1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 03 (Class XII, Semester - 1)
Module Name/Title	Inverse Trigonometric Functions - Introduction ; Properties of Trigonometric Functions- Part 4
Module Id	lemh_10204
Pre-requisites	Basic knowledge about Inverse Trigonometric Functions
Objectives	After going through this lesson, the learners will be able to understand the following: • Properties of Inverse Trigonometric Functions
Keywords	Inverse of Tangent Function, Inverse of Cotangent Function, Domains and Ranges of Inverse Trigonometric Function

2. Development Team

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1. PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS

- Introduction

We will learn about various properties of six inverse trigonometric functions. These properties are very useful in simplifying expressions and solving equations involving inverse trigonometric functions.These results are valid within the principal value branches of the corresponding inverse trigonometric functions and wherever they are defined. Some results may not be valid for all values of the domains of inverse trigonometric functions. In fact, they will be valid only for some values of x for which inverse trigonometric functions are defined.

Property 1:

$\sin^{-1}(\sin x)=x$: $x \in [\frac{-\pi}{2}\frac{\pi}{2}]$

$\cos^{-1}(\cos x)=x$: $x \in [0,\pi]$

$\tan^{-1}(\tan x)=x$: $x \in (\pi/2, \frac{\pi}{2})$

$\cot^{-1}(\cot x)=x$: $x \in (0,\pi)$

$\sec^{-1}(\sec x)=x$: $x \in [0,\pi]-\{\frac{\pi}{2}\}$

$\text{cosec}^{-1}(\text{cosec } x)=x$: $x \in [\frac{-\pi}{2}\frac{\pi}{2}]-\{0\}$

Example: $\sin^{-1}(\sin\frac{\pi}{3})$

Solution: We know that $\sin^{-1}(\sin \theta)= \theta$, if $x \in [\frac{-\pi}{2}\frac{\pi}{2}]$

$\therefore \sin^{-1}(\sin\frac{\pi}{3})=\frac{\pi}{3}$

Example: $\cos^{-1}(\cos\frac{2\pi}{3})$

Solution: We know that $\cos^{-1}(\cos x)=x$ for $x \in [0,\pi]$

$\cos^{-1}(\cos\frac{2\pi}{3})=\frac{2\pi}{3}$

Property 2- We have learnt that if $f: A \rightarrow B$, B is a bijection, then f^{-1}:B→A exists such that $fof^{-1}(x)=x$ or, $f(f^{-1}(x))=x$ for all $x \in B$. Applying this property on various trigonometric functions and their inverses, we obtain the following property:

$\sin(\sin^{-1} x)=x$	:	$x \in[-1,1]$
$\cos(\cos^{-1} x)=x$	:	$x \in[-1,1]$
$\tan(\tan^{-1} x)=x$	:	$x \in R$
$\cot(\cot^{-1} x)=x$	:	$x \in R$
$\sec(\sec^{-1} x)=x$	:	$x \in R-(-1,1)$
$\text{cosec}(\text{cosec}^{-1} x)=x$	:	$x \in R-(-1,1)$

More to Learn

Obtain the expression and express it in the form $f(g^{-1}(x))$, where f and g are trigonometric functions.

Express $g^{-1}x$ in terms of f^{-1} by using the following results:

$$\sin^{-1}\left(\frac{p}{h}\right)=\cos^{-1}\left(\frac{b}{h}\right)=\tan^{-1}\left(\frac{p}{b}\right)=\text{cosec}^{-1}\left(\frac{h}{p}\right)=\sec^{-1}\left(\frac{h}{b}\right)=\cot^{-1}\left(\frac{b}{p}\right)$$

where p, b and h denote respectively the perpendicular, base and hypotenuse of a right triangle.

Let $g^{-1}(x)=f^{-1}(y)$.

Replace $g^{-1}(x)$ by $f^{-1}(y)$ in $f(g^{-1}(x))$ and use property-II to get $f(g^{-1}(x)) = f(f^{-1}(y))=y$.

Example: Evaluate sin[$\sin^{-1}\left(\frac{5}{13}\right)$]

Using sin($\sin^{-1}$x)=x, xϵ[-1,1], we obtain sin[$\sin^{-1}\left(\frac{5}{13}\right)$] =$\frac{5}{13}$

Example: Evaluate sin[$\cos^{-1}\left(\frac{4}{5}\right)$]

In order to express $\cos^{-1}\left(\frac{4}{5}\right)$ in terms of $\sin^{-1}$,we know

$\cos^{-1}\left(\frac{b}{h}\right)=\sin^{-1}\left(\frac{p}{h}\right)$ $\therefore \cos^{-1}\left(\frac{4}{5}\right)=\sin^{-1}\left(\frac{3}{5}\right)$

Hence, sin[$\cos^{-1}\left(\frac{4}{5}\right)$=sin[$\sin^{-1}\left(\frac{3}{5}\right) = \frac{3}{5}$

