1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics - Part 1, Class XI)
Module Name/Title	Unit 2, Module 5, **Solving problems using Equation of motion** Chapter 3, Motion in a Straight Line
Module Id	Keph_10305_eContent
Pre-requisites	Students should have knowledge of Equations of motion.
Objectives	**After going through this module, the learners will be able to:** Solve graphical problems based on equations of motion for uniformly accelerated motion
Keywords	Equations of motion, uniformly accelerated motion solving problems using graphs

2. Development Team

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1. UNIT SYLLABUS

Chapter 3: Motion in a straight line

Frame of reference, motion, position –time graph Speed and velocity

Elementary concepts of differentiation and integration for describing motion, uniform and non-uniform motion, average speed and instantaneous velocity, uniformly accelerated motion, velocity –time and position time graphs relations for uniformly accelerated motion - equations of motion (graphical method).

Chapter 4: Motion in a plane

Scalar and vector quantities, position and displacement vectors, general vectors and their notations, multiplication of vectors by a real number , addition and subtraction of vectors, relative velocity, unit vector, resolution of a vector in a plane, rectangular components ,scalar and vector product of vectors

Motion in a plane, cases of uniform velocity and uniform acceleration projectile motion uniform circular motion.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 10 Modules

The above unit is divided into 10 modules for better understanding.

Module 1	• Introduction to moving objects • Frame of reference, • limitations of our study • treating bodies as point objects
Module 2	• Motion as change of position with time • Distance travelled unit of measurement • Displacement negative, zero and positive • Difference between distance travelled and displacement • Describing motion by position time and displacement time graphs
Module 3	• Rate of change of position • Speed • Velocity • Zero , negative and positive velocity • Unit of velocity • Uniform and non-uniform motion • Average speed • Instantaneous velocity • Velocity time graphs • Relating position time and velocity time graphs
Module 4	• Accelerated motion • Rate of change of speed, velocity • Derivation of Equations of motion
Module 5	• Application of equations of motion • Graphical representation of motion • Numerical
Module 6	• Vectors

	• Vectors and physical quantities • Vector algebra • Relative velocity • Problems
Module 7	• Motion in a plane • Using vectors to understand motion in 2 dimensions' projectiles • Projectiles as special case of 2 D motion • Constant acceleration due to gravity in the vertical direction zero acceleration in the horizontal direction • Derivation of equations relating horizontal range vertical range velocity of projection angle of projection
Module 8	• Circular motion • Uniform circular motion • Constant speed yet accelerating • Derivation of $a = \frac{v^2}{r} or\ \omega^2 r$ • Direction of acceleration • If the speed is not constant? • Net acceleration
Module 9	• Numerical problems on motion in two dimensions • Projectile problems
Module 10	• Differentiation and integration • Using logarithm tables

Module 5

3. WORDS YOU MUST KNOW

- **Distance travelled**: Distance is length of the actual path traversed by a body. It is a scalar quantity. Its S.I. unit is m.
- **Displacement**: Change in the position of an object in a fixed direction. It has both magnitude and direction and therefore it is a vector quantity. Its S.I. Unit is m.
- **Speed:** Speed is rate of change of position. Its S.I. unit is m/s.
- **Average speed**: Total path length /Total time interval. S.I. unit is m/s.
- **Velocity**: Rate of change of position in a particular direction. Its S.I. Unit is m/s.
- **Instantaneous velocity**: The velocity of a body at a particular instant of time.
- **Uniform motion**: Object covers equal distance in equal interval of time.
- **Non uniform motion**: Object covers unequal distance in equal interval of time and vice versa.
- **Acceleration**: Rate of change of velocity with time. Its S.I. unit is m/s^2.
- **Velocity may change due to**: Change in its magnitude or Change in its direction or Change in both magnitude and direction.
- **Equations of motion** relation between initial final velocities, u, v or $v(t_1)$ and $v(t_2)$ acceleration(a), time elapsed (t_2-t_1) and distance travelled (s).

$$v = v_0 + at$$

$$x = v_0 t + \frac{1}{2}at^2$$

$$v^2 = {v^2}_0 + 2ax$$

4. INTRODUCTION

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities.

