1. Details of module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics - Part 1, Class XI)
Module Name/Title	Unit 2, Module 10, **Mathematical tools for solving problems** Chapter 4, Motion in a plane
Module Id	Keph_10405_eContent
Pre-requisites	Basic Arithmetic's, graphs dependent and independent variables, basic and derived units
Objectives	**After going through this module, the learners will be able to:** • Understand the concept need for new mathematical operations • Understand the need for basic differentiation and basic integration • Apply differentiation and integration to solve simple problems in kinematics • Use logarithm tables for calculation in physics
Keywords	Differential calculus, integral calculus, logarithms.

2. Development Team

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TABLE OF CONTENTS

1. UNIT SYLLABUS

Chapter 3: Motion in a straight line

Frame of reference, motion, position –time graph Speed and velocity

Elementary concepts of differentiation and integration for describing motion, uniform and non-uniform motion, average speed and instantaneous velocity, uniformly accelerated motion, velocity –time and position time graphs relations for uniformly accelerated motion - equations of motion (graphical method).

Chapter 4: Motion in a plane

Scalar and vector quantities, position and displacement vectors, general vectors and their notations, multiplication of vectors by a real number , addition and subtraction of vectors, relative velocity, unit vector, resolution of a vector in a plane, rectangular components ,scalar and vector product of vectors

Motion in a plane, cases of uniform velocity and uniform acceleration projectile motion uniform circular motion.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 10 Modules

The above unit is divided into 10 modules for better understanding.

Module 1	• Introduction to moving objects • Frame of reference, • limitations of our study • Treating bodies as point objects
Module 2	• Motion as change of position with time • Distance travelled unit of measurement • Displacement negative, zero and positive • Difference between distance travelled and displacement • Describing motion by position time and displacement time graphs
Module 3	• Rate of change of position • Speed • Velocity • Zero , negative and positive velocity • Unit of velocity • Uniform and non-uniform motion • Average speed • Instantaneous velocity • Velocity time graphs • Relating position time and velocity time graphs
Module 4	• Accelerated motion • Rate of change of speed, velocity • Derivation of Equations of motion
Module 5	• Application of equations of motion • Graphical representation of motion • Numerical
Module 6	• Vectors

	• Vectors and physical quantities • Vector algebra • Relative velocity • Problems
Module 7	• Motion in a plane • Using vectors to understand motion in 2 dimensions' projectiles • Projectiles as special case of 2 D motion • Constant acceleration due to gravity in the vertical direction zero acceleration in the horizontal direction • Derivation of equations relating horizontal range vertical range velocity of projection angle of projection
Module 8	• Circular motion • Uniform circular motion • Constant speed yet accelerating • Derivation of $a = \frac{v^2}{r} or\ \omega^2 r$ • direction of acceleration • If the speed is not constant? • Net acceleration
Module 9	• Numerical problems on motion in two dimensions • Projectile problems
Module 10	• Differentiation and integration • Using logarithm tables

Module 10

3. WORDS YOU MUST KNOW

- Mathematics: A logical method to express any problems in science engineering commerce so as to describe a situation and predict analyze future events that may follow, social science.

- Mathematical operation: Methods following rules taking care that the solutions have a logical inference and can be adapted for multiple situations.

- Addition: An operation that leads to summation of two or more similar quantities, numbers to be added to numbers, vectors to vectors etc.

- Subtraction: An operation that leads to getting a difference between two similar quantities, numbers to be subtracted from numbers, vectors to be subtracted from vectors etc.

- Multiplication: Operation of increase or decrease in proportion, in case of numbers it means adding quantities of same type and value.

- Algebra: Branch of mathematics that generalizes numbers and their operations such that its application is useful in many situations.

- Trigonometry: Branch of mathematics that connects length and angles in a right angle triangle.

- Integer: It may be defined as the (positive) natural numbers, zero, and the negations of the natural numbers.

4. INTRODUCTION

In our study of kinematics so far you have come across a large number of quantities. We can categories them as constant, variable and function a variable that depends on another variable.

But first of all we should know the meaning of following words i.e., constant, variable and function.

A quantity that remains unchanged is called a constant. Like…2,3 ,…so on… these are constants and as in the universal law of gravitation we say capital 'G' as gravitational constant, why because it has a fixed value i.e., 6.67×10^{-11} Nm^2kg^{-2} which remains fixed always.

