1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 03 (Class XII, Semester - 1)
Module Name/Title	Matrices – Part 1
Module Id	lemh_10301
Pre-requisites	Knowledge about Number, solving of equations.
Objectives	After going through this lesson, the learners will be able to understand the following: 1. Concept, notation, construction of the Matrices 2. Types of Matrices • Column Matrix • Row Matrix • Square Matrix • Diagonal Matrix • Scalar Matrix • Identity Matrix • Zero Matrix • Upper and lower triangular Matrix 3. Equality of Matrices 4. Summary
Keywords	Matrix, Construction, Types, Equality

2. Development Team

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1. Introduction

Matrices are one of the most powerful tools in mathematics. This mathematical tool simplifies our work to a great extent when compared with other straight forward methods. The evolution of concept of matrices is the result of an attempt to obtain compact and simple methods of solving system of linear equations. Matrices are not only used as a representation of the coefficients in system of linear equations, but utility of matrices far exceeds that use. Matrix notation and operations are used in electronic spreadsheet programs for personal computer, which in turn is used in different areas of business and science like budgeting, sales projection, cost estimation, analysing the results of an experiment etc. Also, many physical operations such as magnification, rotation and reflection through a plane can be represented mathematically by matrices. Matrices are also used in cryptography. This mathematical tool is not only used in certain branches of sciences, but also in genetics, economics, sociology, modern psychology and industrial management.

2. Matrix and its construction

A *matrix* is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements or the entries of the matrix. A matrix having *m* rows and *n* columns is called a matrix of ***order*** $m \times n$ or simply $m \times n$ matrix (read as an m by n matrix). The number of elements in an $m \times n$ matrix will be equal to mn.

In general, an $m \times n$ matrix has the following rectangular array:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Or $A = [a_{ij}]_{m \times n}, 1 \le i \le m, 1 \le j \le n$ where i,j belongs to Natural Numbers. The elements a_{ij} belongs to the i^{th} row and j^{th} column and is called as $(i,j)^{th}$ element of the matrix $A = [a_{ij}]_{m \times n}$

Following are some of the examples of matrices:

Example1: $A = \begin{bmatrix} 22 & -1 & 5 \\ 6 & 7 & 1/2 \end{bmatrix}$ is a matrix of order 2×3 .

Example2: $B = \begin{bmatrix} -\sin x & \tan 2x \\ \cos x & \cos y \end{bmatrix}$ is a matrix of order 2×2

Construction:

In order to construct a matrix we must define its order and its elements either by a general formula

Example1: construct a 3x2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = i + j$

Solution: we have $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

Where $a_{ij} = i + j$

Therefore,

$$a_{11} = 1+1 = 2, a_{12} = 1+2 = 3, a_{21} = 2+1 = 3, a_{22} = 2+2 = 4, a_{31} = 3+1 = 4, a_{32} = 3+2 = 5$$

Hence, $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{bmatrix}$

3. Types of Martices:

- **Column matrix:** A matrix is said to be a *column matrix* if it has only one column.

For example, $A = \begin{bmatrix} 1 \\ \sqrt{13} \\ \frac{3}{5} \end{bmatrix}$ is a column matrix of order 3×1

In general, $B = [b_{ij}]_{m \times 1}$ is a matrix of order $m \times 1$

- **Row Matrix:** A matrix is said to be a *row matrix* if it has only one row.

  For example, $A=\begin{bmatrix} \sin x & -e & 14\sqrt{3} & 9 \end{bmatrix}$ is a row matrix of order 1×4

  In general, $B = [b_{ij}]_{1\times m}$ is a matrix of order $1\times m$.

- **Square Matrix:** A matrix in which the number of rows is equal to the number of columns, is said to be a *square matrix*. Thus an $m \times n$ matrix is said to be a square matrix if $m = n$ and is known as a square matrix of order 'm'.

  For example, $A=\begin{bmatrix} 9 & 5 & 11 \\ 1 & 0 & 0 \\ -3 & 7 & 8 \end{bmatrix}$ is a square matrix of order 3.

  In general, $B = [b_{ij}]_{m\times m}$ is a square matrix of order m.

- **Diagonal matrix:** A square matrix $B = [b_{ij}]_{m\times m}$ is said to be a *diagonal matrix* if all its non-diagonal elements are zero, that is a matrix $B = [b_{ij}]_{m\times m}$ is said to be a diagonal matrix if $b_{ij} = 0$, when $i \neq j$.

  For example, $A=\begin{bmatrix} 2 & 0 \\ 0 & \sqrt[3]{5} \end{bmatrix}$ is a diagonal matrix of order 2.

- **Scalar Matrix:** A diagonal matrix is said to be a *scalar matrix* if its diagonal elements are equal, that is, a square matrix $B = [b_{ij}]_{m\times m}$ is said to be a scalar matrix if $bij = 0$, when $i \neq j$ $bij = k$, when $i = j$, for some constant k.

  For example, $A=\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$ is a scalar matrix of order 3.

- **Identity matrix:** A square matrix in which elements in the diagonal are all 1 and rest are all zero is called an *identity matrix*. In other words, the square matrix

  A = $[aij]_{n\times n}$ is an identity matrix, if $a_{\mathrm{ij}}=\begin{cases} 1 \text{ if } i=j \\ 0 \text{ if } i\neq j \end{cases}$ .We denote the identity matrix of order n by I_n.

  For example, $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are Identity matrix of order 3 and 2 respectively.

- **Zero matrix:** A matrix is said to be *zero matrix* or *null matrix* if all its elements are zero.