There are, some questions in, 'Motion in a straight line', which can be solved, quickly, by making graphs.

In this module we will learn to solve problems using equations of motion and graphs .

5. EQUATIONS OF MOTION FOR UNIFORMLY ACCELERTED MOTION

If a body starts with velocity 'u' and after time 't' its velocity changes to 'v', if the uniform acceleration is 'a' and the distance travelled in time 't' in 's', then the following relations are obtained, which are called equations of uniformly accelerated motion.

$$\mathbf{v = v_0 + at}$$

$$x = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v^2{}_0 + 2ax$$

Motion under Gravity

If an object is falling freely (u = 0) under gravity, then equations of motion are taken as

(i) $\mathbf{v = u + g\,t}$

(ii) $h = ut + \frac{1}{2}gt^2$

(iii) $v^2 = u^2 + 2gh$

6. GRAPHICAL PROBLEMS

EXAMPLE

Rahul is driving to his office at 25m/s and begins to accelerate (decelerate) at a constant rate of - 1m/s^2 and on reaching the office parking; he comes to a complete stop.

(a) Plot a velocity-time graph showing Rahul's motion.

(b) Use the velocity-time graph to determine the distance travelled by Rahul while decelerating.

(c) Also, use kinematic equations to calculate this distance.

SOLUTION:

(a) Initial velocity u = 25 m/s , final velocity = 0 , a = -1 m/s^2

Using equation of motion

v = u + at

$$0 = 25 + (-1)t$$

We get, t = 25 sec

This means Rahul comes to a complete stop after 25 sec.

Using equation

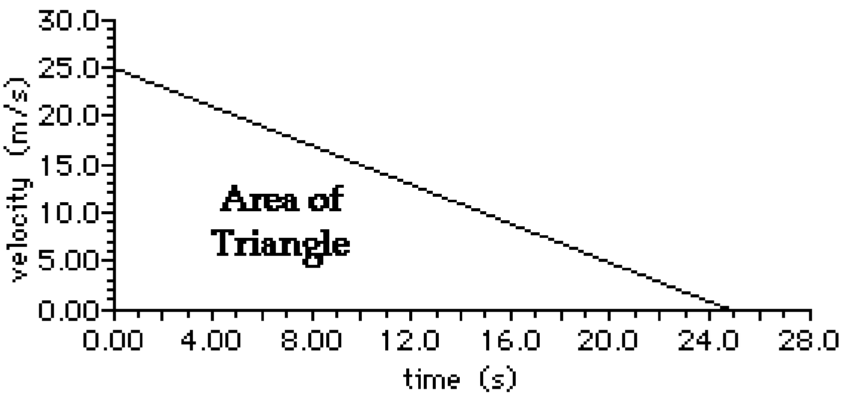
v = u + at which in our case will be

$$v = 25 + (-1)t$$

We get the following data by putting different value of t from 0-25 s, the duration of considered motion and finding the instantaneous velocities

t (in sec)	0	5	10	15	20	25
v (in m/s)	25	20	15	10	5	0

Using the above data the velocity-time graph for the motion is:



(b) The distance travelled is given by area under the velocity time graph.

S = area of the triangle

$= \frac{1}{2} \times \text{base} \times \text{height}$

= 0.5 × b × h = 0.5 (25 s) (25 m/s) = **313 m**

(c) The distance travelled can also be calculated using a kinematic equation.

$$u = 25\ ms^{-1}, \qquad v = 0\ ms^{-1}, \qquad a = -1 ms^{-2}$$

s =??