And variable is a quantity that changes, that is unfixed with time. Like value of x, y etc. or like at t = 0, distance, S is equal to 5 m and at t = 5 sec, S becomes 15 m. so here S is called a variable.

Now what is a function?

Let's take an example: suppose a car moving on a road. Its speed, in general, is constantly changing. Speed may change from 0 to, say, 10 m/s in say, 30s. The average rate of change of speed of the car is then (10 /30 m/s) per second, i.e. 0.33 m/s^2.

It can be said that the speed of the car is a function of time.

If the value of a quantity y depends on the value of another quantity x, then y is said to be a function **of x.**

We can use the notation y = f(x) to represent this mathematically.

In this notation, the quantity y is called the dependent variable, and the quantity x is called the independent variable.

5. DIFFERENTIAL CALCULUS

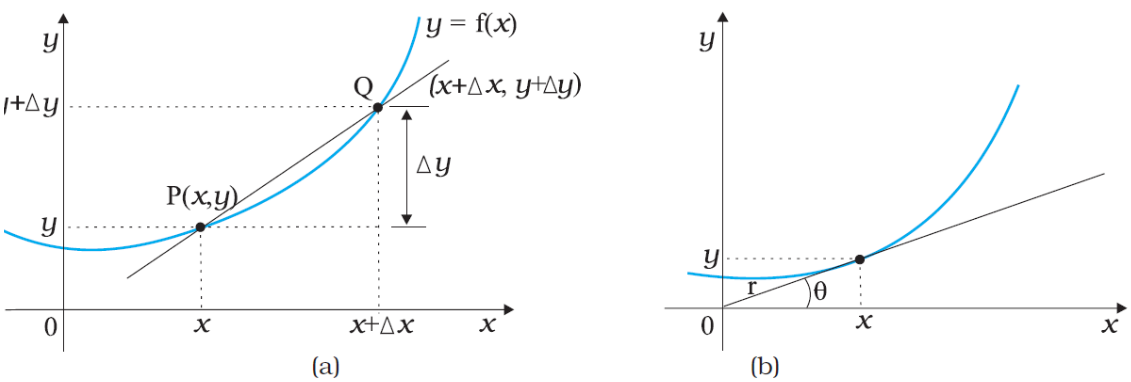
Using the concept of 'differential coefficient' or 'derivative', we can easily define velocity and acceleration. Though you will learn in detail in mathematics about derivatives, we shall introduce this concept in brief in this module 10, so as to facilitate its use in describing physical quantities involved in kinematics.

Suppose we have a quantity *y* whose value depends upon a single variable *x*, and is expressed by an equation defining *y* as some specific function of *x*.

This is represented as:

$$y = f(x)$$

This relationship can be visualised by drawing a graph of function $y = f(x)$ regarding *y* and *x* as Cartesian coordinates, as shown in the graphs fig (a).



This function does not change linearly, (like: y = mx + C)

The slope to the curve is changing so if we were to calculate the value if change in y with very small change in x that would be a difficult job.

Consider the point P on the curve $y = f(x)$ whose coordinates are (x, y) and another point Q where coordinates are $(x + \Delta x, y + \Delta y)$. The slope of the line joining P and Q is given by:

$$tan\theta = \frac{\Delta y}{\Delta x} = \frac{(y + \Delta y) - y}{\Delta x}$$

Suppose now that the point Q moves along the curve towards P. In this process, Δy *and* Δx decrease and approach zero; but their ratio $\frac{\Delta y}{\Delta x}$ will not necessarily vanish.

What happens to the line PQ as Δy and Δx approach 0?

You can see that this line becomes a tangent to the curve at point P as shown in Fig (b). This means that tan θ approaches the slope of the tangent at P, denoted by *m*:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{(y + \Delta y) - y}{\Delta x}$$

The limit of the ratio dy/dx as dx approaches zero is called the derivative of y with respect to x and is written as d*y*/d*x*. It represents the slope of the tangent line to the curve $y = f(x)$ at the point (x, y).

Since $y = f(x)$ and $y + \Delta y = f(x + \Delta x)$, we can write the definition of the derivative as:

$$\frac{dy}{dx} = \frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left[\frac{f(x+\Delta x) - f(x)}{\Delta x}\right]$$

So, we can also say differentiation refers to the rate of change.

The limiting value, of the rate of change of a quantity y (the dependent variable) with respect to the quantity x (the independent variable), is called the differential coefficient of y with respect to x. It is also known as the derivative of y with respect to x. It is denoted by the symbol $\frac{dy}{dx}$ or $\frac{d}{dx}(y)$. Remember $\left(\frac{d}{dx}\right)$ is one complete symbol; we cannot cancel the 'd' in the numerator and denominator of this symbol.

The term $\frac{dy}{dx}$, implying the rate of change of y with respect to x, is calculated mathematically by finding the value of the ratio $\left(\frac{\Delta y}{\Delta x}\right)$ (read as delta y by delta x). Δy represents the very very small change caused in the value of y, when x is changed by a very very small amount Δx.

Given below are some elementary formulae for derivatives of functions.

In these $u(x)$ and $v(x)$ represent arbitrary functions of *x*, and 'a' and 'b' denote constant quantities that are independent of *x*.

Derivatives of some common functions are also listed.

- $\frac{d(ay)}{dx} = a\,\frac{dy}{dx}$
- $\frac{d(yz)}{dx} = y\frac{dz}{dx} + \,z\frac{dy}{dx}$
- $\frac{dy}{dt} = \frac{dy}{dx}.\frac{dx}{dt}$
- $\frac{d}{dx}\left(\frac{y}{z}\right) = \frac{z\frac{dy}{dx} - y\frac{dz}{dx}}{z^2}$
- $\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}}$
- $\frac{d}{dx}(sinx) = \cos x$: $\frac{d}{dx}(cosx) = -\sin x$
- $\frac{d}{dx}(secx) = tanx\sec x$: $\frac{d}{dx}(\tan x) = sec^2 x$
- $\frac{d}{dx}(cotx) = -\operatorname{cosec}^2\ x$: $\frac{d}{dx}(\operatorname{cosec}^2\ x) = -\cot\ x\, cosec\, x$
- $\frac{d}{dx}(y)^n = \mathrm{n}y^{n-1}\frac{dy}{dx}$
- $\frac{d}{dx}(\ln y) = \frac{1}{y}$
- $\frac{d}{dx}(e)^y = e^y$

Now you can appreciate in terms of derivatives, instantaneous velocity and acceleration are defined as:

- $v = \lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t} = \frac{dx}{dt}$
- $a = \lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

The basic rules of differentiation:

i) **Derivative of a constant function:**

The derivative of f(x) = c, where c is a constant, is given by f'(x) = 0.

Example: f(x) = 2 then f'(x) = 0.

ii) Derivative of a power function(power rule):

The derivative of $f(x) = x^n$, where n is a constant real number is given by $f'(x) = n\, x^{(n-1)}$.

Example: $f(x) = x^4$ then $f'(x) = 4\, x^3$.

iii) Derivative of a function multiplied by a constant:

The derivative of $f(x) = c\, y(x)$ is given by $f'(x) = c\, y'(x)$.

Example: $f(x) = 2\, x^3$ then $f'(x) = 2 \times (3\, x^2) = 6\, x^2$.

iv) Derivative of the sum of functions (sum rule):

The derivative of $f(x) = y(x) + z(x)$ is given by $f'(x) = y'(x) + z'(x)$

Example: $f(x) = 3x^3 + 2x$ then $f'(x) = 9x^2 + 2$.

v) Derivative of the difference of functions (difference rule):

The derivative of $f(x) = y(x) - z(x)$ is given by $f'(x) = y'(x) + z'(x)$

Example: $f(x) = 3x^3 - 2x$ then $f'(x) = 9x^2 - 2$.

vi) Derivative of the product of two functions (product rule):

The derivative of $f(x) = y(x).z(x)$ is given by $f'(x) = y(x)\, z'(x) + y'(x)\, z(x)$

Example: $f(x) = (x^3 + 1).(x - 4)$

then $f'(x) = (x^3 + 1)\,(1) + (3x^2)\,(x-4)$

$= x^3 + 1 + 3x^3 – 12\, x^2$

vii) Derivative of the quotient of two functions (quotient rule):

