1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 03 (Class XII, Semester - 1)
Module Name/Title	Determinant - Part 5
Module Id	lemh_10405
Pre-requisites	Basic knowledge about linear Equations
Objectives	After going through this lesson, the learners will be able to understand the following: • Solution of Simultaneous linear Equations
Keywords	Simultaneous linear Equations, Consistent System, Inconsistent System, Homogeneous and Non–Homogenous System

2. Development Team

Role	Name	Affiliation
National MOOC Coordinators (NMC)	Prof. Amarendra P. Behera	CIET, NCERT, New Delhi
Program Coordinator	Dr. Indu Kumar	CIET, NCERT, New Delhi
Course Coordinator	Prof. Til Prasad Sharma	DESM, NCERT, New Delhi
Subject Coordinator	Anjali Khurana	CIET, NCERT, New Delhi
Subject Matter Expert (SME)	Dr. Monika Sharma	Shiv Nadar University, Noida
Revised By	Manpreet Kaur Bhatiya	IITNM College, GGSIP University
Review Team	Prof. Bhim Prakash Sarrah Prof. V.P. Singh (Retd.) Prof. SKS Gautam (Retd.)	Assam University, Tezpur DESM, NCERT, New Delhi DESM, NCERT, New Delhi

TABLE OF CONTENTS

1. Solution of Simultaneous Linear Equation

- **Consistent System** – A system of equations is said to be consistent if its solution (one or more) exists.

For example, the system of equations

$2x + 3y = 37$

$6x-5y = -43$

has the solution $x= 2$ and $y=11$. So the above system of equations is a consistent one.

- **Inconsistent System** - A system of equations is said to be inconsistent if its solution does not exists.

For example, the system of equations

$2x - 3y = 4$

$6x - 9y = 11$

Does not has any valid solutions. So the above system of equations is an inconsistent one.

- **Homogeneous and Non –Homogeneous System:**

A system of equations AX=B is called a homogeneous system if B=0. Otherwise , it is called non-homogeneous system of equations.

For example, the system of equations;

$2x+5y=0$

$5x-y=0$

is a homogeneous system of linear equations, whereas the system of equations given by;

$2x+5y=3$

$5x-y=8$

is a non-homogeneous.

2. Matrix Method for the Solution of a Non-Homogeneous System

- **Solution of system of linear equations using inverse of matrix:** Let us express the system of linear equations as **matrix equations** and solve them using inverse of the matrix.

Consider the system of equations

$a_1x+b_1y+c_1z=d_1$

$a_2x+b_2y+c_2z=d_2$

$a_3x+b_3y+c_3z=d_3$

Let A= $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B= $\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

Then the system of equations can be written as AX=B i.e.

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

The matrix A is called the **coefficient matrix** of the system of linear equations.

Theorem 1: If A is a non-singular matrix , then the system of equations given by AX=B has the unique solution given by

$X = A^{-1}B$

Proof: If A is nonsingular matrix, then its inverse exists. Now

$AX=B$

$A^{-1}(AX)=A^{-1}B$ (pre-multiplying by A^{-1})

$(A^{-1}A)X= A^{-1}B$ (by associative property)

$I X= A^{-1}B$

$X= A^{-1}B$

This matrix equation provides unique solution for the given system of equation as inverse of a matrix is unique. This method of solving system of equations is known as **Matrix Method.**

3. System of linear equations in two or three variables and criterion for checking the consistency of the system of linear equation.

Let AX = B be a system of n-linear equations in n unknowns.

(i) If |A| is not equal to zero, then the system is consistent and has the unique solution given by $X= A^{-1}B$

(ii) If A singular matrix, then |A|=0,In this case, we calculate (adj A)B.

(a) If (adj A)B≠0, (0 being zero matrix), then solution does not exist and the system

of equations is called inconsistent.

(b) If (adj A)B=0,then system may be either consistent or inconsistent according as the system have either infinitely many solutions or no solution.

Example: Solve the system of equations

2x + 5y = 1

3x + 2y = 7

Solution: The system of equations can be written in the form AX=B, where

$$A=\begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}, X=\begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B=\begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

Now |A|=-11≠0, Hence, A is nonsingular matrix and so has a unique solution.

