1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 03 (Class XII, Semester - 1)
Module Name/Title	Application of Derivatives - Part 3
Module Id	lemh_10603
Pre-requisites	Basic knowledge about Slopes and Equations of Tangents and Normals
Objectives	After going through this lesson, the learners will be able to understand the following: • Slopes of Tangents and Normals • Equations of Tangents and Normals • Some Particular cases • Use differentials to find approximate values
Keywords	Slopes and Equations of Tangents and Normals, Some Particular cases

2. Development Team

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1. Introduction

We know that derivatives have wide range of applications in various disciplines in real life situations. It has been discussed in previous modules how derivatives can be used to determine rate of change of various quantities and how for a given function we can use derivatives to find the intervals in which the function is increasing or decreasing.

This module explains the use of differentiation to find the equations of the **tangent and normal** to a curve.

Let us recall what do we mean by a tangent and a normal line to a curve. The **tangent** is a straight line which just touches the curve at a given point rather we say that it cuts the curve at two coincident points and the **normal** to a curve is a straight line which is perpendicular to the **tangent** and passes through the point of tangency.

See the figure below,

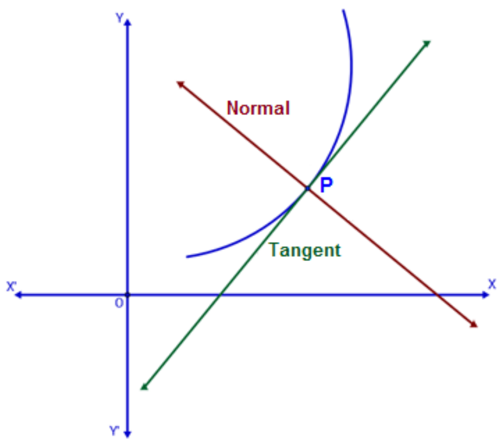

Fig-1

We know to write the equation of a line in two-dimensional plane. In the following figure the line is making angle θ with positive direction of x-axis in anticlockwise direction.

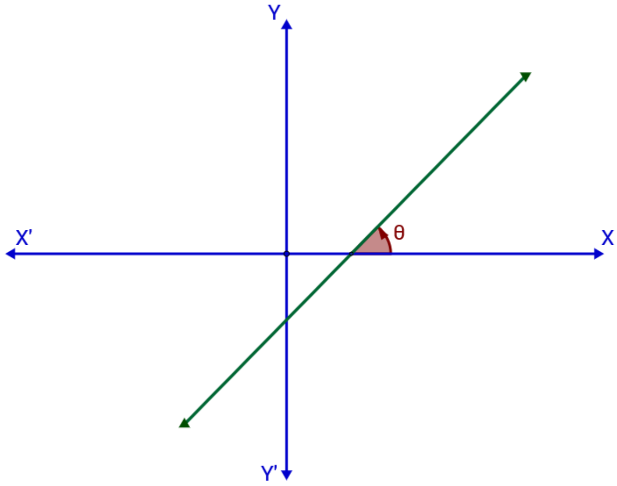

Fig-2

The trigonometrical tangent of the angle θ, i.e., $\tan\theta$ is known as slope or gradient of the line and we generally denoted it by m.

Recall that the equation of a straight line passing through a given point (x_o, y_o) having finite slope m, is given by

$$y - y_o = m\,(x - x_o)$$

2. Slopes of Tangents and Normals

Let, $y = f(x)$ be the equation of a continuous curve and P (x_o, y_o) be a point on it,

then $\left(\frac{dy}{dx}\right)_{(x_o, y_o)}$ is the slope of the tangent to the curve $y = f(x)$ at point P (x_o, y_o). Thus, slope of the tangent at P,

