth. No satellite can have a time period less than 84minutes.

7. NEWTON'S UNIVERSAL LAW OF GRAVITATION:

The key argument of Isaac Newton which led to the Universal law of gravitation in 1665 was that he considered that the movement of the celestial bodies and the free fall of the objects were due to the same force.

The story of the falling apple is significant because Newton was able to relate the acceleration of apple and the acceleration of the moon to their distances from the centre of the earth.

7.1 NEWTON'S ANALYSIS OF THE FALLING APPLE:

Newton argued that the apple fell on the earth due to a force experienced by it towards the earth. Similarly the moon was orbiting around the earth in almost a circular path due to the force experienced by it towards the earth. He could find the acceleration of the moon using this logic.

Acceleration of the free falling apple, a_{apple}= 9.8 m/s^2

Centripetal acceleration of the moon going around the earth $a_{moon} = \omega^2 r$

Where

ω is the angular velocity of the moon and

r is the distance of the moon from the centre of the earth.

Orbital period of moon (T) = 27.3 days, angular velocity,

$\omega = \frac{2\pi}{T}$

and

orbital radius, r_{moon} = 384000km

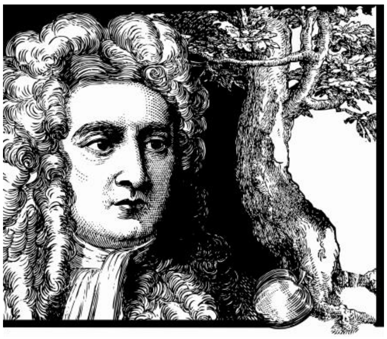
Acceleration of the moon is calculated from the above data:$\omega^2 r$= 0.0027m/s^2

So we have accelerations: a_{apple}= 9.8m/s^2 and a_{moon} = 0.0027m/s^2 , and the distance from centre of earth :r_{apple}= 6400km and r_{moon} = 384000km

Ratio : a_{apple}/ a_{moon} = 9.8/ 0.0027= 3600 (approx..)

Ratio : r_{apple} / r_{moon}= 6400/384000 = 1/60

So we can see here that distance of the moon from the centre of the earth is 60 times more than the distance of the apple from the centre of the earth while the acceleration of the moon is 3600 times less than the acceleration of the apple.



From these calculations Newton arrived at the famous result

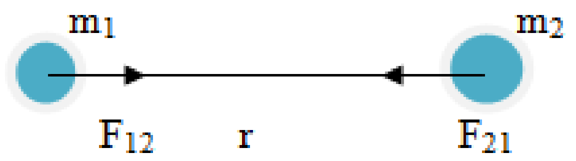
$$\frac{acceleration_{apple}}{acceleration_{moon}} = \left(\frac{\text{distance from centre of earth}_{moon}}{\text{distance from centre of earth}_{apple}}\right)^2$$

This established the **INVERSE SQUARE LAW**which stated that:

The force of gravitational attraction between two masses is inversely proportional to the square of the distance between them.

The inverse square law and the fact that force depends directly of the mass of the body helped Newton formulate his famous law:

Newton's universal law of gravitation states that every particle in the universe attracts every magnitude of the other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.



$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = G\frac{m_1 m_2}{r^2}$$

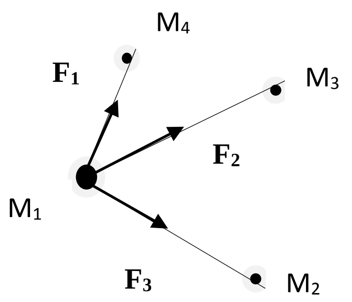
- **Here F represents the magnitude of gravitational force between the two masses separated by a distance r.**
- **This force acts along the line joining their centres if the masses are uniform. Generally, the masses are taken to be point masses to overcome this difficulty.**
- **G is the universal gravitational constant whose value is constant everywhere in the universe.**
- **The value of G is very small and is equal to 6.673 x 10^{-11} Nm^2kg^{-2}. This means that the gravitational force between two masses of 1kg separated by a distance of 1m will only be 6.673 x 10^{-11} N. This is reason why gravitational forces are weaker in magnitude than the electromagnetic forces.**
- **As forces always occur in pairs, mass m_1 experiences a force F_{12}towards m_2 and mass m_2 experiences a force F_{21} towards m_1 along the line joining**

them. These two forces are equal in magnitude and opposite in direction according to the Newton's third law of action and reaction.

$\mathbf{F_{12} = -F_{21}}$

7.2 PRINCIPLE OF SUPERPOSITION:

According to this principle, if we have a collection of point masses, the force on any one of them is the vector sum of the gravitational forces exerted by the other point masses.