Note that $A^{-1}= -\frac{1}{11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$,

Therefore, $X= A^{-1}B = -\frac{1}{11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \end{bmatrix}= -\frac{1}{11}\begin{bmatrix} 2-35 \\ -3+14 \end{bmatrix}= \begin{bmatrix} 33/11 \\ -11/11 \end{bmatrix}=\begin{bmatrix} 3 \\ -1 \end{bmatrix}$.

i.e., x=3 and y= -1

Example: Solve the system of equations

2x + 3y + 5z = 11

3x + 2y - 4z = -5

x + y -2z = -3

Solution: The system of equations can be written in the form AX=B, where

$$A=\begin{bmatrix}2 & -3 & 5\\3 & 2 & -4\\1 & 1 & -2\end{bmatrix} X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{and } B=\begin{bmatrix}11\\-5\\-3\end{bmatrix}$$

Now |A|=-1 ≠ 0, Hence, A is nonsingular matrix and so has a unique solution.

Note that $A^{-1}=\frac{1}{-1}\begin{bmatrix}0 & 1 & -2\\2 & -9 & 23\\1 & -5 & 13\end{bmatrix}=\begin{bmatrix}0 & -1 & 2\\-2 & 9 & -23\\-1 & 5 & -13\end{bmatrix}$

Therefore, $X= A^{-1}B=\begin{bmatrix}0 & -1 & 2\\-2 & 9 & -23\\-1 & 5 & -13\end{bmatrix}\begin{bmatrix}11\\-5\\-3\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$

So, x=1 y=2 and z=3

Example : The sum of three numbers is 6 . If we multiply third number by 3 and add second number to it , we get 11. By adding first and third numbers , we get double of the second number . Represent it algebraically and find the numbers using matrix method .

Solution : Let first , second and third numbers be denoted by x, y and z , respectively.

Then , according to given conditions , we have

x+y +z=6

y+3z = 11

x+ z= 2y or x-2y + z =0

This system can be written as AX=B , where

$$A=\begin{bmatrix}1 & 1 & 1\\0 & 1 & 3\\1 & -2 & 1\end{bmatrix},\ X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}6\\11\\0\end{bmatrix}$$

Here, |A| = 1(1+6) – (0 -3) + (0-1) = 9 ≠ 0.

Now we find adj A.

A_{11}= 1(1+6) =7, A_{12}= -(0-3) =3, A_{13}= -1

A_{21}=- (1+2) =-3, A_{22}= 0 , A_{23}= - (-2-1) =3

A_{31}= (3-1) =2, A_{32}=- (3-0) =-3, A_{33}= (1-0) =1

Hence adj A = $\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$

Thus adj A== $(A)^{-1}$= $\frac{1}{|A|}$adj(A) =$\frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$

X= A^{-1}B

X= $\frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

or $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ = $\frac{1}{9}\begin{bmatrix} 42-33+0 \\ 18+0+0 \\ -6+33+0 \end{bmatrix}$ = $\frac{1}{9}\begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix}$= $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Thus x= 1, y = 2 , z= 3.

Example : Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

x-y+ 2z =1

2y-3z=1

3x-2y+4z=2

Solution : Consider the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

$$\begin{bmatrix} -2-9+12 & 0-2+2 & 1+3-4 \\ 0+18-18 & 0+4-3 & 0-6+6 \\ -6-18+24 & 0-4+4 & 3+6-8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1}$ = $\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

Now , given system of equations can be written , in matrix form, as follows

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2+0 & +2 \\ 9+2 & -6 \\ 6+1 & -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

Hence x=0,y=5 and z=3

4. Summary:

- If $a_1x+b_1y+c_1z=d_1$

  $a_2x+b_2y+c_2z=d_2$

  $a_3x+b_3y+c_3z=d_3$

  Then these equations can be written as AX=B

  Where A= $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B= $\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$
- A system of equations is said to be consistent if its solution (one or more) exists.
- A system of equations is said to be inconsistent if its solution does not exists.
- A system of equations AX=B is called a homogeneous system if B=0 else a non-homogeneous system of equations.
- The unique solution of system of equations Ax = B is given by X= $A^{-1}B$
- For a system of equations AX = B,

  (i) If $|A| \neq 0$, then the has a unique solution

  (ii) If |A|=0 and (adjA)B≠0,then solution does not exist

  (iii) If |A|=0 and (adjA)B=0,then system may or may not be consistent.