$$m = \left(\frac{dy}{dx}\right)_P$$

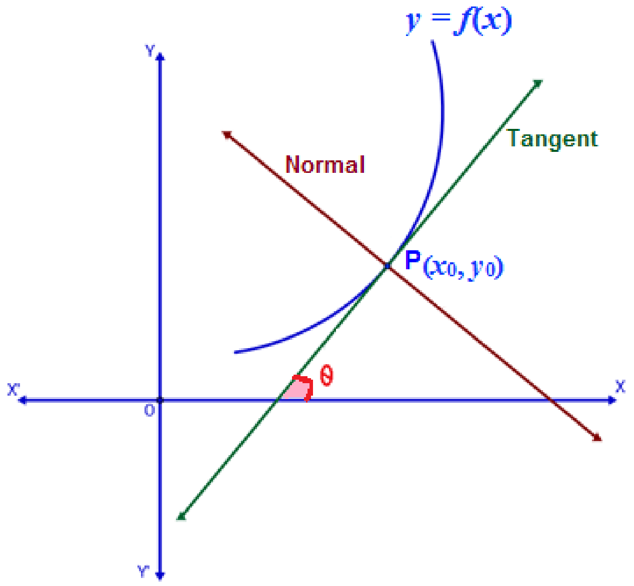

Fig-3

i.e. $$m = tan\ \theta = \left(\frac{dy}{dx}\right)_{(x_o, y_o)} = f'(x_0)$$

where, $\boldsymbol{\theta}$ is the angle made by the tangent with positive direction of x-axis in anticlockwise direction.

The normal to the curve $y = f(x)$ at point P (x_o, y_o) is a line which passes through the point P and is perpendicular to the tangent at P

Hence, Slope of the normal at P = $\dfrac{-1}{Slope\ of\ the\ tangent\ at\ P}$

$$= \frac{-1}{\left(\frac{dy}{dx}\right)_P} = -\left(\frac{dx}{dy}\right)_{(x_0, y_0)}$$

$$= \frac{-1}{f'(x_0)}$$

when, $f'(x_o) \neq 0$

Example 1: Find the slope of tangent and normal to the curve $y = 3x^4 - 4x$ at $x = 4$.

Solution:

We have $y = 3x^4 - 4x$

Differentiating both the sides w.r.t. x, we get

$$\frac{dy}{dx} = 3.4x^3 - 4 = 12\,x^3 - 4$$

The slope of the tangent at $x = 4$,

$$m = \left(\frac{dy}{dx}\right)_{x=4} = 12\,(4)^3 - 4$$

$$= 768 - 4 = 764$$

Thus, the slope of tangent to the curve at $x = 4$ is,

$$m = 764.$$

Now, slope of normal to the curve (at $x = 4$)

$$= \frac{-1}{m} = \frac{-1}{\left(\frac{dy}{dx}\right)_{x=4}}$$

$$= \frac{-1}{764}$$

Example 2: Show that the tangents to the curve $y = 2x^3 - 3$ at the points (2, 13) and (–2, –19) are parallel.

Solution:

The equation of the given curve is,

$$y = 2x^3 - 3$$

Differentiating both the sides w.r.t. x, we get

$$\frac{dy}{dx} = 2.3x^2 = 6\,x^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

The slope of the tangent at point (2, 13),

$$m_1 = \left(\frac{dy}{dx}\right)_{(2,13)} = \left(\frac{dy}{dx}\right)_{x=2}$$

$$= 6\,(2)^2 = 24 \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

The slope of the tangent at point (–2, –19),

$$m_1 = \left(\frac{dy}{dx}\right)_{(-2,-19)} = \left(\frac{dy}{dx}\right)_{x=-2}$$

$$= 6\,(-2)^2 = 24 \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(iii)}$$

Form (ii) and (iii),

$$m_1 = m_2$$

Thus, the tangents to the curve $y = 2x^3 - 3$ at the points (2, 13) and (–2, –19) are parallel.

Example 3: Find points at which the tangents to the curve $y = x^3 - 3x^2 - 9x + 7$ are parallel to the x-axis.

Solution:

The equation of the given curve is,

$$y = x^3 - 3x^2 - 9x + 7$$

Differentiating both the sides w.r.t. x, we get

$$\frac{dy}{dx} = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3)$$

$$= (x - 3)(x + 1)$$

Let the required point be P (x_1, y_1) at which the tangent to the curve is parallel to x-axis.

The slope of the tangent at point (x_1, y_1) is,

$$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = (x_1 - 3)(x_1 + 1)$$

Since, the tangent at point (x_1, y_1) is parallel to x-axis,

Therefore,

$$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 0 \quad \Rightarrow \quad (x_1 - 3)(x_1 + 1) = 0$$

$$\Rightarrow \quad x_1 = 3, -1$$

Since, point (x_1, y_1) lies on the given curve,

$\therefore$ for $x_1 = 3$,

$$y_1 = (3)^3 - 3(3)^2 - 9(3) + 7 = 27 - 27 - 27 + 7 = -20$$

for $x_1 = -1$,

$$y_1 = (-1)^3 - 3(-1)^2 - 9(-1) + 7 = -1 - 3 + 9 + 7 = 12$$

Thus, the required points are (3, – 20) and (– 1, 12).

Example 4: Find the slope of normal to the curve,

$$x = a\cos^3\theta,\ y = a\sin^3\theta \text{ at } \theta = \frac{\pi}{4}$$

Solution:

We are given, $x = a\cos^3\theta$ and $y = a\sin^3\theta$

Differentiating, $x = a\cos^3\theta$, w.r.t. ' θ ' we get

$$\frac{dx}{d\theta} = a\,(3\cos^2\theta).\ \frac{d(\cos\theta)}{d\theta}$$

$$= 3a\cos^2\theta.(-\sin\theta)$$

$$= -3a\sin\theta\cos^2\theta \quad \ldots\ldots\ldots\ldots\ldots(i)$$

Now differentiating, $y = a\sin^3\theta$, w.r.t. ' θ ' we get

$$\frac{dy}{d\theta} = a\,(3\sin^2\theta).\ \frac{d(\sin\theta)}{d\theta}$$

$$= 3a\sin^2\theta.\cos\theta \quad \ldots\ldots\ldots\ldots\ldots(ii)$$

From (i) and (ii) we get,

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$= \frac{3a\sin^2\theta.\cos\theta}{-3a\sin\theta\cos^2\theta}$$

$$= -\tan\theta$$

Now, Slope of normal $= \dfrac{-1}{\left(\dfrac{dy}{dx}\right)}$

$$= \frac{-1}{(-\tan\theta)}$$

$\therefore$ at $\theta = \dfrac{\pi}{4}$,

Slope of normal $= \dfrac{-1}{(-\tan\frac{\pi}{4})} = 1$

3. Equations of Tangents and Normals

We know that the equation of a straight line passing through a given point (x_o, y_o) and having finite slope m is given by

$$y - y_o = m\,(x - x_o)$$

Slope of the tangent to the curve $y = f(x)$ at point P (x_o, y_o) is,

$$m = \left(\frac{dy}{dx}\right)_{(x_o, y_o)} = f'(x_o)$$

Hence, the equation of the tangent to the curve $y = f(x)$ at point P (x_o, y_o) is given by,

$$y - y_o = f'(x_o)\,(x - x_o)$$

And since the normal is perpendicular to the tangent, so the slope of the normal to the curve $y = f(x)$ at point P (x_o, y_o) is,

$$\frac{-1}{f'(x_o)}, \text{ if } f'(x_o) \neq 0$$

And the equation of the normal to the curve $y = f(x)$ at (x_o, y_o) is

$$y - y_o = \frac{-1}{f'(x_o)}\,(x - x_o)$$

or $$(y - y_o).\, f'(x_o) + (x - x_o) = 0$$

4. Some Particular cases:

<u>Case 1:</u> If tangent is parallel to the x-axis, we have

$$\theta = 0 \Rightarrow \tan\theta = 0$$

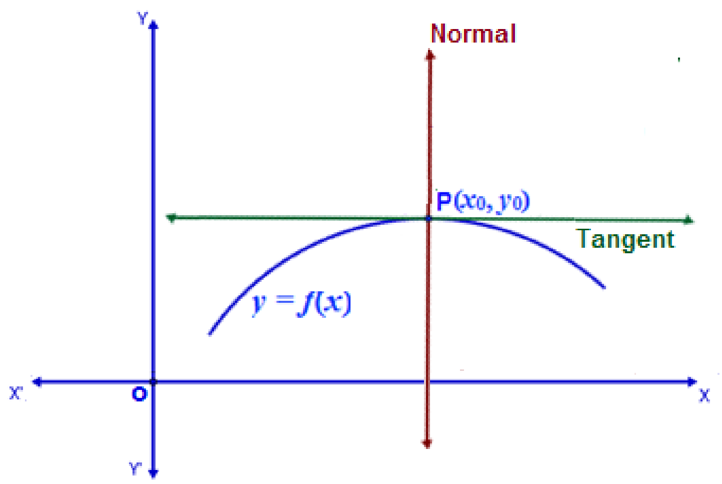

fig-4

Slope of the tangent line to the curve $y = f(x)$ at point P (x_o, y_o) is zero. Observe in the fig-4, in such a case when tangent line is parallel to x-axis, the normal line will be parallel to y-axis.

Therefore, at point P (x_o, y_o), the equation of tangent line will be $y = y_o$ and the equation of normal line will be $x = x_o$.

Case 2: When tangent line to the curve is parallel to y-axis,

i.e., when $\theta \to \frac{\pi}{2}$, we get $\frac{dy}{dx} = \tan\theta \to \infty$,

thus, the slope of the tangent line in such case will not be defined.

See the figure below,

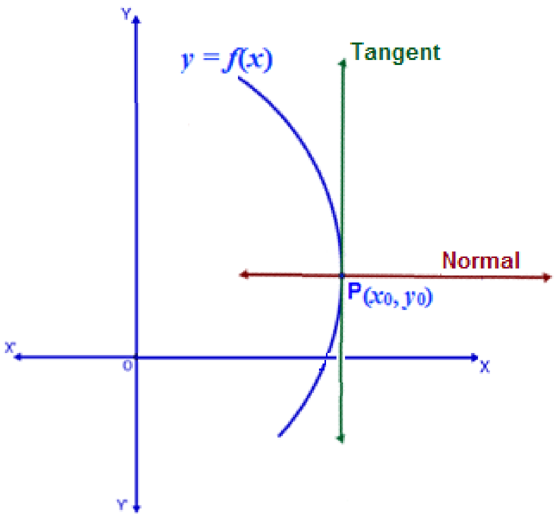

Fig-5

In such case, the equation of the tangent line will be $x = x_o$ and the equation of the normal line will be $y = y_o$, at point P (x_o, y_o).

Example 5: Find equation of the tangent to the curve $y = x^3 - x$ at a point whose x-coordinate is 2.

Solution:

The equation of the given curve is,

$$y = x^3 - x \quad \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

Differentiating both the sides w.r.t. x, we get

$$\frac{dy}{dx} = 3x^2 - 1$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{x=2} = 3(2)^2 - 1 = 11$$

Thus, slope of the tangent to the curve at $x = 2$,

$$m = 11 \quad \text{................}(ii)$$

Putting, $x = 2$ in equation (i),

$$y = (2)^3 - 2 = 6$$

Hence, we have to find tangent at point (2, 6).

Using $\quad y - y_o = f'(x_o)(x - x_o)$

Equation of the required tangent is,

$$y - 6 = 11.(x - 2)$$

or $\quad 11x - y - 16 = 0$

Example 6: Find the point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4) also find equation of the tangent.

Solution:

The given equation is $\quad y = (x - 2)^2 \quad$ .... (i)

Differentiating,

Slope of tangent = $\dfrac{dy}{dx} = 2(x - 2) \quad$ .... (ii)

Slope of the chord joining the points (2, 0) and (4, 4)

$$= \frac{4-0}{4-2} = 2 \qquad \left[slope = \frac{y_2 - y_1}{x_2 - x_1}\right] \quad \text{.......}(iii)$$

Since tangent is parallel to the chord joining the given points

$$\therefore \quad 2(x - 2) = 2 \quad \text{..............} \text{ from } (ii) \text{ and } (iii)$$

$$\Rightarrow x = 3$$

putting $x = 3$ in equation (i), we get

$$y = (3 - 2)^2 = 1$$

Thus, the required point is (3, 1).

The tangent parallel to the chord joining the points (2, 0) and (4, 4) passes through the point (3, 1) and its slope is 2, Therefore, the required equation is,

$$y - 1 = 2\,(x - 3)$$

or $$2x - y - 5 = 0$$

Example 7: Find the equation of tangent and normal to the curve given by

$$x = 1 - \cos\theta \text{ and } y = \theta - \sin\theta \text{ at } \theta = \frac{\pi}{4}.$$

Solution:

Let, (x_1, y_1) be the coordinates of the point at $\theta = \frac{\pi}{4}$, then

$$x_1 = 1 - \cos\frac{\pi}{4} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

and $$y_1 = \frac{\pi}{4} - \sin\frac{\pi}{4} = \frac{\pi}{4} - \frac{1}{\sqrt{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Given curve is given by $x = 1 - \cos\theta$ and $y = \theta - \sin\theta$

Differentiating both the sides w.r.t. θ, we get

$$\frac{dx}{d\theta} = \frac{d(1-\cos\theta)}{d\theta} = \sin\theta$$

$$\frac{dy}{d\theta} = \frac{d(\theta-\sin\theta)}{d\theta} = 1 - \cos\theta$$

Now $$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{1-\cos\theta}{\sin\theta}$$

$$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = \frac{1-\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} = \frac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$$

$$= \sqrt{2} - 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

The equation of the tangent is given by,

$$y - y_1 = \frac{dy}{dx}(x - x_1)$$

from (i), (ii), and (iii) we get,

$$y - \left(\frac{\pi}{4} - \frac{1}{\sqrt{2}}\right) = \left(\sqrt{2}-1\right)\left[x - \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)\right]$$

and equation of the normal is given by,

$$y - y_1 = \frac{-1}{\left(\frac{dy}{dx}\right)} \quad (x - x_1)$$

Thus from (*i*), (*ii*), and (*iii*), equation of the normal is,

$$y - \left(\frac{\pi}{4} - \frac{1}{\sqrt{2}}\right) = \frac{-1}{\left(\sqrt{2}-1\right)} \cdot \left[x - \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)\right]$$

Example 8: Find the equation of tangent to the curve given by $y = \dfrac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.

Solution:

The equation of the given curve is,

$$y = \frac{x-7}{(x-2)(x-3)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

or $\qquad (x-2)(x-3)\,y - (x-7) = 0$

on x-axis $y = 0$, putting it in equation (*i*) we get $x = 7$.

Thus, the curve cuts the x-axis at (7, 0).

Differentiating eq. (*i*) w.r.t. x, we get,

$$(x-2)(x-3)\frac{dy}{dx} + y\,(2x-5) - 1 = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Putting $x = 7$ and $y = 0$ in eq. (*ii*) we get,

$$\left(\frac{dy}{dx}\right)_{(7,0)} = \frac{1}{20}$$

So, the tangent passes through the point (7, 0) and the slope of the tangent is $\dfrac{1}{20}$.

Thus, the required equation of the tangent is,

$$y - 0 = \frac{1}{20}(x-7)$$

or $\qquad x - 20y - 7 = 0$

Notes:

1. If two curves $y = f(x)$ and $y = g(x)$ intersect at point $P(x_o, y_o)$, then the angle between the tangents to the two curves at point $P(x_o, y_o)$ is defined as the angle of intersection of the two curves.

   If m_1 and m_2 are the slopes of the two tangents at the point of intersection $P(x_o, y_o)$ then the angle θ between them is given by

$$tan\theta = \pm \frac{m_2 - m_1}{1 + m_1 m_2}$$

2. Two curves intersect at right angle if the tangents to the curves at the point of intersection are perpendicular to each other.
3. When $\theta = \frac{\pi}{2}$, the curves are said to be orthogonal at the point of intersection
4. when $\theta = 0$, the two curves touch each other at the point of intersection and then they have a common tangent at that point.

Example 9: Show that the curves, $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 - 2y = 0$ cut orthogonally at the point (0, 0).

Solution:

The given curves are,

$$x^2 + y^2 - 2x = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

and

$$x^2 + y^2 - 2y = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

It is obvious from the equations that the point (0, 0) lies on both the curves, hence the two curves cut each other at (0, 0).

Differentiating equation (i) w.r.t. x, we get,

$$2x + 2y \frac{dy}{dx} - 2 = 0$$

$$\Rightarrow \quad 2y \frac{dy}{dx} = 2 - 2x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1 - x}{y}$$

$\therefore$ Slope of the tangent to the curve (i) at point (0, 0) is,

$$\left(\frac{dy}{dx}\right)_{(0,0)} = \frac{1-0}{0}\ ,\quad \text{which is not defined.}$$

Hence, the tangent to the curve (*i*) at (0, 0) is parallel to *y*-axis.

……………… (*iii*)

Now differentiating equation (*ii*) w.r.t. *x*, we get,

$$2x + 2y\frac{dy}{dx} - 2\frac{dy}{dx} = 0$$

$$\Rightarrow \quad 2(y-1)\frac{dy}{dx} = -2x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-x}{y-1}$$

∴ Slope of the tangent to the curve (*ii*) at point (0, 0) is,

$$\left(\frac{dy}{dx}\right)_{(0,0)} = 0$$

Hence, tangent to the curve (*ii*) at (0, 0) is parallel to *x*-axis. ……………… (*iv*)

From (*iii*) and (*iv*) the tangents to the two curves at point (0, 0) are perpendicular to each other, hence the two curves cut orthogonally.

5. Approximations

In this section we are going to learn a useful technique to calculate small changes (or errors) in dependent variable corresponding to small changes (or errors) in independent variable using differentials.

This technique is of great importance in the theory of errors in Engineering, Physics, Statistics and in many other branches of Science. So, we are going to learn use differentials to find approximate values of certain quantities.

Let, $y = f(x)$ be a function of *x*,

$$f : A \to B\ ,\quad A \subset B$$

Let Δx be a small increment in the value of independent variable *x* and the corresponding increment in the value of dependent variable *y* be Δy. Then,

$$\Delta y = f(x + \Delta x) - f(x)$$

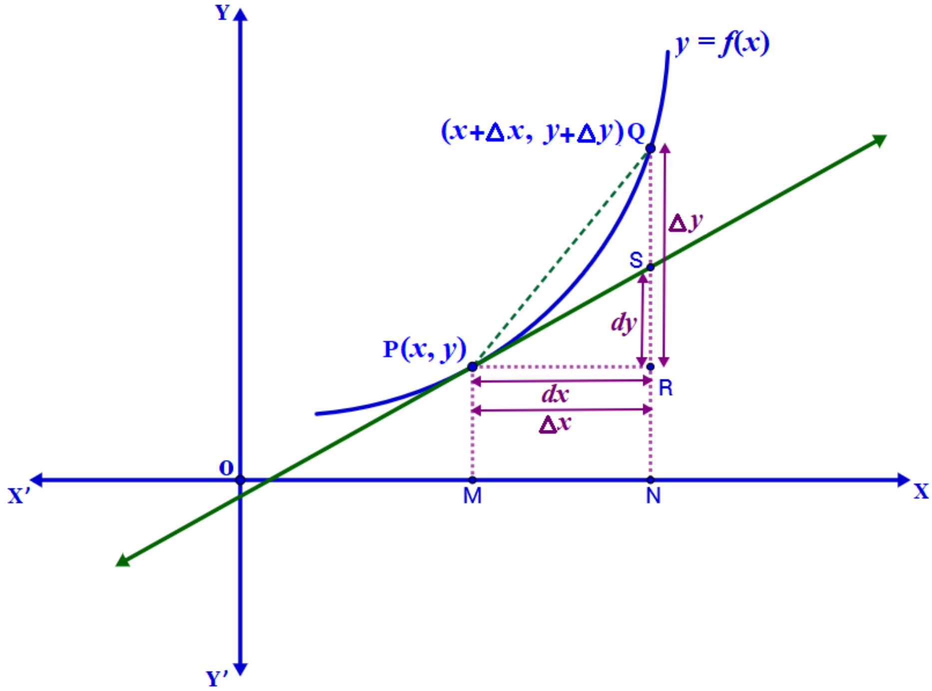

Fig-6

Here we define,

(i) The differential of x, denoted by dx, as

$$dx = \Delta x$$

(ii) The differential of y, denoted by dy, as

$$dy = f'(x)\, dx$$

$f'(x)$ is slope of the tangent to the curve at point P(x, y)

then we have $dy = f'(x)\, \Delta x$ $\quad (\because dx = \Delta x)$

$$\Rightarrow \quad dy = \left(\frac{dy}{dx}\right) . \Delta x \quad \ldots\ldots\ldots\ldots\ldots\ldots. (i)$$

Look at the figure to understand the geometrical meaning of Δx, Δy, dx and dy. It is clear that the differential of the dependent variable dy is not equal to the increment of dependent variable Δy (caused due to the change in independent variable) *i.e.*, $dy \neq \Delta y$. But the differential of independent variable x is equal to the increment of the independent variable ($dx = \Delta x$).

Now from the figure,

Slope of the line through segment PQ = $\dfrac{\Delta y}{\Delta x}$

When $\Delta x \to 0$, line PQ becomes tangent to the curve at point P

and then $\Delta y \to dy$.

So, when increment Δx ($= dx$) is very small compared with x, dy is a good approximation of Δy *i.e.* $dy \approx \Delta y$.

In many problems it is easier to compute dy, hence for small change Δx in x we can compute approximate change Δy in the dependent variable y.

Note:

While learning differentiation we have defined $\frac{dy}{dx}$ as derivative of y w. r. t. x as limit of the ratio

$\frac{\Delta y}{\Delta x}$ when $\Delta x \to 0$ and considered $\frac{dy}{dx}$

as a symbol not as a quotient. Here we have defined symbols dx and

dy as differentials of x and y in such a way that original meaning of the symbol $\frac{dy}{dx}$ coincides with the quotient dy divided by dx.

Absolute error: The error Δx in x is called the absolute error in x.

Relative error: If Δx is an error in x, then $\frac{\Delta x}{x}$ is called the relative error in x.

Percentage error: If Δx is an error in x, then $\left(\frac{\Delta x}{x} x\, 100\right)$ is called the percentage error in x.

Example 10: Use differentials and find approximate value of $\sqrt{25.3}$

Solution:

Let $y = f(x) = \sqrt{x}$ ……………………. (i)

$x = 25$ and $x + \Delta x = 25.3$ then $\Delta x = 0.3$

and $\Delta y = \sqrt{x+\Delta x} - \sqrt{x}$

$= \sqrt{25.3} - \sqrt{25}$

$= \sqrt{25.3} - 5$

$\therefore \sqrt{25.3} = 5 + \Delta y$ ………………… (ii)

Now $\Delta y \approx dy$ ………………… (iii)

and $\quad dy = \left(\frac{dy}{dx}\right).\Delta x$

$= \left(\frac{1}{2\sqrt{x}}\right)\Delta x \qquad (\because y = \sqrt{x})$

$= \left(\frac{1}{2\sqrt{25}}\right).(0.3) \qquad$ (putting values of x Δx)

$= \left(\frac{1}{10}\right).(0.3)$

$= 0.03$ ………………… (iv)

From (ii), (iii) and (iv), $\quad \sqrt{25.3} \approx 5 + 0.03$

≈ 5.03

Thus, the approximate value of $\sqrt{25.3}$ is 5.03

Example 11: Find the approximate value of $f(2.01)$, where $f(x) = 4x^2 + 5x + 2$.