Using equation of motion,

$$v^2 = u^2 + 2as$$

$$0^2 = 25^2 + 2 \times (-1) \times s$$

$$0 = 625 - 2s$$

2s = 625

s = 313 m

EXAMPLE

Riya is driving her car at 25 ms^{-1}. She accelerates at 2ms^{-2} for 5 seconds, and then maintains a constant velocity for 10 more seconds.

a. **Plot a velocity-time graph representing the motion of this journey.**
b. **Use the graph to determine the distance that Riya travelled.**
c. **Use kinematic equations to calculate the total distance travelled.**

SOLUTION:

(a) u = 25 m/s, a = 2 ms^{-2}, t = 5 sec

Using equation of motion

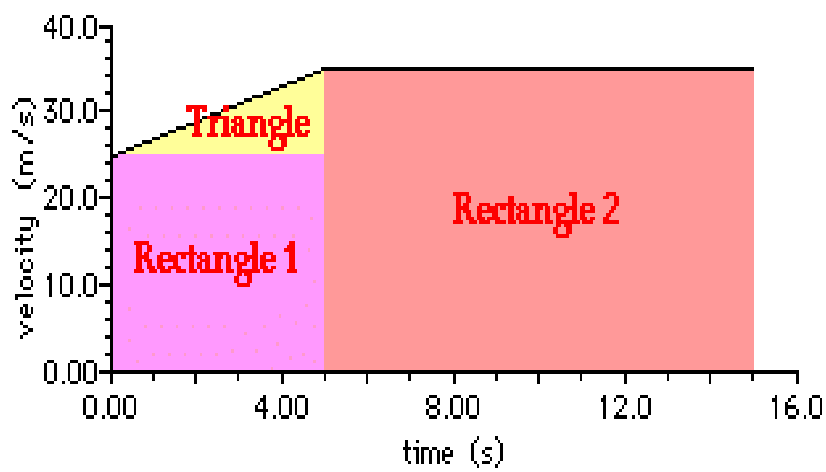
$v = u + at$

$v = 25 + 2\times5$

$\mathbf{v = 35\ ms^{-1}}$

After 5 sec Riya moves with velocity 35 ms^{-1} for the next 10 sec.

The velocity-time graph for the motion is:



(b) The distance travelled can be found by a calculation of the area under the velocity time graph.

This area would be the area of the trapezium plus the area of rectangle.

Distance = area = area of trapezium + area of rectangle

Distance = Area = $\frac{1}{2}$ × (sum of parallel sides) × height + length x breadth

Distance = Area = 0.5 × (25 m s^{-1} + 35 ms^{-1}) x 15 sec + (10.0 s) x (35.0 m/s)

s = Area = 150m + 350 m

Distance = 500 m

(c) The distance traveled can be calculated using a kinematic equation.

First part of the journey (for t = 0 to 5 sec)

U = 25.0 ms^{-1}, t = 5.0 s, a = 2.0 ms^{-2}

s =??

Using equation of motion

$s = u\,t + \frac{1}{2}\,a\,t^2$

s = (25.0 ms^{-1}) (5.0 s) + 0.5 (2.0 m s^{-2}) $(5.0\ s)^2$

s = 125 m + 25.0 m

Distance 1 = 150 m

Second part of the journey (for the next 10 sec)

u = 35.0 m s^{-1} t = 10.0 s a = 0.0 ms^{-2}

s =??

Using equation of motion

$$s = ut + \frac{1}{2}at^2$$

$$s = 35 \times 10 + \frac{1}{2} \times 0 \times 10^2$$

$$s = 350 + 0 = 350\ m$$

Distance 2 = 350 m

The total distance for the 15 seconds of motion is the sum of these two distances

Distance 1 + Distance 2 = (150 m + 350 m):

Distance = 500 m

EXAMPLE

Sumit was driving his car at 45m/s. He looks ahead and observes an accident that results in a pileup in the middle of the road. By the time Sumit applied the brakes, he is 50m from the pileup. He slows down at a rate of 10.0 m/s^2.