Force on mass, M_1 = $\mathbf{F_1 + F_2 + F_3}$

(Bold letters denote vectors)

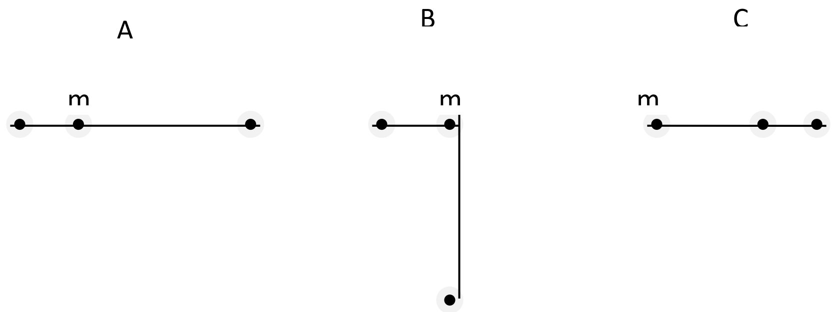
- Resultant force on mass M_1 is a **vector sum of all the forces** due to every other point mass along the line joining them.

- Mistake of only adding the magnitude of forces without considering the angle between the forces should not be done.

THINK ABOUT THIS:

EXAMPLE:

Figure shows three identical masses in different arrangements. Rank the given figures according to magnitude of the force experienced by the point mass marked as m in ascending order.

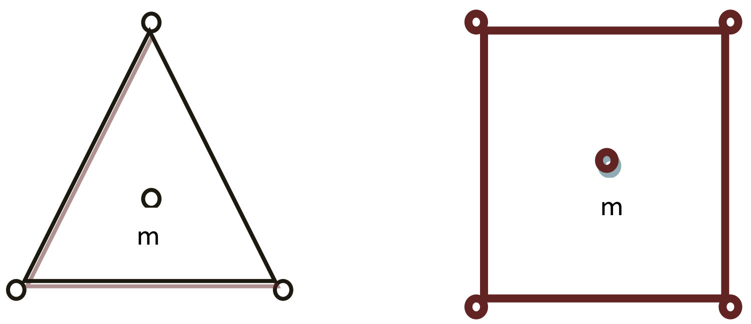


ANSWER:

A, B, C

EXAMPLE:

Figure shows identical point masses kept on the vertices of a equilateral triangle and square. What is the force experienced by a particle of mass 'm' kept at the geometrical centre of the figures?



ANSWER:

Zero.

EXAMPLE:

A satellite of mass m orbits around a planet of mass M in a nearly circular orbit of radius r with a time period T with velocity v.

a. **What provides the centripetal force for the circular motion of the satellite?**
b. **On what factors does the time period of the satellite depend?**
c. **How would time period change if a satellite of greater mass was placed in the same orbit.**

ANSWER:

a. Gravitational force provides the centripetal force.

$$\frac{\mathrm{GMm}}{\mathrm{r}^2} = \frac{\mathrm{mv}^2}{\mathrm{r}}$$

b. velocity

$$\mathrm{v} = \frac{2\pi \mathrm{r}}{\mathrm{T}}$$

So

$$\frac{\mathrm{GMm}}{\mathrm{r}^2} = \frac{\mathrm{mv}^2}{\mathrm{r}} = \frac{m4\pi^2 r^2}{rT^2}$$

$$\mathbf{T^2 = \frac{4\pi^2}{GMr^3}}$$

Hence, the time period of the satellite depends on the orbital radius and the mass of the planet.

Note that: we have arrived at the Kepler's law of periods

c. The time period of the satellite is independent of the mass of the satellite.

Video showing gravity well:

https://www.youtube.com/watch?v=cHySqQtb-rk

9. SUMMARY:

In this module we have learnt about the Kepler's laws of planetary motion. They are particularly important as shed the beliefs which were prevalent for thousands of years.

- Kepler's laws:

i. Law of orbits: All planets move in elliptical orbits with the Sun situated at one of the foci of the ellipse.

ii. Law of Areas: The line that joins any planet to the sun sweeps equal areas in equal intervals of time.

iii. Law of periods: The Square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet.

Although Kepler formulated his laws of planetary motion early in the 17^{th} century, they were properly mathematically derived by Isaac Newton later.

- Newton's Universal Law of Gravitation:

  It states that the gravitational force between two point masses is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

- If we have a collection of point masses, the force on any one of them is the vector sum of the gravitational forces exerted by the other point masses in accordance with the Principle of superposition: