1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics - Part 1, Class XI)
Module Name/Title	Unit 2, Module 4, **Accelerated motion in one dimension** Chapter 3, Motion in a Straight Line
Module Id	Keph_10304_eContent
Pre-requisites	Motion in a straight line, gravity, instantaneous speed, average speed, distance, displacement
Objectives	**After going through this module, the learners will be able to:** • Understand rate of change of speed or velocity is acceleration • Derive Equations of motion (which relate initial and final velocities, time elapsed to do so, distance /displacement, and acceleration) • Apply equations of motion to solve simple examples from daily life • Recognize Free fall • Distinguish between velocity and relative velocity
Keywords	Acceleration, average acceleration, instantaneous acceleration, equations of motion free fall graph for accelerated motion, relative velocity

2. Development Team

Role	Name	Affiliation
National MOOC Coordinator (NMC)	Prof. Amarendra P. Behera	Central Institute of Educational Technology, NCERT, New Delhi
Programme Coordinator	Dr. Mohd Mamur Ali	Central Institute of Educational Technology, NCERT, New Delhi
Course Coordinator / PI	Anuradha Mathur	Central Institute of Educational Technology, NCERT, New Delhi
Subject Matter Expert (SME)	Anuradha Mathur	Central Institute of Educational Technology, NCERT, New Delhi
Review Team	Prof. V. B. Bhatia (Retd.) Associate Prof. N.K. Sehgal (Retd.) Prof. B. K. Sharma (Retd.)	Delhi University Delhi University DESM, NCERT, New Delhi

TABLE OF CONTENTS:

1. UNIT SYLLABUS

Chapter 1: Motion in a straight line

Frame of reference, motion, position –time graph Speed and velocity
Elementary concepts of differentiation and integration for describing motion, uniform and non-uniform motion, average speed and instantaneous velocity, uniformly accelerated motion, velocity –time and position time graphs relations for uniformly accelerated motion - equations of motion (graphical method).

Chapter 2: Motion in a plane
Scalar and vector quantities, position and displacement vectors, general vectors and their notations, multiplication of vectors by a real number , addition and subtraction of vectors, relative velocity, unit vector, resolution of a vector in a plane, rectangular components ,scalar and vector product of vectors
Motion in a plane, cases of uniform velocity and uniform acceleration projectile motion uniform circular motion.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 10 Modules

The above unit is divided into 10 modules for better understanding.

Module 1	• Introduction to moving objects • Frame of reference, • limitations of our study • treating bodies as point objects
Module 2	• Motion as change of position with time • Distance travelled unit of measurement • Displacement negative, zero and positive • Difference between distance travelled and displacement • Describing motion by position time and displacement time graphs
Module 3	• Rate of change of position • Speed • Velocity • Zero , negative and positive velocity • Unit of velocity • Uniform and non-uniform motion • Average speed • Instantaneous velocity • Velocity time graphs • Relating position time and velocity time graphs
Module 4	• Accelerated motion • Rate of change of speed, velocity • Derivation of Equations of motion
Module 5	• Application of equations of motion • Graphical representation of motion • Numerical
Module 6	• Vectors

	• Vectors and physical quantities • Vector algebra • Relative velocity • Problems
Module 7	• Motion in a plane • Using vectors to understand motion in 2 dimensions' projectiles • Projectiles as special case of 2 D motion • Constant acceleration due to gravity in the vertical direction zero acceleration in the horizontal direction • Derivation of equations relating horizontal range vertical range velocity of projection angle of projection
Module 8	• Circular motion • Uniform circular motion • Constant speed yet accelerating • Derivation of $a = \frac{v^2}{r} or\ \omega^2 r$ • direction of acceleration • If the speed is not constant? • Net acceleration
Module 9	• Numerical problems on motion in two dimensions • Projectile problems
Module 10	• Differentiation and integration • Using logarithm tables

Module 4

3. WORDS YOU MUST KNOW

Let us remember the words we have been using in our study of this physics course

- Rigid body: An object for which individual particles continue to be at the same separation over a period of time.

- Point object: If the position of an object changes by distances much larger than the dimensions of the body the body may be treated as a point object.

- Frame of reference: Any reference frame the coordinates (x, y, z), which indicate the change in position of object with time.

- Inertial frame: It is a stationary frame of reference or one moving with constant speed.

- Observer: Someone who is observing objects.

- Rest: A body is said to be at rest if it does not change its position with surroundings with time.

- Motion: A body is said to be in motion if it changes its position with respect to its surroundings with the passage of time.

- Time elapsed: Time interval between any two observations of an object.

- Motion in one dimension: When the position of an object can be shown by change in any one coordinate out of the three (x, y, z), also called motion in a straight line.

- Motion in two dimension: When the position of an object can be shown by changes any two coordinate out of the three (x, y, z), also called motion in a plane.

- Motion in three dimension: When the position of an object can be shown by changes in all three coordinate out of the three (x, y, z).

- Distance travelled: The distance an object has moved from its starting position SI unit m, this can be zero, or positive.

- Displacement: The distance an object has moved from its starting position moves in a particular direction.SI unit: m, this can be zero, positive or negative.

- Path length: Actual distance is called the path length.

- Position time, distance time, displacement time graph: These graphs are used for showing at a glance the position, distance travelled or displacement versus time elapsed.
- Speed: Rate of change of distance is called speed. Its SI unit is m/s.

- Average speed: Total path length divided total time taken for the change in position.
- Velocity: Rate of change of position in a particular direction is called velocity, it can be zero, negative and positive, and its SI unit is m/s.
- Instantaneous velocity velocity of an object at any instant of time

- Velocity time graph: Graph showing change in velocity with time, this graph can be obtained from position time graphs.

4. INTRODUCTION

We are studying motion of objects. We were dealing with motion of rigid bodies along a straight line, where the position of an object in any coordinate system was given by change in any one of x, y or z in (x, y, z). The change in position was the displacement or distance travelled, the rate at which the body changed its position was velocity or speed, and this value could be constant or variable. We also understood the meaning of, average and instantaneous velocity and speed. We learnt to represent these quantities mathematically and graphically.

When the velocity of a body changes, we call this accelerated motion.

Now, in this module,

- **We will understand the meaning of acceleration, constant and variable acceleration with suitable examples.**
- **We will understand zero, positive and negative acceleration. We will find a relation between initial and final velocities, the time in which the change in velocity takes place, the distance covered during the motion.**

- **These relations between the said physical quantities initial velocity, final velocity, acceleration, distance travelled and time are called equations of motion. We will also learn to solve simple problems using the equations of motion**
- **We will also study the concept of free fall, a special case of body moving with constant acceleration in daily life. We will consider relative motion.**

5. ACCELERATION

Rate of change of velocity is called acceleration.

As we have studied earlier

$$\text{speed} = \frac{\text{distance travelled}}{\text{time}}$$

$$= \frac{X_2 - X_1}{t_2 - t_1}$$

Or we can write the same in terms of

position at instant 0 x (0), and position at instant t x(t)

$$= \frac{X(t) - X(0)}{t - 0}$$

In the same way we can say that at instant t, the velocity of an object may be v(0) initial velocity, at instant zero.

We can represent the velocity at a later instant by v(t)

IMPORTANT

In your earlier courses you referred to initial velocity as u and final velocity as v. We are now describing it in terms of time instant at which the velocity is observed.

So **change in velocity** will be given by **v(t) – v(0)** in **time interval (t-0)**

So time rate of change of velocity will be

$$= \frac{v(t) - v(0)}{t - 0}$$

Notice we are sometimes saying speed and sometimes velocity in one dimensional motion or motion in a straight line because the direction remains constant and it is the magnitude of speed that changes

Acceleration occurs when the velocity changes with time this can be due to

i. **Change in magnitude**
ii. **Change in direction**
iii. **Both change in magnitude and direction**

Acceleration should be reported with direction so we represent velocity with vector notation, $\vec{v}$, so when we say a car is travelling with a speed of 50 km/h we mean speed , but if we say that the car is moving towards the north at 50 km/h we mean velocity.

https://www.youtube.com/watch?v=VDaHWm2DD8A

The above videos show cyclists and cars moving with different speeds, different velocities

Though change in velocity can be described in two ways

i. **change in velocity with distance**

ii. **change in velocity with time**

We are only considering change in velocity with time.

Its SI unit is unit ms^{-2} and the dimensional formula is

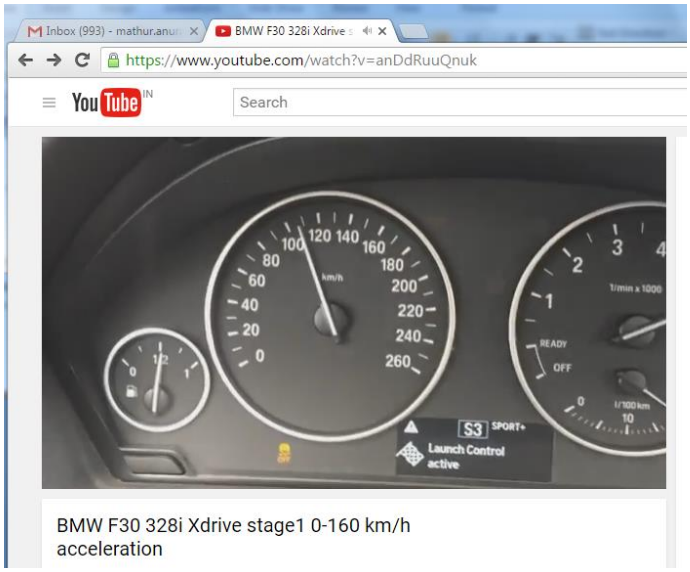
6. AVERAGE ACCELERATION

In real life, the rate of change of velocity due to change in speed in vehicles is observed only with **speedometer**. Since this value may be changing during the motion we describe average acceleration. This only indicates the general acceleration over a period of time and not at different instants of time.

Let us use the video and plot a graph.

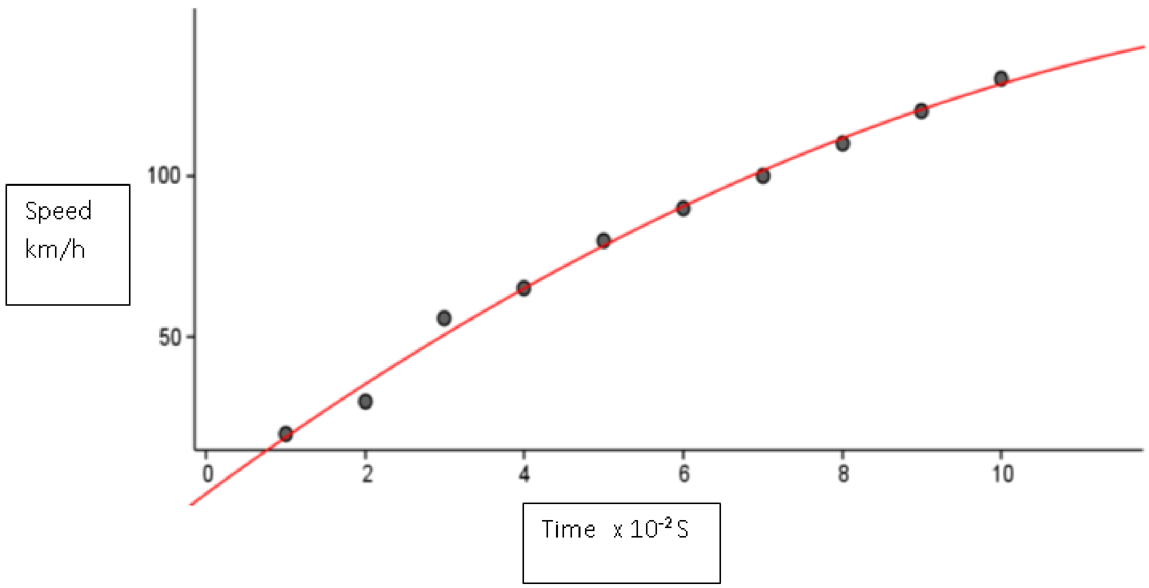
https://www.youtube.com/watch?v=42iLg6eXGUE

The speedometer values along with time as shown by the video playing bar



Using Geo Gebra or on a graph sheet plot the graph and answer the questions that follow

Time in s	Speedometer reading km/h
0.01	20
0.02	30
0.03	56
0.04	65
0.05	90
0.06	100
0.07	110



1. **Is the speed constant?**
2. **Is the car in uniform motion?**
3. **Is the change in speed constant?**

View this video on the given link
https://www.youtube.com/watch?v=znV8QtUIcag

We have plotted one graph as an example; you can plot a similar graph with other readings and compare the nature of acceleration. **This exercise is done by car makers to study the ability of their engine design.**

Since, we defined Acceleration as change of velocity with time

- **acceleration is constant if the change in velocity is equal in equal intervals of time also called constant or uniform acceleration**
- **non-uniform acceleration occurs in case the rate of change of velocity is changing with time**
- **The above graph shows non-uniform acceleration.**

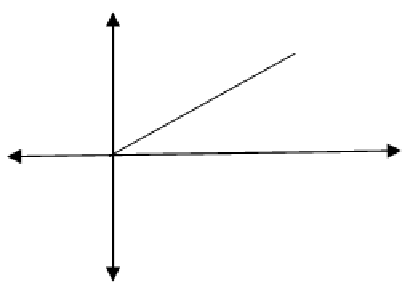
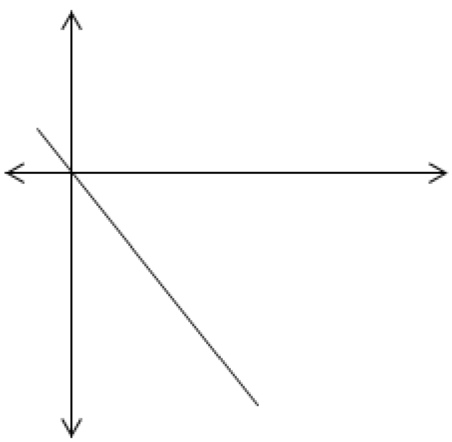
- **average acceleration is the change in velocity divided by the time interval** $\overline{a} = \frac{\Delta v}{\Delta t}$

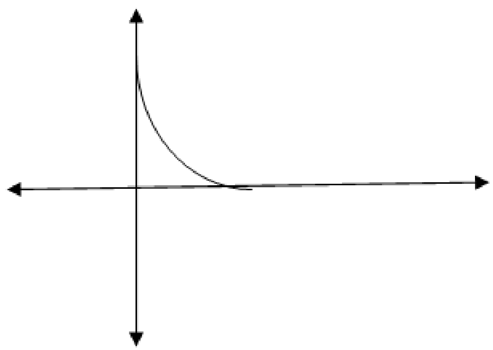
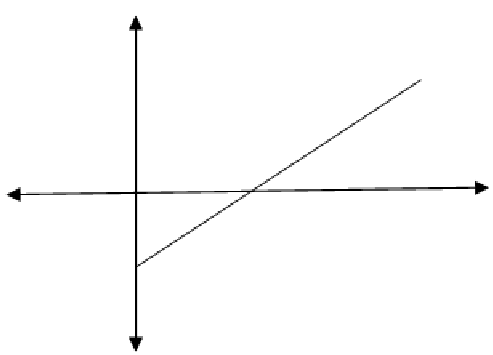
  As said earlier, average acceleration gives an idea of how the velocity changes over a period of time

TRY YOUR SELF:-

Check these <u>velocity-time</u> graphs out, identify zero, negative, positive, uniform and non-uniform acceleration give reason for your answer.

Note: on x axis time is plotted while on y-axis velocity is plotted

Graph	Nature of acceleration with reason
	
	

UNIFORM ACCELERATION

Is thus defined over a period of time interval

$$a = \frac{V - V_0}{t - 0} = \frac{\Delta v}{\Delta t}$$

Or

$$a = \frac{v(t) - v(0)}{t - 0} = \frac{\Delta v}{\Delta t}$$

Where V is the final velocity and V_0 is the initial velocity

Or v(t) is the velocity at time instant t and v(0) is the velocity at instant 0, which is the instant we decide to consider the velocity of a moving object.

Over a time, interval of Δt

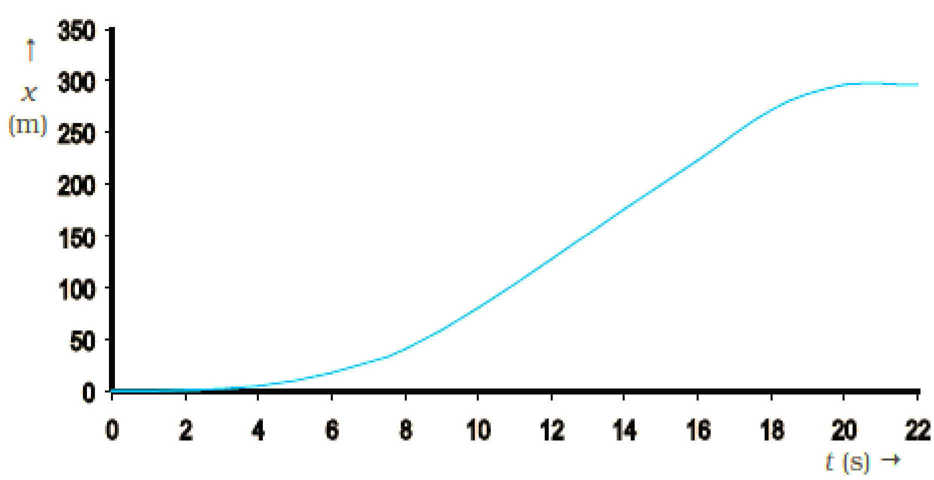
The **three graphs** given are for

i) **position time,**

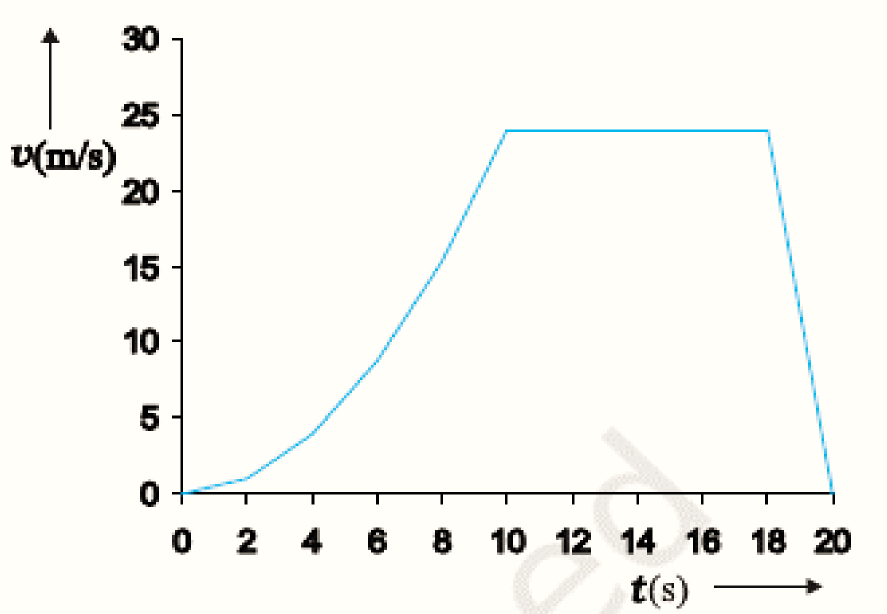
ii) **velocity time** and

iii) **acceleration time**

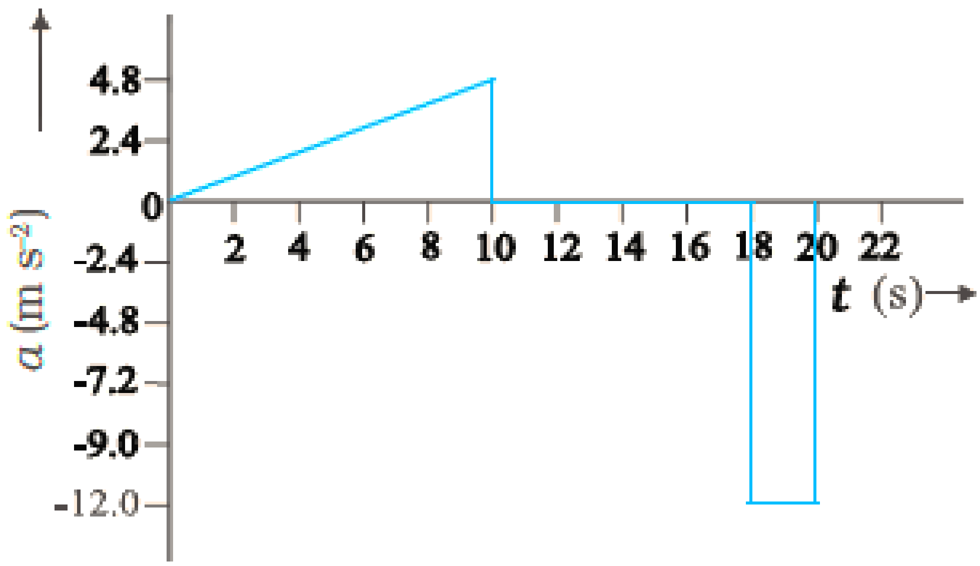
For an object, keeping all three in mind one can see the position, velocity or acceleration at any instant of time.



Position-time graph of a car



Velocity–time graph corresponding to motion



Acceleration as a function of time for motion

Notice the values in the three graphs at the same time instant, you may draw them on graph sheet

7. INSTANTANEOUS ACCELERATION

Instantaneous acceleration is defined in the same way as the instantaneous velocity:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

The acceleration at an instant is the slope of the tangent to the v–t curve at that instant.

For the v- t curve as shown above, we can obtain acceleration at every instant of time.

The resulting a-t curve as shown above

We see that the acceleration is non-uniform over the period 0 s to 10 s. It is zero between 10 s and 18 s and is constant with value –12 m s^{-2} between 18 s and 20 s.

8. POSITIVE, NEGATIVE AND ZERO ACCELERATION

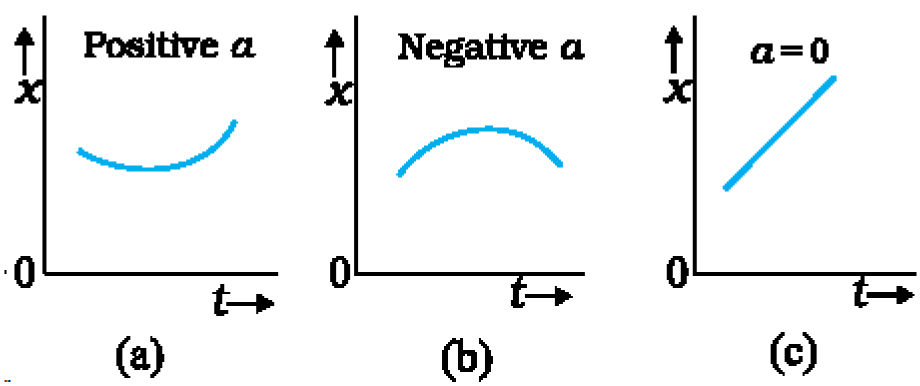
When the acceleration is uniform, obviously, it equals the average acceleration over that period. Since velocity is a quantity having both magnitude and direction, a change in velocity may involve either or both of these factors. Acceleration, therefore, may result from a change in speed (magnitude), a change in direction or changes in both. Like velocity, acceleration can also be positive, negative or zero.

Position-time graphs for motion with

Positive acceleration Fig.(a),

Negative acceleration Fig.(b) and

Zero acceleration Fig.(c).



Note that the graph curves upward for positive acceleration; downward for negative acceleration and it is a straight line for zero acceleration

Although acceleration can vary with time,

Our study in this chapter will be restricted to motion with constant acceleration.

In this case, the **average acceleration** equals the constant value of acceleration during the time interval.

If the velocity of an object is v_o at $t = 0$ and v at time t, we have

$$\bar{a} = \frac{v - v_o}{t - 0} \quad \text{or} \quad v = v_o + at$$

9. GRAPHICAL STUDY OF ACCELERATION

We will take up an interesting example to understand average and instantaneous acceleration

We are using the time record with distance of Usain Bolt a Jamaican athlete of note four time Olympic (gold medalist) champions in 100m 200m like Carl Lewis.

We can find the velocity for each of the time intervals as also the acceleration

The table is generated to show time interval to travel a distance of 10m, velocity for the time interval is calculated, and change in velocity at intervals of 10m and hence acceleration is calculated using the above formula for rate of change of velocity.

It is interesting to see from the last column that the acceleration is not uniform. The world record holder for 100m sprint initially accelerates at a very high rate and closes with actually slowing down.

Record of position and time for Usain bolt

Find out more about Usain Bolt

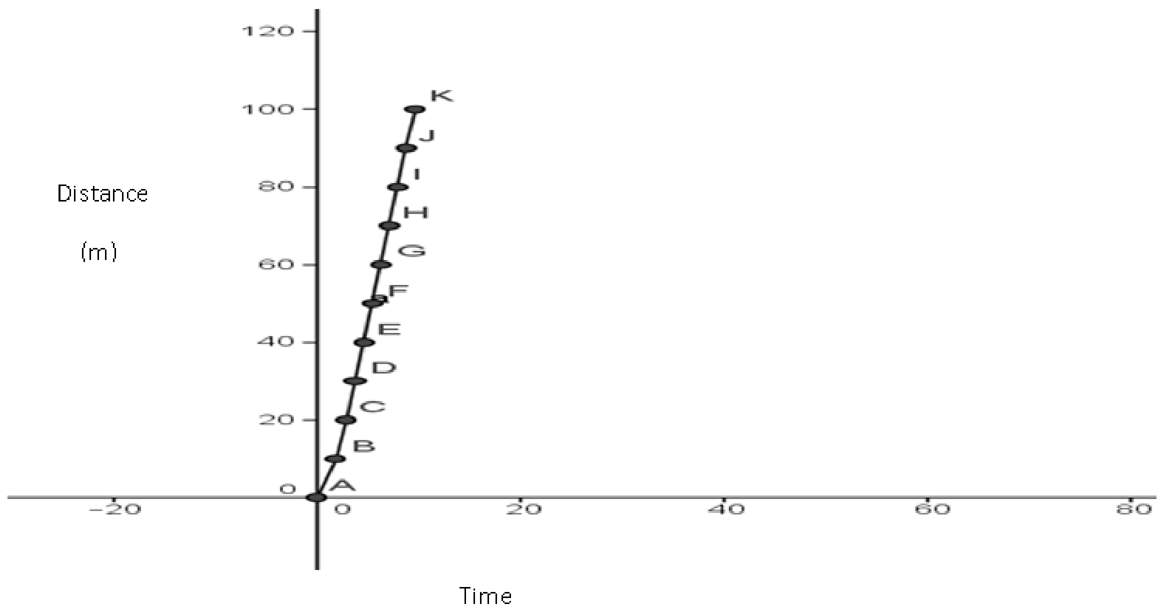
X	T
0	0
10.0	1.85
20.0	2.87
30.0	3.78
40.0	4.65
50.0	5.50
60.0	6.32
70.0	7.14
80.0	7.96
90.0	8.79
100.0	9.69

This table is available on the internet

TABLE WITH CALCULATIONS

X	**T**	**Δt for 10m**	$\mathbf{V = \frac{distance}{time}}$ **(m/s)**	**Δv for 10m**	$\mathbf{A = \frac{Velocity}{time}}$ **(m/s^2)**
0	0	0	0	0	0
10.0	1.85	1.85	5.40	5.40	2.9
20	2.87	1.02	9.80	4.40	4.31
30	3.78	0.91	10.98	1.18	1.29
40	4.65	0.87	11.49	0.59	0.67
50	5.50	0.85	11.76	0.27	0.003
60	6.32	0.82	12.19	0.43	0.003
70	7.14	0.82	12.19	0	0
80	7.96	0.82	12.19	0	0
90	8.79	0.83	12.04	-0.15	-0.18
100	9.69	0.90	11.11	-0.93	-1.03

Distance-time graph



Average velocity = $\frac{100}{9.68} = 10.33\ ms^{-1} = 33\ km\ h^{-1}$

Need for describing instantaneous acceleration from graph

By calculation of average velocity, we find the value 10.33 m/s, this tells us the speed over a given interval of time but, does not tell us how fast he moved at say 7th second, for this we define instantaneous acceleration in the same way as instantaneous velocity

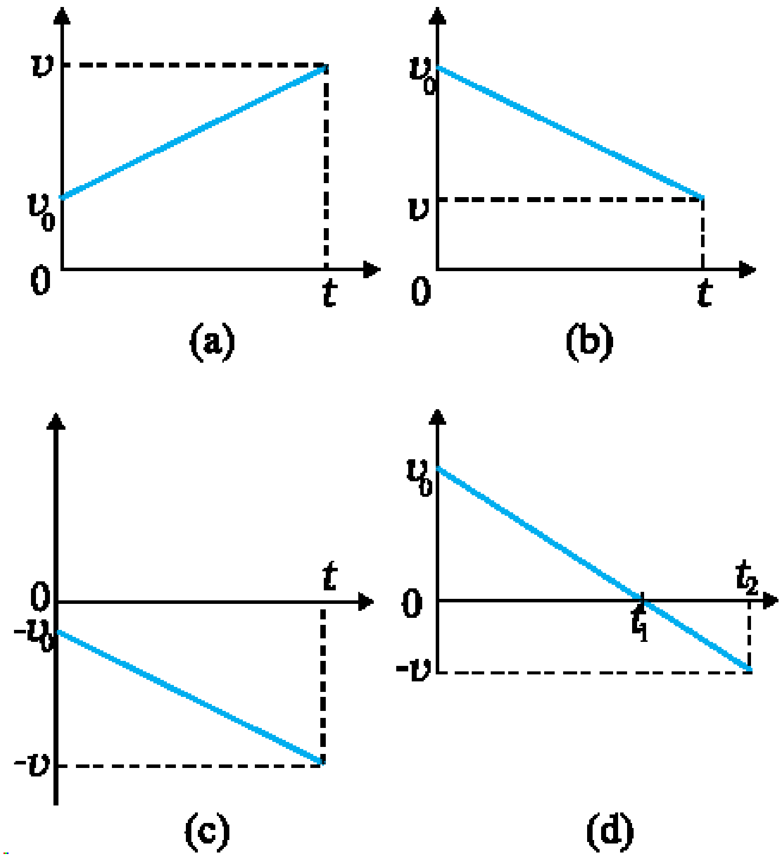
Instantaneous acceleration or acceleration at any instant

It is given by the slope of velocity time graph at an instant

Or $a = \lim_{\Delta t \to 0} \bar{a} = \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

Acceleration may vary with time but we will currently only study situations of constant acceleration.

Just to recall the velocity - time graphs for constant acceleration.



Acceleration and velocity of an object do not change abruptly at an instant, changes are always continuous.

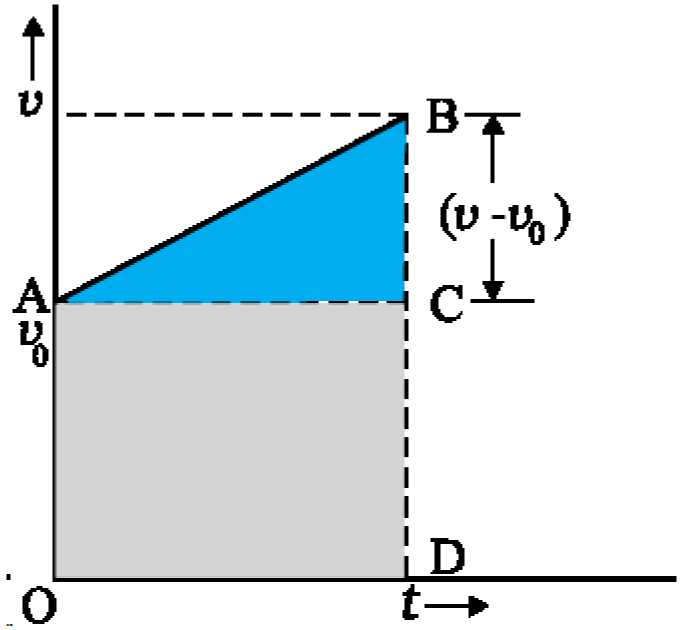
Constant acceleration can be negative or positive as we have seen.

10. DERIVATION OF EQUATIONS OF MOTION

For uniformly accelerated motion we can derive relation between the following physical quantities

1. **Initial velocity**
2. **Final velocity**
3. **Distance travelled**
4. **Acceleration**
5. **Time interval for the distance travelled**

The process of finding the relation between related physical quantities is called Derivation.



v = **final velocity**

v_0= **initial velocity**

t = time interval for velocity to change from v_0 ***to*** v

a = constant acceleration

From our definition of acceleration

$$a = \frac{v - v_0}{t}$$

Or

$$\mathbf{v = v_0 + at}$$

This is the **first equation of motion.**

The area under the velocity time graph gives the distance /displacement s (distance travelled = area of the colored portion of the graph, which has two sections grey and blue

= area of rectangle OACD + area of triangle ABC

$$x = v_0 t + \frac{1}{2}(v - v_0)t$$

Using first equation of motion $v = v_0 +$ at

$$\mathbf{x = v_0 t + \frac{1}{2} a t^2}$$

The equation is true only for constant acceleration which may be zero, positive or negative

This is also our **second equation of motion**.

x is a vector and could be zero, positive or negative.

We know average velocity $\bar{v} = \frac{v_0 + v}{2}$

And $x = \bar{v}t$

Using the value of above average velocity and the value of t from the first equation we have

$$\boldsymbol{x} = \left(\frac{\boldsymbol{v_0} + v}{\mathbf{2}}\right)\left(\frac{\boldsymbol{v} - \boldsymbol{v_0}}{\boldsymbol{a}}\right) = \frac{\boldsymbol{v^2} - {v_0}^2}{\mathbf{2}\boldsymbol{a}}$$

$$\boldsymbol{v^2} = \boldsymbol{v_0}^{\mathbf{2}} + \mathbf{2ax}$$

This is our **third equation of motion**.

Depending on the given physical quantities, the above three equations can be modified as follows

1. v = $\boldsymbol{v_0}$ + at
2. x = $\boldsymbol{\bar{v} t}$
3. $\mathbf{x = v_0 t + \frac{1}{2} a t^2}$
4. $\mathbf{v^2 = v_0^2 + 2ax}$
5. $\mathbf{x = vt - \frac{1}{2} a t^2}$

6. Distance travelled in the nth second

$$= \mathbf{x_n} = \mathbf{v_0} + \frac{\mathbf{a}}{2}(\mathbf{2n - 1})$$

In the above Kinematic equations of motion, the various quantities are algebraic.

They can be taken as vector equations, if we choose the identify directions with the physical quantities.

Remember the direction is taken according to the Cartesian coordinate system… Initial point is taken as (0, 0), all physical quantities in +x or + y direction to be positive.

Also in case the acceleration is not constant, the above equations will be more complex as instantaneous velocity and instantaneous positions would have to be considered.

11. APPLICATION OF EQUATIONS OF MOTION

Objects Falling under gravity.

The ball and the feather experiment in the world's largest vacuum chamber

https://www.youtube.com/watch?v=E43-CfukEgs

This means that all objects hollow or solid, big or small should fall at the same rate. According to a story, Galileo dropped different objects from the top of the Leaning Tower of Pisa in Italy to prove the same.

http://physics.bu.edu/~duffy/HTML5/Galileos_ramp.html

Try the simulation

Pumpkin and the ball:

Consider the video of a freely falling body the speed of falling object is changing in the same way or the acceleration must be independent of mass of the body

We have learnt that the earth attracts objects towards it. This is due to the gravitational force. Whenever objects fall towards the earth under this force alone, we say that the objects are in free fall. Is there any change in the velocity of falling objects? While falling, there is no change in the direction of motion of the objects. But due to the earth's attraction, there will be a change in the magnitude of the velocity. Any change in velocity involves acceleration.

12. FREE FALL MOTION UNDER GRAVITY

Whenever an object falls towards the earth, acceleration is involved. This acceleration is called the acceleration due to the gravitational force of the earth (or acceleration due to gravity).

It is denoted by g.

The unit of g is the same as that of acceleration, that is, m s^{-2}.

Its average value near the surface of the earth is 9.8 ms^{-2}.

As g is independent of mass of the freely falling object, the ball and pumpkin reached the ground simultaneously.

For freely falling bodies ($V_0 = 0$) falling through heights small compared to the radius of the earth g can be taken as constant = 9.8 ms^{-2}.

You will learn in the unit of Gravitation. The value of 'g' may change due to different factors, but for now the value of acceleration due to gravity is constant.

EXAMPLE

A ball is thrown vertically upwards with a velocity of 20 m s^{-1} from the top of a multistory building. The height of the point from where the ball is thrown is 25.0 m from the ground.

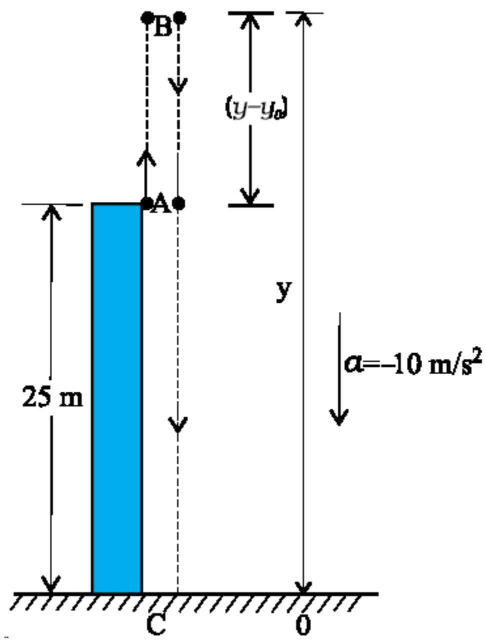
(a) How high will the ball rise?

(b) How long will it be before the ball hits the ground?