- **a. Plot a velocity-time plot for Sumit's motion.**
- **b. Use the plot to determine the distance that Sumit would travel prior to reaching a complete stop (if he did not collide with the pileup).**
- **c. Use kinematic equations to determine the distance that Sumit would travel prior to reaching a complete stop (if he did not collide with the pileup).**
- **d. Will Sumit hit the cars in the pileup?**

SOLUTION:

u = 45 ms^{-1}, a = -10 ms^{-2}, s = 50 m, v = 0

Using

v = u + at

0 = 45 -10 t

t = 4.5 s

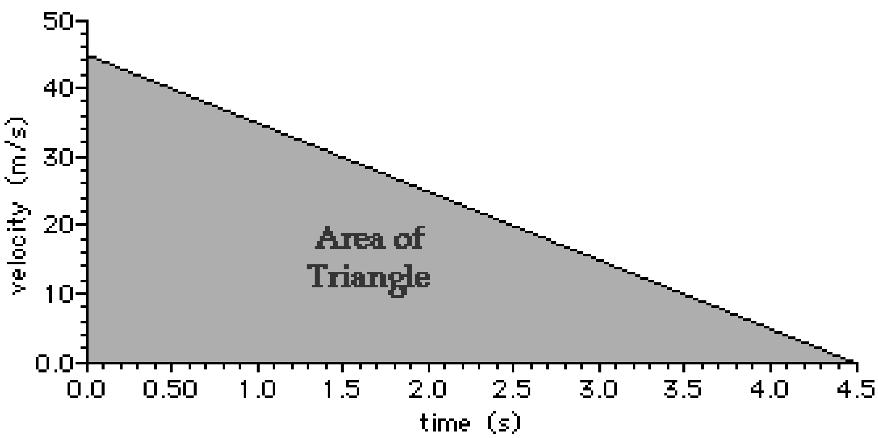
Sumit's car will stop after 4.5 sec.

Using equation v = u + at

We get the following data

t(in sec)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
v (m/s)	45	40	35	30	25	20	15	10	5	0

The velocity-time graph for the motion is:



(b)**The distance travelled can be found by a calculation of the area under the velocity time graph**

Distance = area of the triangle = $\frac{1}{2} \times base \times height$

Distance = Area of the triangle = 0.5 b x h

= 0.5 (4.5 s) × (45m/s)

Distance = Area of the triangle = 101 m

Note you might argue 'how can distance between two points be equal to the area?'

You must remember the context in motion with constant acceleration the area under the velocity time graph on measure is equal to the distance travelled. Also, the unit of the area from the graph is that of distance as

$= \frac{1}{2} \times$ base (time) $\times$ height (velocity)

$$= s \times \frac{m}{s} = m$$

(c)

u = 45.0 ms^{-1} v= 0.0 ms^{-1} a = -10.0 ms^{-2}

s =??

Using equation of motion

$v^2 = u^2 + 2\ a\ s$

$(0)^2 = (45)^2 + 2\ (-10)\ s$

$0 = 2025 - 20\ s$

$$s = \frac{2025}{20} = 101\ m$$

Distance = **101 m**

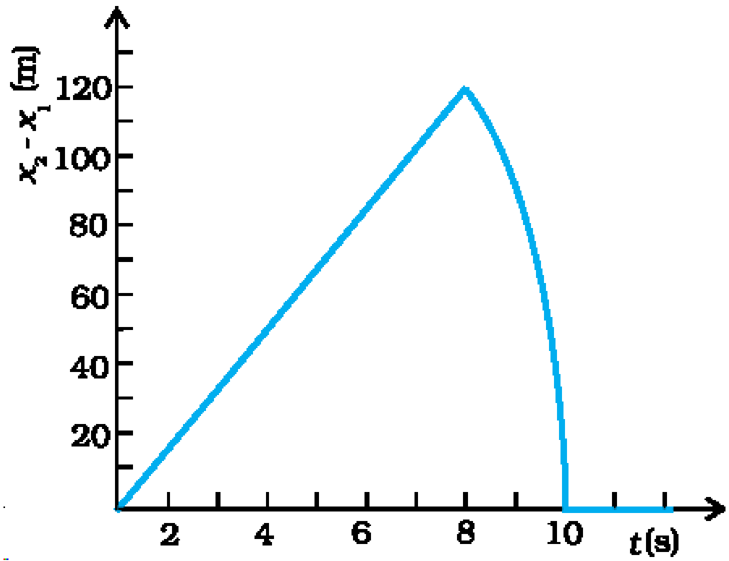
(d) Since the accident pileup is less than 101 m from Sumit, he will hit the pileup before completely stopping.