The derivative of $f(x) = y(x)/z(x)$ is given by

$$f'(x) = \frac{[\,y'(x)\, z(x) - y(x)\, z'(x)]}{v(x)^2}$$

Example: $f(x) = \frac{x+1}{x-2}$

$$\text{Then } f'(x) = \frac{(1)(x-2) - (x+1)(1)}{(x-2)^2} = \frac{(x-2) - (x+1)}{(x-2)^2} = \frac{-3}{(x-2)^2}$$

viii) Chain rule:

If we have y = f(u) and u = g(x) then $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$

Example: if y = 4 tan u and u = $5x^2$

Then $\frac{dy}{dx} = 4sec^2u.(10\,x) = 40\,x\,sec^2u = 40\,x\,sec^2(5x^2)$

EXAMPLE

The position of an object moving along x-axis is given by x = a + bt² where a = 8.5 m, b = 2.5 m s⁻² and t is measured in seconds.

What is its velocity at t = 0 s and t = 2.0 s?

What is the average velocity between t = 2.0 s and t = 4.0 s?

SOLUTION

In notation of differential calculus, the velocity is rate of change of position with time or

$$\frac{dx}{dt}$$

Now relation between x and t as given in the problem is:

$x = a + bt^2$

So we use method of differentiation the right hand side has two terms so we will differentiate according to the rules:

$v = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 2bt = 5.0\,t\,m/s$

At t = 0 s

v = 0 m/s

t = 2.0 s

v = 10 m/s

Average Velocity = $\frac{x\,(4.0) - x\,(2.0)}{4.0 - 2.0}$

$$= \frac{a + 16\,b - a - 4b}{2.0}$$

$$= 6.0\ \times b$$

$$= 6.0 \times 2.5$$

$$= 15\ m/s$$

So, we can summarize differentiation: it is the rate of change in function and physically it gives us formulae. Like differentiation of displacement is called velocity i.e.,$\frac{dx}{dt} = v$

And differentiation of velocity is called acceleration i.e. $\frac{dv}{dt}$ is equal to a. or we can say acceleration is the differentiation of velocity with respect to time.

6. INTEGRAL CALCULUS

Let us first understand the physical meaning of the term integration. Consider the mass of a particle which is very small. Next, consider the mass of a collection of a very-very large number of such particles. It would, as we know, be a reasonable and measurable quantity. We are, in a way, here adding a very-very large number of quantities, each of which is very-very small. Integration is the mathematical technique that enables us to find such sums- the sum of a very-very large number of quantities each of which is very-very small. We could in a rather crude way, think of it as finding the product of (0) with (infinity). Mathematicians use a special symbol for integration, this symbol is $\int$. It is interesting to note that the symbol for integration, in a way, resembles the letter 'S' that has been stretched out both ways. This may be linked with the physical meaning of integration- sum of a very-very large number of very-very small terms. The very very small terms, which are added up through integration, are denoted, mathematically, by the symbol f(x) dx.

Here f(x) could be any function of x.

For example: we could have

$f(x) = x^4+1$

Or

f(x)= a cos x

Or

f(x)= log x

and

So on….

The addition of such small terms is known as integration and is expressed through the complete symbol

$\int f(x)dx$.

The symbol then implies the addition of a very very large number of very very small terms.

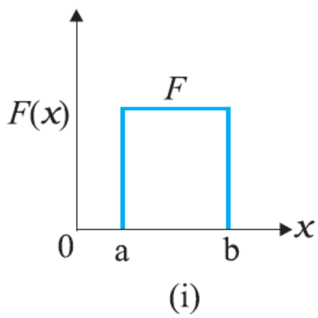
The concept of Integration, mathematically speaking, is the "Inverse" of the concept of differentiation. This implies that if we first differentiate a function, say f(x), and get the result $f'(x)$, the integration of $f'(x)$, would give us back the function f(x). This, in a way, is similar to addition and subtraction or multiplication and division. Suppose we add 1 and 10, we get the result 11. If we now subtract 1 from 11, we get back the number 10. Similarly, if we multiply, say 6 by 7, we get 42. On dividing 42 by 6, we get back the number 7.

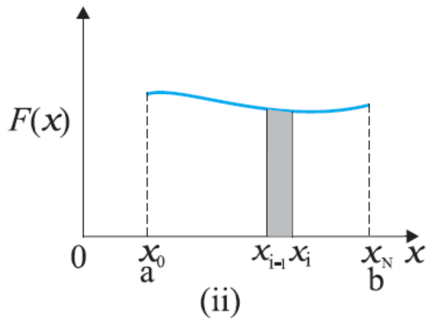
EXAMPLE TO UNDERSTAND THE PHYSICAL MEANING OF INTEGRATION

Suppose a variable force $f(x)$ acts on a particle in its motion along x - axis from x = a *to* x = b The problem is to determine the work done (W) by the force on the particle during the motion.

The graphs show the variation of $F(x)$ with x.

If the force were constant, work (given by F x S) would be simply the area {F x (b-a)} as shown in Fig. (i). But in the general case, force is varying





To calculate the area under this curve Fig.(ii) let us employ the following trick. Divide the interval on *x-axis* from *a to b* into a large number (*N*) of small intervals:

x_0(=a) to x_1 , x_1 to x_2 ; x_2 to x_3,............................... x_{N-1} to $x_{N=b}$.

The area under the curve is thus divided into *N* strips.

Each strip is approximately a rectangle, since the variation of *F*(*x*) over a strip is negligible.

The area of the ith strip shown is then approximately:

$$\Delta A_i = F(x_i)(x_i - x_{i-1}) = F(x_i)\Delta x$$

Where, Δx is the width of the strip which we have taken to be the same for all the strips. You may wonder whether we should put $F(x_{i}-1)$ or the mean of $F(x_i)$ and $F(x_i-1)$ in the above expression.

If we take *N* to be very very large (*N* tending to infinity), it does not really matter, since then the strip will be so thin that the difference between $F(x_i)$ and $F(x_i-1)$ is vanishingly small.

The total area under the curve then is:

A= $\sum_{i=1}^{N} \Delta A_i = \sum_{i=1}^{N} F(x_i)\Delta x$

The limit of this sum is known as the integral of *F*(*x*) over *x* from a to b. It is given a special symbol as shown below:

A = $\int_a^b F(x)dx$

The integral sign $\int$ looks like an elongated S, reminding us that it basically is the limit of the sum of an infinite number of terms.

A most significant mathematical fact is that integration is, in a sense, an inverse of differentiation.

The integration of '0'

It is important to remember that if y = constant say (c), the rate of change of y, would have to be zero. We can, therefore, say that here $\frac{dy}{dx} = \frac{d(c)}{dx} = 0$.

Remembering that integration is the inverse of differentiation, this result implies $\int f(0)dx$ = c = a constant.

We can thus say "The integration of '0', would give us a 'constant' as its result." This result will hold irrespective of the actual value of the constant (c).

We, therefore say;

The integration of '0', gives an indeterminate constant as its answer. Therefore, the result of integration, of any given function, does not give us a unique/definite answer. To the result, we can always add a (arbitrary) constant.

We now quote the mathematical results for integration of some standard functions.

Remember that the term, (+c), in these results, is due to the above stated, and discussed, property of the integration of zero.

Some common functions of integration:

Integration of some common functions:

Derivatives **Integrals (Anti derivatives)**

(i) $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$ $\qquad$ $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

Particularly, we note that

$$\frac{d}{dx}(x) = 1; \qquad \int dx = x + C$$

(ii) $\frac{d}{dx}(\sin x) = \cos x;$ $\qquad$ $\int \cos x \, dx = \sin x + C$

(iii) $\frac{d}{dx}(-\cos x) = \sin x;$ $\qquad$ $\int \sin x \, dx = -\cos x + C$

(iv) $\frac{d}{dx}(\tan x) = \sec^2 x;$ $\qquad$ $\int \sec^2 x \, dx = \tan x + C$

(v) $\frac{d}{dx}(-\cot x) = cosec^2 x;$ $\qquad$ $\int cosec^2 x \, dx = -\cot x + C$

(vi) $\frac{d}{dx}(\sec x) = secx \, tanx;$ $\qquad$ $\int \sec x \tan x \, dx = \sec x + C$

(vii) $\frac{d}{dx}(-\operatorname{cosec} x) = cosec \, x \cot x\,;$ $\qquad$ $\int cosec \, x \cot x \, dx = -\operatorname{cosec} x + C$