Take g = 10 m s^{-2}

SOLUTION

(a) Let us take the y-axis in the vertically upward direction with zero at the ground, as shown below.



Now $v_o = +\,20\ m\ s^{-1}$,

$a = -\,g = -10\ m\ s^{-2}$,

$v = 0\ m\ s^{-1}$

If the ball rises to height y from the point of launch, then using the equation

$$v^2 = {v_0}^2 + 2a\,(y - y_0)$$

We get

$0 = (20)^2 + 2(-10)\,(y - y_0)$

Solving, we get, $(y - y_0) = 20$ m.

(b) We can solve this part of the problem in two ways. Note, carefully the methods used.

FIRST METHOD: In the first method, we split the path in two parts: the upward motion (A to B) and the downward motion (B to C) and calculate the corresponding time taken t_1 and t_2. Since the velocity at B is zero, we have:

$v = v_o + at$

$0 = 20 - 10t_1$

Or, t_1 = 2 s

This is the time in going from A to B. From B, or the point of the maximum height, the ball falls freely under the acceleration due to gravity. The ball is moving in negative y direction. We use equation

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

We have, y_0 = 45 m, y = 0, v_0 = 0, a = – g = –10 m s^{-2}

$$0 = 45 - \frac{1}{2} \times (-10) \times t_2^2$$

Solving, we get t_2 = 3 s

Therefore, the total time taken by the ball before it hits the ground = $t_1 + t_2$ = 2 s + 3 s = 5 s.

SECOND METHOD: The total time taken can also be calculated by noting the coordinates of initial and final positions of the ball with respect to the origin chosen and using equation

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

Now, y_0 = 25 m, y = 0 m

v_o = 20 m s^{-1},

a = –10m s^{-2},

t =?

$$0 = 25 + 20\,t + \frac{1}{2} \times (-10) \times t^2$$

$$\text{Or, } 5t^2 - 20t - 25 = 0$$

Solving this quadratic equation for t, we get

$$t = 5\text{s}$$

Note that the second method is better, since we do not have to worry about the path of the motion as the motion is under constant acceleration.

DISCUSSION

An object released near the surface of the Earth is accelerated downward under the influence of the force of gravity. The magnitude of acceleration due to gravity is represented by g. If air resistance is neglected, the object is said to be in free fall. If the height through which the object falls is small compared to the earth's radius, g can be taken to be constant, equal to 9.8 m s^{-2}. Free fall is thus a case of motion with uniform acceleration.

We assume that the motion is in y-direction, more correctly in -y-direction because we choose upward direction as positive. Since the acceleration due to gravity is always downward, it is in the negative direction and we have

$$a = -g = -9.8 \text{ m s}^{-2}$$

The object is released from rest at y = 0. Therefore, $v_0 = 0$ and the equations of motion become:

$$v = 0 - g\,t = -9.8\,t \text{ m s}^{-1}$$

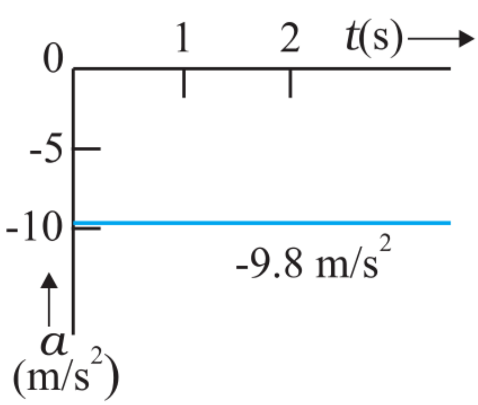
$$y = 0 - \frac{1}{2}\,gt^2 = -4.9\,t^2 \text{ m}$$

$$v^2 = 0 - 2\,g\,y = -19.6\,y \text{ m}^2\text{ s}^{-2}$$

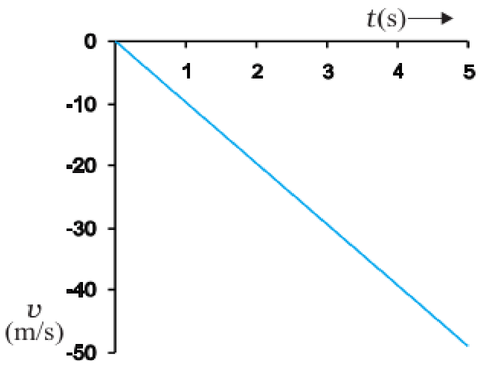
These equations give the velocity and the distance travelled as a function of time and also the variation of velocity with distance.

EXAMPLE

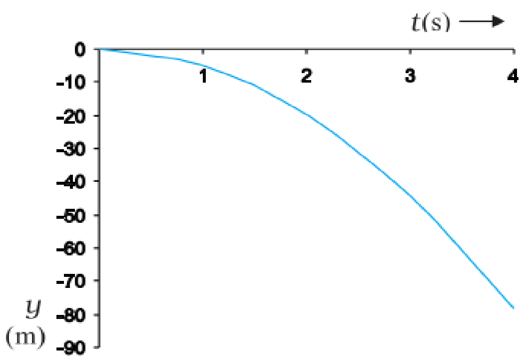
The variation of acceleration, velocity, and distance, with time have been plotted in Fig (a), (b) and (c).



(a)



(b)



(c)

Motion of an object under free fall

(a) Variation of acceleration with time.

(b) Variation of velocity with time.

(c) Variation of distance with time.

13. RELATIVE VELOCITY

You must be familiar with the experience of travelling in a train and being overtaken by another train moving in the same direction as you are. While that train must be travelling faster than you to be able to pass you, it does seem slower to you than it would be to someone standing on the ground and watching both the trains. In case both the trains have the same velocity with respect to the ground, then to you the other train would seem to be not moving at all. To understand such observations, we now introduce the concept of relative velocity.

Consider two objects A and B moving uniformly with average velocities v_A and v_B in one dimension, say along x-axis. (Unless otherwise specified, the velocities mentioned in this chapter are measured with reference to the ground).

If $x_A(0)$ and $x_B(0)$ are positions of objects A and B, respectively at time t = 0, their positions $x_A(t)$ and $x_B(t)$ at time t are given by:

$x_A(t) = x_A(0) + v_A t$

$x_B(t) = x_B(0) + v_B t$

Then, the displacement from object A to object B is given by

$$x_{BA}(t) = x_B(t) - x_A(t) = [\ x_B(0) - x_A(0)\ ] + (v_B - v_A)\ t.$$

It tells us that as seen from object A, object B has a velocity $v_B - v_A$ because the displacement from A to B changes steadily by the amount $v_B - v_A$ in each unit of time.