https://youtu.be/gTeMqej48Kw

(Reading position time graph)

EXAMPLE

Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 m s^{-1} and 30 m s^{-1}. Verify that the graph shown below correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take g = 10 m s^{-2}. Give the equations for the linear and curved parts of the plot.



SOLUTION

As we know

$$X(t) = x(0) + v(0)\,t + \frac{1}{2} g t^2$$

On choosing origin for position measurement on the ground, the positions of two stones at any instant t will be

$$x_1 = 200 + 15t - \frac{1}{2} 10 t^2 \qquad \text{..............(1)}$$

$$x_2 = 200 + 30t - \frac{1}{2} 10 t^2 \qquad \text{..............(2)}$$

Subtracting equation 1 from equation 2, we get the relative position of second stone w.r.t first

$$x_2 - x_1 = 15t$$

When first stone hits the ground,

$$x_1 = 0$$

$$200 + 15t - \frac{1}{2} 10t^2 = 0$$

or $\quad 200 + 15\,t - 5\,t^2 = 0$

or $\quad 5\,t^2 - 15\,t - 200 = 0$

On solving this equation we get

t = 8 sec or t = – 5 s

As time cannot be negative, we get

t = 8 s

This means the first stone hits the ground after 8 sec.

The relative position of second stone w.r.t. first is: $x_2 - x_1 = 15t$, shows that there is linear relationship between $x_2 - x_1$ and t, so the graph is straight line up to t = 8 s.

After t = 8 s, only the second stone is in motion. So the graph is parabolic as per the quadratic equation,

$$x_2 = 200 + 30t - \frac{1}{2}10t^2$$

The second stone will hit the ground, when $x_2 = 0$

So putting $x_2 = 0$, the above equation becomes

$200 + 30t - \frac{1}{2} 10\, t^2 = 0$

On solving this equation, we get

t = 10 sec

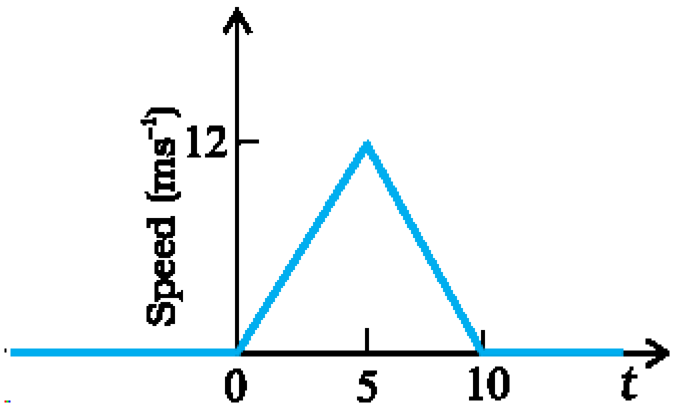
After t = 10 sec, the separation between the balls is zero.

For interpretation of motion graphs follow the following links

https://youtu.be/7GJ_SYM8cyU

EXAMPLE

The speed-time graph of a particle moving along a fixed direction is shown in Fig. Obtain the distance traversed by the particle between t = 0 s to 10 s.



What is the average speed of the particle over the interval?

SOLUTION

(1) Distance travelled by the particle between t = 0 to 10 sec is given by

S = Area of the triangle

$= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 10 \times 12$

$= 60$ m

Average speed = Total distance covered /total time taken

$= \frac{60}{10}$

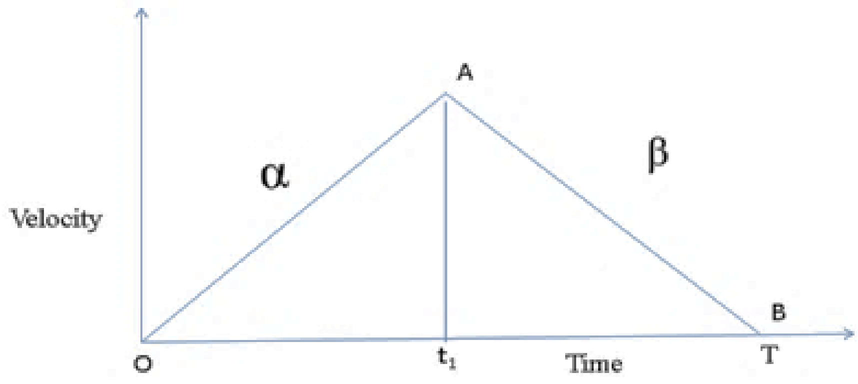
= 6 m s^{-1}

For Graphical Analysis of One-Dimensional Motion

http://philschatz.com/physics-book/contents/m42103.html

EXAMPLE

A car accelerates from rest, at a constant rate, α for some time, after which, it decelerates at a constant rate β, to come to rest. If the total time elapsed is T, then, find the maximum velocity, acquired by the car, and the distance covered.



SOLUTION

The graph shows that the car, accelerates till time t, and then, during the duration of $(T-t_1)$, it decelerates. Let, the maximum velocity acquired, in this process, is v.

We know that, the slope of the velocity v/s time graph, gives acceleration, and, the area gives the distance covered.

The slope of the line OA is,

$$\alpha = \frac{v}{t_1}$$

This can be written as:

$v = \alpha t_1$………. (1)

Similarly, the slope of line AB is given by

$$\beta = \frac{v}{(T-t_1)}$$

This can be written as:

$v = \beta (T - t_1)$ …… (2)

Equating, equations (1) and (2), we get

$\alpha t_1 = \beta (T - t_1)$ if we open the bracket the equation will become

$\alpha\, t_1 = \beta\, T - \beta t_1$

On rearranging the equation we get

$\alpha\, t_1 + \beta t_1 = \beta\, T$

On solving the equation we get the value of t_1

This is

$$\mathbf{t_1} = \frac{\beta\, T}{(\alpha + \beta)} \;\ldots\ldots\; (3)$$

Now, substituting the value of t_1

From equation (3) in equation (1) which is

$v = \alpha\, t_1$, we get

$$\boldsymbol{v} = \frac{\alpha\, \beta\, T}{(\alpha + \beta)} \qquad \ldots\ldots\ldots (4)$$

This is the expression for the maximum velocity acquired by the car.