Property 3:

$\sin^{-1}(-x)=-\sin^{-1}x$	:	$x \epsilon[-1,1]$
$\cos^{-1}(-x)= \pi-\cos^{-1}x$	:	$x \epsilon[-1,1]$
$\tan^{-1}(-x)= - \tan^{-1}x$	:	$x \epsilon R$
$\cot^{-1}(-x)= \pi -\cot^{-1}x$	:	$x \epsilon R$
$\sec^{-1}(-x)= \pi - \sec^{-1}x$:	:	$x \epsilon R-(-1,1)$
$\mathrm{cosec}^{-1}(-x)= - \mathrm{cosec}^{-1}x$	:	$x \epsilon R-(-1,1)$

Example: Evaluate cos{$\sin^{-1}(\frac{-5}{13})$}

$\cos\{\sin^{-1}(\frac{-5}{13})\}=\cos(-\sin^{-1}\frac{5}{13})=\cos(-\sin^{-1}\frac{5}{13})=(\cos^{-1}\frac{12}{13}) = \frac{12}{13}$

Property 4.

$\sin^{-1}\left[\frac{1}{x}\right] = \text{cosec}^{-1} x \quad : \quad x \in R-(-1,1)$

$\cos^{-1}\left[\frac{1}{x}\right] = \sec^{-1} x \quad : \quad x \in R-(-1,1)$

$\tan^{-1}\left[\frac{1}{x}\right] = \cot^{-1} x \quad : \quad x>0$

$= -\pi + \cot^{-1} x \quad : \quad x<0$

Example: Prove that $\tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right) = \{ \pi/2$, if $x>0$ & $- \pi/2$, if $x<0$

We know $\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x$ for $x>0$

$= -\pi + \cot^{-1} x$ for $x<0$

$\therefore \tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}x + \cot^{-1} x = \pi/2,$ if $x>0$

$= \tan^{-1}x + \cot^{-1} x - \pi = \pi/2 - \pi = -\pi/2$ if $x<0$

Property 5: $\sin^{-1}x+ \cos^{-1}x=\frac{\pi}{2} \quad : \quad x \in [-1,1]$

$\tan^{-1}x+ \cot^{-1}x=\frac{\pi}{2} \quad : \quad x \in R$

$\sec^{-1}x+ \text{cosec}^{-1}x =\frac{\pi}{2} \quad : \quad x \in R-[-1,1]$

Example: Find the value of $\cot(\tan^{-1}a+ \cot^{-1}a)$

We know that $\tan^{-1}x+ \cot^{-1}x=\frac{\pi}{2}$

$\therefore \quad \cot(\tan^{-1}a+ \cot^{-1}a)=\cot \frac{\pi}{2}=0$

Example : If $\sin[\sin^{-1}(\frac{1}{5})+\cos^{-1}x]=1$,then find the value of x.

Solution : We have $\sin[\sin^{-1}(\frac{1}{5})+\cos^{-1}x]=1$

$\therefore \quad \sin^{-1}(\frac{1}{5})+\cos^{-1}x=\sin^{-1}1$

$\sin^{-1}(\frac{1}{5})+\cos^{-1}x=\frac{\pi}{2}$

$\cos^{-1}x=\pi/2-\sin^{-1}(\frac{1}{5})$

$\cos^{-1}x=\cos^{-1}(\frac{1}{5})$ using $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

$x=\frac{1}{5}$

Property 6: $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left[\frac{x+y}{1-xy}\right]$: $xy<1$

$\tan^{-1}x-\tan^{-1}y=\tan^{-1}\left[\frac{x-y}{1+xy}\right]$: $xy>-1$

Example : Prove that : $\tan^{-1}(\frac{2}{11})+\tan^{-1}(\frac{7}{24})=\tan^{-1}\left(\frac{1}{2}\right)$

Solution : We have,

$\tan^{-1}(\frac{2}{11})+\tan^{-1}(\frac{7}{24}) = \tan^{-1}\left[\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\times\frac{7}{24}}\right]$ using $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left[\frac{x+y}{1-xy}\right]$ if $xy<1$

$=\tan^{-1}\left[\frac{48+77}{264-14}\right]$

$=\tan^{-1}\left[\frac{125}{250}\right]$

$= \tan^{-1}\left[\frac{1}{2}\right]$

Property 7: $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$: $-1 \le x \le 1$

$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$: $x \ge 0$

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$: $-1 < x < 1$

Example : Evaluate : $\tan[2\tan^{-1}\frac{1}{5})]$

Solution : We have

$\tan[2\tan^{-1}\frac{1}{5})] = \tan(\tan^{-1}\frac{2*1/5}{1-1/25}) = \tan(\tan^{-1}\frac{5}{12}) = \frac{5}{12}$

Example : Prove that: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\frac{1-x}{1+x})$,x $\epsilon[0,1]$

Solution : We have $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

$= \frac{1}{2} \times 2\tan^{-1}\sqrt{x}$

$= \tan^{-1}\sqrt{x}$.

2. **Summary**

- **The properties of the Inverse Trigonometry Functions:**

1. $\sin^{-1}(\sin x)=x$: $x \in [\frac{-\pi}{2}, \frac{\pi}{2}]$

$\cos^{-1}(\cos x)=x$: $x \in [0,\pi]$

$\tan^{-1}(\tan x)=x$: $x \in (\frac{-\pi}{2}, \frac{\pi}{2})$

$\cot^{-1}(\cot x)=x$: $x \in (0,\pi)$

$\sec^{-1}(\sec x)=x$: $x \in [0,\pi]-\{\frac{\pi}{2}\}$

$\text{cosec}^{-1}(\text{cosec}\, x)=x$: $x \in [\frac{-\pi}{2}, \frac{\pi}{2}]-\{0\}$

2. $\sin(\sin^{-1} x)=x$: $x \in [-1,1]$

$\cos(\cos^{-1} x)=x$: $x \in [-1,1]$

$\tan(\tan^{-1} x)=x$: $x \in R$

$\cot(\cot^{-1} x)=x$: $x \in R$

$\sec(\sec^{-1} x)=x$: $x \in R-(-1,1)$

$\text{cosec}(\text{cosec}^{-1} x)=x$: $x \in R-(-1,1)$

3. $\sin^{-1}\left[\frac{1}{x}\right] = \text{cosec}^{-1} x$: $x \in R-(-1,1)$

$\cos^{-1}\left[\frac{1}{x}\right] = \sec^{-1} x$: $x \in R-(-1,1)$

$\tan^{-1}\left[\frac{1}{x}\right] = \cot^{-1} x$: $x>0$

$= -\pi + \cot^{-1} x$: $x<0$

4. $\sin^{-1}(-x) = -\sin^{-1}x$: $x \in [-1,1]$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$: $x \in [-1,1]$

$\tan^{-1}(-x) = -\tan^{-1}x$: $x \in R$

$\cot^{-1}(-x) = \pi - \cot^{-1}x$: $x \in R$

$\sec^{-1}(-x) = \pi - \sec^{-1}x$: $x \in R-(-1,1)$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$: $x \in R-(-1,1)$

5. $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$: $x \in [-1,1]$

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$: $x \in R$

$\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$: $x \in R-[-1,1]$

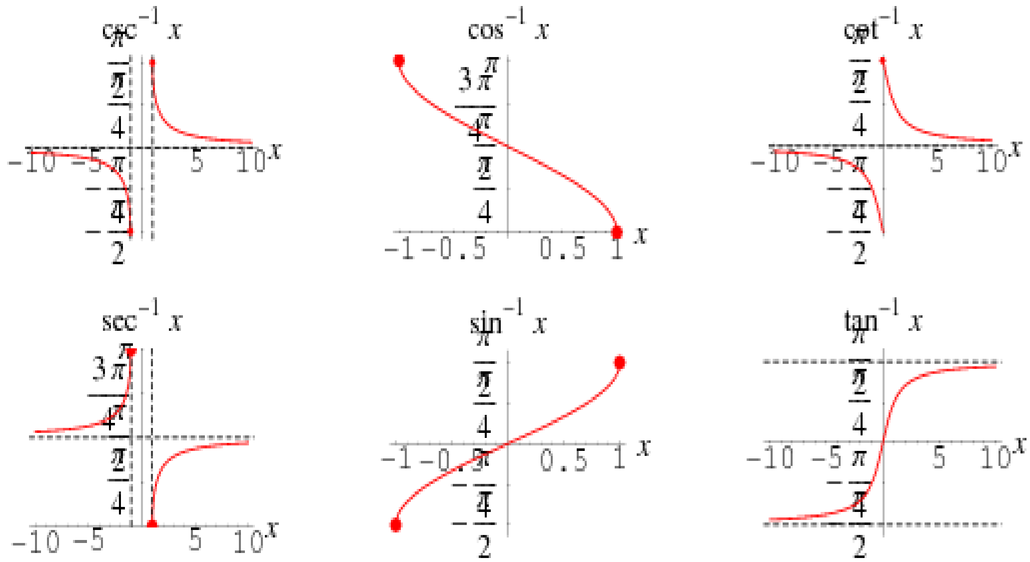
6. $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$: $xy<1$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left[\frac{x-y}{1+xy}\right]$: $xy>-1$

7. $2\tan^{-1}x = \sin^{-1}\left[\frac{2x}{1+x^2}\right]$: $-1 \leq x \leq 1$

$2\tan^{-1}x = \cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$: $x \geq 0$

$2\tan^{-1}x = \tan^{-1}\left[\frac{2x}{1-x^2}\right]$: $-1 < x < 1$


csc^-1 x
cos^-1 x
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