We say that the velocity of object B relative to object A is $v_B - v_A$:

$v_{BA} = v_B - v_A$

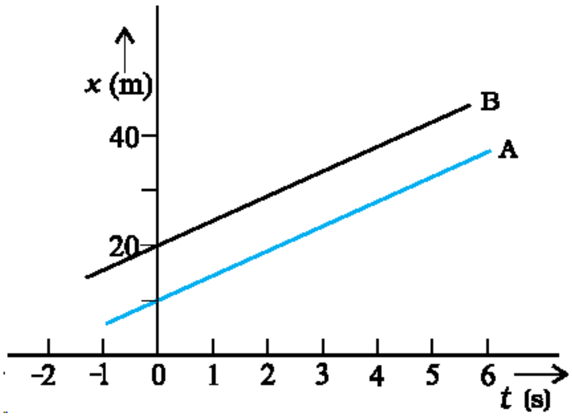
Similarly, velocity of object A relative to object B is:

$v_{AB} = v_A - v_B$

This shows: $v_{BA} = -v_{AB}$

SOME INTERESTING GRAPHS

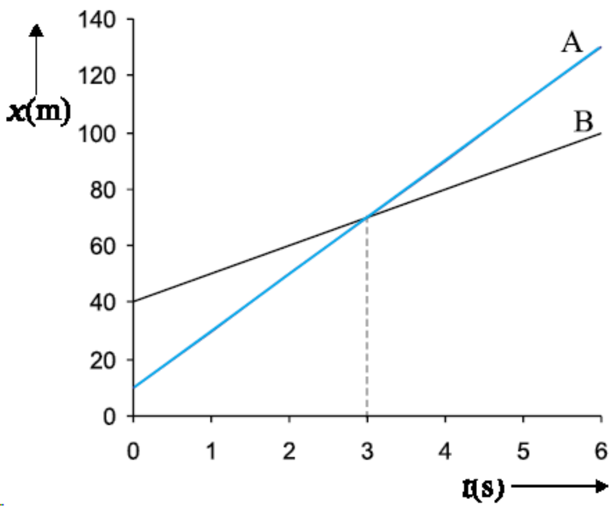
.



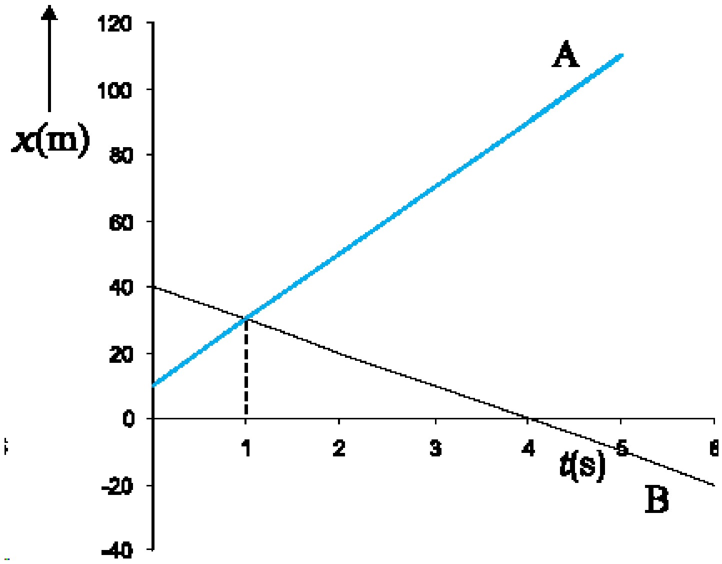
Position-time graphs of two objects with equal velocities

EXAMPLE

Interpret the following



Position-time graphs of two objects with unequal velocities, showing the time of meeting.



Position-time graphs of two objects with velocities in opposite directions, showing the time of meeting.

EXAMPLE

Two parallel rail tracks run north-south. Train A moves north with a speed of 54 km h^{-1}, and train B moves south with a speed of 90 km h^{-1}. What is the:

(a) Velocity of B with respect to A?

(b) Velocity of ground with respect to B?

(c) velocity of a monkey running on the roof of the train A against its motion (with a velocity of 18 km h^{-1} with respect to the train A) as observed by a man standing on the ground ?

SOLUTION

Choose the positive direction of x-axis to be from south to north. Then,

$$v_A = +54\ km\ h^{-1} = 15\ ms^{-1}$$

$$v_B = -90\ km\ h^{-1} = -25\ ms^{-1}$$

Relative velocity of B with respect to A = $v_B - v_A = -40\ ms^{-1}$, i.e. the train B appears to A to move with a speed of 40 m s^{-1} from north to south.

Relative velocity of ground with respect to B = 0 – v_B = 25 m s^{-1}.

In (c), let the velocity of the monkey with respect to ground be v_M. Relative velocity of the monkey with respect to A,

$$v_{MA} = v_M - v_A = -18 \text{ km h}^{-1} = -5 \text{ ms}^{-1}.$$

Therefore,

$$\boldsymbol{v_M = (15 - 5) \text{ m s}^{-1} = 10 \text{ m s}^{-1}.}$$

14. SUMMARY

In this module we have learnt:

- Acceleration is time rate of change of velocity.
- Velocity can change due to Change in magnitude of speed or change in direction of motion, so an object moving with constant speed can accelerate.
- Average acceleration is the change in velocity divided by the time interval during which the change occurs :

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

- Instantaneous acceleration is defined as the limit of the average acceleration as the time interval Δt goes to zero:

$$a = \lim_{\Delta t \to 0} \bar{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- The acceleration of an object at a particular time is the slope of the velocity-time graph at that instant of time. For uniform motion, acceleration is zero and the x-t graph is a straight line inclined to the time axis and the v-t graph is a straight line parallel to the time axis. For motion with uniform acceleration, x-t graph is a parabola while the v-t graph is a straight line inclined to the time axis.

- The area under the velocity-time curve between times t_1 and t_2 is equal to the displacement of the object during that interval of time.

- For objects in uniformly accelerated rectilinear motion, the five quantities, displacement x, time taken t, initial velocity v_0, final velocity v and acceleration a are related by a set of simple equations called kinematic equations of motion :

$$v = v_0 + at$$

$$x = v_0 t + \frac{1}{2} a t^2$$

$$v^2 = {v^2}_0 + 2ax$$

- If the position of the object at time t = 0 is 0. If the particle starts at $x = x_0$, x in above equations is replaced by $(x – x_0)$.