Solution:

Given $\quad f(x) = 4x^2 + 5x + 2$

Let $\quad x = 2$ and $\Delta x = 0.01$

Then $\quad f(2.01) = f(x + \Delta x)$

Suppose $\quad y = f(x)$ ……………. (i)

then $\quad \Delta y = f(x + \Delta x) - f(x)$

or $\quad \Delta y = f(2.01) - f(2)$

or $\quad f(2.01) = f(2) + \Delta y$ …………………….. (ii)

Now $\quad \Delta y \approx dy$ ………………… (iii)

and $\quad dy = \left(\frac{dy}{dx}\right).\Delta x$

$= f'(x).\Delta x \quad .\Delta x$

$= (8x + 5).\ \Delta x$

$= (8 * 2 + 5).\ (0.01) = 0.21$ ………………… (iv)

(putting values of x and Δx)

$f(2) = 4(2)^2 + 5(2) + 2 = 28$ ………………… (v)

substituting the values in (ii)

$$f(2.01) \approx 28 + 0.21$$
$$\approx 28.21$$

Hence, the approximate value of $f(2.01)$ is 28.21.

Example 12: If radius of a circular metal plate is measured as 10 cm with an error of 0.02 cm, then find the approximate error in calculating its area.

Solution:

Let r be the radius of the circular metal plate then its area A is given by

$$A = \pi r^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Radius of the plate is measured as 10 cm with an error of 0.02 cm

$$\therefore \quad r = 10 \text{ cm} \quad \text{and} \quad \Delta r = 0.02 \text{ cm} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

The error in calculating area (ΔA) is approximately equal to dA

Hence, approximate error in calculating area,

$$dA = \left(\frac{dA}{dr}\right).\Delta r$$
$$= (2\pi r).\Delta r$$
$$= [2\pi(10)].(0.02) \quad \text{(putting values of } r \text{ and } \Delta r\text{)}$$
$$= 0.4\pi \text{ cm}^2$$

<u>**Summary:**</u>

1. A tangent to a curve is a straight line which cuts the curve just at one point.
2. If $y = f(x)$ be the equation of a continuous curve then $\left(\frac{dy}{dx}\right)_{(x_o, y_o)}$ is the slope of the tangent to the curve at a
   point P (x_o, y_o) on the curve.
3. The equation of the tangent to the curve $y = f(x)$ at point P (x_o, y_o) is,
   $$y - y_o = f'(x_o)(x - x_o)$$
4. Since normal is perpendicular to the tangent at the point of contact, so the slope of the normal to the curve $y = f(x)$ at point P (x_o, y_o) is,
   $$\frac{-1}{f'(x_o)}, \text{ if } f'(x_o) \neq 0$$

5. If tangent line is parallel to the x-axis, then slope of the tangent at point P (x_o, y_o) is zero, *i.e.*

$$\left(\frac{dy}{dx}\right)_{(x_o, y_o)} = 0$$

and the equation of the tangent line to the curve at point P (x_o, y_o) will be $y = y_o$.

6. If $\left(\frac{dy}{dx}\right)_{(x_o, y_o)}$ does not exist, then the tangent to the curve at point P (x_o, y_o) is parallel to the y-axis and its equation is $x = x_o$.
7. Two curves will intersect at right angle if the tangents to the curves at the point of intersection are perpendicular to each other and then they are said to cut each other orthogonally.

8. The equation of the normal to the curve $y = f(x)$ at (x_o, y_o) is given by,

$$y - y_o = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_o, y_o)}} (x - x_o)$$

9. Let $y = f(x)$ and Δx be a small increment in x, Δy be the increment in y corresponding to the increment in x,

   i.e., $\Delta y = f(x + \Delta x) - f(x)$

   then dy given by,

$$dy = \left(\frac{dy}{dx}\right) . \Delta x$$

dy is a good approximation for Δy when $dx = \Delta x$ is relatively small compared to x and we denote it by $\Delta y \approx dy$.