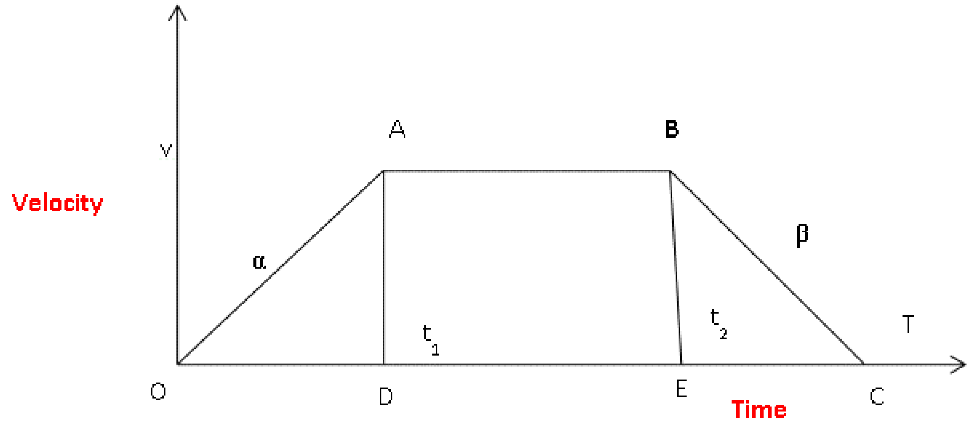
EXAMPLE

The velocity of a train, increases at a constant rate α, from 0 to v, and then, remains constant for some time interval, and then finally decreases to zero; at a constant rate β.If the total distance covered by the particle is S, then show that the total time taken will be

$$\boldsymbol{T = \frac{s}{v} + \frac{v}{2} + \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)}$$

SOLUTION

The v-t graph for given motion is



The line OA, represents motion with, constant acceleration α.

Straight line AB, parallel to the time axis, represents the motion, with uniform velocity.

The line BC, represents the motion with, constant acceleration β.

We know that, the slope of v-t graph, gives the acceleration.

The slope of the line OA is

$\alpha = \frac{v}{OD}$ which can be written as

$OD = \frac{v}{\alpha}$ ……. (1)

Similarly, the slope of the line BC is

$\beta = \frac{v}{EC}$ which can be written as

$EC = \frac{v}{\beta}$ ……… (2)

The distance from vt graph can be calculated by taking area under the curve

Distance = area under the v-t graph

In our graph, S = Area of the trapezium

$$S = \frac{1}{2} \times (AB + OC) \times v$$

As, in the graph AB = DE, AD = v ad OC = OD +EC + DE

We can write the above equation as,

$$S = \frac{1}{2} \times (DE + OD + EC + DE) \times AD$$

$$S = \frac{1}{2} \times (2DE + OD + EC) \times AD$$

Substituting the value of OD and EC, we get

$$S = \frac{1}{2} \times (2DE + \frac{v}{\alpha} + \frac{v}{\beta}) \times v \text{ (on rearrangement it becomes)}$$

$$\frac{2S}{v} = (2DE + \frac{v}{\alpha} + \frac{v}{\beta})$$

$$\frac{2S}{v} - \frac{v}{\alpha} - \frac{v}{\beta} = 2DE$$

$$DE = \frac{1}{2} \times \left(\frac{2S}{v} - \frac{v}{\alpha} - \frac{v}{\beta}\right) \quad \text{.....(3)}$$

IN THE GRAPH Total time

T = OD + DE + EC putting their values we get

$$= \frac{v}{\alpha} + \frac{1}{2} \times \left(\frac{2S}{v} - \frac{v}{\alpha} - \frac{v}{\beta}\right) + \frac{v}{\beta}$$

$$= \frac{v}{\alpha} + \frac{S}{v} - \frac{v}{2\alpha} - \frac{v}{2\beta} + \frac{v}{\beta}$$

$$= \frac{S}{v} + \frac{v}{2\alpha} + \frac{v}{2\beta}$$

$$T = \frac{S}{v} + \frac{v}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

Hence, we proved the required result.

https://youtu.be/NKOzGf1nNfc

(Interpreting velocity time graph)

7. SUMMARY

If a body starts with velocity u and after time t its velocity changes to v, if the uniform acceleration is a and the distance travelled in time t in s, then the following relations are obtained, which are called equations of uniformly accelerated motion.

(i) $\mathbf{v = u + at}$

(ii) $\boldsymbol{s = ut + \frac{1}{2}at^2}$

(iii) $\mathbf{v^2 = u^2 + 2as}$

Motion under Gravity:

If an object is falling freely (u = 0) under gravity, then equations of motion are taken as:

(i) $\mathbf{v = u + g\,t}$

(ii) $\boldsymbol{h = ut + \frac{1}{2}gt^2}$

(iii) $\mathbf{v^2 = u^2 + 2gh}$