Rules for Integration:

$$\int \mathbf{a\, f(x)dx} = \mathbf{a} \int \mathbf{f(x)dx}$$

$$\int [\,\mathbf{f(x)} + \mathbf{g(x)}]\mathbf{dx} = \int \mathbf{f(x)dx} + \mathbf{g(x)dx}$$

$$\int [\,\mathbf{f(x)} - \mathbf{g(x)}]\mathbf{dx} = \int \mathbf{f(x)dx} - \mathbf{g(x)dx}$$

$$\int [\,\mathbf{af(x)} + \mathbf{bg(x)}]\mathbf{dx} = \mathbf{a} \int \mathbf{f(x)dx} + \mathbf{b} \int \mathbf{g(x)d\,x}$$

Example: $\int [\mathbf{3x^3} + \mathbf{x^{-2}} - \mathbf{4x}]\, \mathbf{dx}$

$$= \frac{3x^4}{4} + \frac{x^{-1}}{(-1)} - \frac{4x^2}{2} + C$$

$$= \frac{3x^4}{4} - \frac{1}{x} - 2x^2 + C$$

Example:

$$\int 5 \sec x \tan x \; dx \qquad = 5 \int \sec x \tan x \, dx = 5 \sec x + C$$

We can summarize integration as it is inverse of differentiation and can be used to the sum of a very large number of quantities each of which is very very small.

7. USING LOGARITHMS

Now if I ask you to multiply 8764590 to 3456732 without using a calculator. Will you do this?

Or it frightens you for a while. Yes, you will do it but this will consume a lot of time and energy. So what is that method by which it can be done easily without consuming much time? This will done by using logarithms.

So, now, we shall discuss about logarithms and learn about how we can use log tables.

To begin with, we will define logarithms and use them to solve simple exponential equations. What does logarithm mean?

Logarithms are a tool originally designed to simplify complicated arithmetic calculations. They were extensively used before the advent of calculators. Logarithms transform multiplication and division processes to addition and subtraction processes which are much simpler.

Logarithms have a precise mathematical definition as under:

For a positive number b (called the base);

If $b^a = x$;

then

$\log_b x = a$

The above expression is read as: "logarithm (or simply log) of a given number (x), to a base (b), is the power (a), to which the base has to the raised, to get the number x".

For example:

$10^4 = 10,000$

So

$\log_{10} 10,000 = 4$

i.e. logarithm or log of the number (10,000) to the base (10), is the power (=4) to which the base (10) has to be raised to get the number (10,000).

Remember, logarithms will always be related to exponential equations. For a very clear understanding of logarithm, it is important that we learn how to convert an exponential equation to its logarithm form and also to convert logarithmic expression to exponential form.

Exponential to logarithmic form:

Example: **Write $5^2 = 25$ in logarithmic form.**

Solution: Here number is 25; base is 5; power is 2

$\therefore \log_5 25 = 2$; this is read as: "log of 25 to base 5 is 2."

Solving Exponential and logarithmic equations:

Example: **Find the value of x if $\log_3 81 = x$.**

Solution: By writing the given equation in exponential form; we get:

$3^x = 81$ Also, $3^4 = 81$

Comparing we get, $x = 4$.

Expressing Logarithimic Expression in Exponential Form:

We will now take up some examples to express a given logarithimic expressions in its equivalent in its equivalent exponential form.

Example: **$\log_2 16 = 4$, in logarithmic form, it can be written as $2^4 = 16$ in exponential form.**

Example: Evaluate $\log_3 9^2$.

Solution: let $\log_3 9^2 = y$

i.e., $\log_3 9^2 = y$

$9^2 = 3^y$

$(3^2)^2 = 3^y$

$(3)^4 = 3^y$

Or $y = 4$

Equality of logarithmic functions:

For $b > 0$ and $b \neq 1$

$\mathrm{Log}_b\, x = \log_b y$ if and only if, $x = y$.