For example, $A=\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}$ is a zero matrix.

- **Upper Triangular Matrix:** A square matrix $A=[a_{ij}]$ is called an upper triangular matrix if $a_{ij}=0$ for all $i>j$. Thus, in an upper triangular matrix, all elements below the main diagonal are zero.

For example, $A=\begin{bmatrix}9 & 5 & 11\\ 0 & 3 & 0\\ 0 & 0 & 8\end{bmatrix}$ is a upper triangular matrix of order 3.

- **Lower Triangular Matrix:** A square matrix $A=[a_{ij}]$ is called an lower triangular matrix if $a_{ij}=0$ for all $i<j$. Thus, in a lower triangular matrix, all elements above the main diagonal are zero.

For example, $A=\begin{bmatrix}9 & 0 & 0\\ 1 & 5 & 0\\ -3 & 7 & 8\end{bmatrix}$ is a lower triangular matrix of order 3.

4. Equality of Matrices:

Two matrices $A=[a_{ij}]$ and $B=[b_{ij}]$ are said to be equal if

(i) they are of the same order i.e. the number of rows and column of *A* must be equal to the rows and columns of *B* respectively.

(ii) each element of *A* is equal to the corresponding element of *B*, that is $a_{ij}=b_{ij}$ for all *i* and *j*

Example: If $\begin{bmatrix}a-b & c\\ 2a-b & d\end{bmatrix}=\begin{bmatrix}-1 & 4\\ 0 & 5\end{bmatrix}$, find the value of *a,b,c,d*

Solution: The corresponding elements of two equal matrices are equal

$$\therefore \begin{bmatrix}a-b & c\\ 2a-b & d\end{bmatrix}=\begin{bmatrix}-1 & 4\\ 0 & 5\end{bmatrix}$$

$$\Rightarrow a-b=-1 \quad , \quad 2a-b=0 \quad , \quad c=4, d=5$$

$\therefore$ We get, $a=1, b=2$ (on solving the equations $a-b=-1$ and $2a-b=0$), $c=4, d=5$

5. Practical Problems:

Example2: Consider the following information regarding the number of men and women Workers in three factories I, II and III

Factories	**Men workers**	**Women workers**
I	25	31
II	30	28
III	27	15

Represent the above information in the form of a 3 × 2 matrix. What does the entry in the third row and first column represent?

Solution: The information is represented in the form of a 3 × 2 matrix as follows:

$$A=\begin{bmatrix} 25 & 31 \\ 30 & 28 \\ 27 & 15 \end{bmatrix}$$

The entry in the third row and first column represents the number of Men Workers in factory III.

Example 3: 8 trees are to be planted in a form of a rectangular array, what are the possible ways they can be planted? What happen when 7 tress are there?

Solution: we know that the matrix of order *m* × *n* has *mn* elements. Thus, the possible ways in which 8 trees can be planted in a form of a rectangular array is all the possible ordered pair of matrix.

Thus, all possible ordered pairs are (1,8), (8,1), (4,2), (2,4).

When having only 7 trees then there are only two pair .i.e. (1,7), (7,1)

Example 4: The sales figure of two car dealers during January 2007, showed that dealer A sold 5 deluxe, 3 premium and 4 standard cars, while dealer B sold 7 deluxe, 2 premium and 3 standard cars. Total sales over the 2 month period of January- February revealed that dealer A sold 8 deluxe, 7 premium and 6 standard cars. In the same two month period dealer B sold 10 deluxe, 5 premium and 7 standard cars. Write 2×3 matrices summarizing sales data for January and 2 month period for each dealer.

Solution: Given that two month data for two dealers A and B. Matrices $A=\begin{bmatrix}534\\876\end{bmatrix}$ and $B=\begin{bmatrix}7\ 23\\1057\end{bmatrix}$ are summarizing the sales data for January and two month period for dealer A and B respectively. Where entries of the first row of each matrix is representing sales of January month of each dealer and the entries of second row of each matrix is representing two month period sales of each dealer.

Example 5: construct 2×2 matrix where $a_{ij}=|-2i+3j|$

Solution: we have $A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}$ where $a_{ij}=|-2i+3j|$

$\therefore \quad a_{11}=|-2+3|=1 \quad , \quad a_{12}=|-2+6|=4 \quad , \quad a_{21}=|-4+3|=1 \quad , \quad a_{22}=|-4+6|=2$

Hence, $A=\begin{bmatrix}1 & 4\\ 1 & 2\end{bmatrix}$

Example 6: Find the value of *a* and *b* if A = B, where $A=\begin{bmatrix}a+4 & 3b\\ 8 & -6\end{bmatrix}$ and

$B=\begin{bmatrix}2a+2 & b^2+2\\ 8 & b^2-10\end{bmatrix}$

Solution: The corresponding elements are equal of two equal matrices.

Since, $\begin{bmatrix}a+4 & 3b\\ 8 & -6\end{bmatrix}=\begin{bmatrix}2a+2 & b^2+2\\ 8 & b^2-10\end{bmatrix}$

$\therefore$ We get $a+4=2a+2 \Rightarrow a=2$

Also, $3b=b^2+2 \Rightarrow b^2-3b+2=0 \Rightarrow b=2 \text{ or } 1$

And $b^2-10+6=0 \Rightarrow b=\pm 2$

Hence, the value of $a=2$ and $b=2$

6. Summary

After competing this module learners will be able to understand the concept of matrices, order of matrix, how to construct a matrix, types of matrices, equal matrices. They will be able to apply the concept practically.