Example: **Find the value of x: $\log_2 (3x+5) = \log_2 (2x+4)$**

Solution: As the bases on the two sides are equal, we have:

$3x + 5 = 2x + 4$

i.e., $x = -1$

Fundamental Laws of logarithms:

(i) Law of Product

$\text{Log}_a (x.y) = \log_a x + \log_a y$

(ii) Law of quotient

$\text{Log}_a (x/y) = \log_a x - \log_a y$

(iii) Law of power

$\log_a (x^y) = y \log_a x$

Other laws of logarithms:

1. $\log_a 1 = 0$ (since $a^0 = 1$)
2. $\log_a a = a$ (since $a^1 = a$)
3. $\log_a x = \frac{\log_b x}{\log_a b}$ (Base change formula)

Example: **If $a^x = b$; $b^y = c$; $c^z = a$; Find xyz.**

Solution:

Given: $a^x = b$

By taking log to both side, we get

$\log a^x = \log b$

or

$x \log a = \log b$

or

x = $\frac{\log b}{\log a}$

similarly, using $b^y = c$; $c^z = a$, we get

y = $\frac{\log c}{\log b}$

and

z = $\frac{\log a}{\log c}$

therefore, x.y.z = $\frac{\log b}{\log a} . \frac{\log c}{\log b} . \frac{\log a}{\log c}$ = 1

8. APPLICATION OF LOGARITHIM TABLES FOR CALCULATION

Using Common logarithms for calculation/ computation

As we know, logarithms are used to simplify calculations. It is, in general, have an integral as well as a fractional part; not be always an integral number. The logarithm of a number therefore consists of the following two parts:

(a) **Characteristic**: It is the integral part of the logarithm.

(b) **Mantissa**: It is the fractional or decimal part, of the logarithm.

For example: In $\log_{10}$ N = 3.2425, the characteristic is 3 and mantissa is 0.2425.

Note:

1. The integral part of the logarithm i.e., characteristic may be positive, zero or negative.
2. The decimal part i.e., mantissa is always taken as positive.
3. In case the logarithm of a number is negative, the characteristic and mantissa are rearranged, to make the mantissa positive.

For Example: **$\log_{10}$ N = -3.2425**

= (-3) + (-0.2425)

= (-3-1) + (-0.2425 +1)   (add and subtract 1)

= (-4) + (0.7575)

Therefore, the characteristic becomes (-4) and mantissa becomes (+ 0.7575).

We can write it as:

$\log_{10}$ N = $\bar{4}$.7575 $(here, \bar{4}\ implies\ that\ the\ characteristic\ is\ (-4))$

Using log tables:

We use the log tables (to the base 10) for the calculation of:

(i) The logarithms

(ii) The antilogarithms

Log tables are the standard tables, available for the calculations. In general, these are four digit tables.

Now we take the examples for finding characteristic and mantissa using log tables.

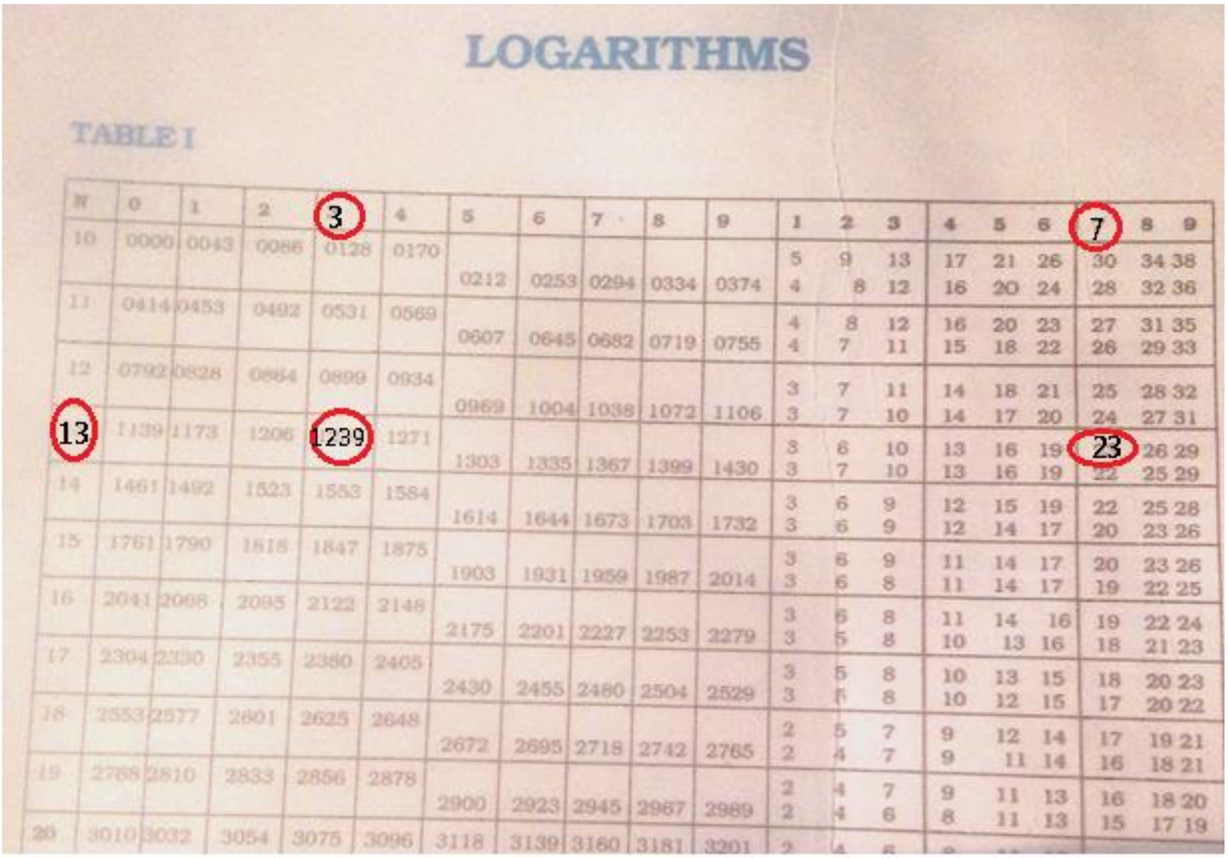
For finding characteristic: express the given number in suitable power of 10.

Example: $2134.2 = 2.1342 \times 10^3$ i.e. the characteristic is 3

Example: $0.02653 = 2.653 \times 10^{-2}$ i.e. the characteristic of this log is (-2) or ($\bar{2}$)

For finding mantissa: the decimal point, the zeros in the beginning, and at the end of the number, is ignored.

Example: **for log (133.7), the mantissa is calculated as:**



LOGARITHMS

TABLE I

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	5 4	9 8	13 12	17 16	21 20	26 24	30 28	34 32	38 36
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 4	8 7	12 11	16 15	20 18	23 22	27 26	31 29	35 33
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 3	7 7	11 10	14 14	18 17	21 20	25 24	28 27	32 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 3	6 7	10 10	13 13	16 16	19 19	23 22	26 25	29 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 3	6 6	9 9	12 12	15 14	19 17	22 20	25 23	28 26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 3	6 6	9 8	11 11	14 14	17 17	20 19	23 22	26 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 3	6 5	8 8	11 10	14 13	16 16	19 18	22 21	24 23
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3 3	5 5	8 8	10 10	13 12	15 15	18 17	20 20	23 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 2	5 4	7 7	9 9	12 11	14 14	17 16	19 18	21 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 2	4 4	7 6	9 8	11 11	13 13	16 15	18 17	20 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

(i) The given number is rewritten as 1.337×10^2 (here characteristic is 2).

(ii) For mantissa, ignore decimal point and take the number as 1337.

(iii) Take the first two digits (i.e. 13) and locate it in the first column of log table as shown in fig. above.

(iv) Follow the horizontal row beginning with the first two digits (i.e. 13) and look for the column under the third digit (i.e. 3) of the four figure log table and record number i.e. 0.1239 as shown in fig. above.

(v) Continue in the same horizontal row, and record the mean difference under the fourth digit (i.e. 7) and add the mean difference i.e 23 to the above number i.e 0.1239

(vi) Therefore, the mantissa is :

= 0.1239 + 0.0023 = 0.1262

Therefore, log (133.7) = characteristic + mantissa = 2 + 0.1262 = 2.1262

ix) SUMMARY

So logarithms can be summarized as: they are used to simplify calculations. It consists two parts:

First one is characteristic which the integral part of logarithm is and another one is mantissa which is the decimal part of logarithm.

Now, you can practice problems based on integration, differentiation and also of logarithmic types.