1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 03 (Class XII, Semester - 1)
Module Name/Title	Inverse Trigonometric Functions - Introduction ; Properties of Trigonometric Functions- Part 3
Module Id	lemh_10203
Pre-requisites	Basic knowledge about Inverse of Tangent Function and Inverse of Cotangent Function
Objectives	After going through this lesson, the learners will be able to understand the following: • Principal value branches of six inverse trigonometric functions • Inverse of Tangent Function • Inverse of Cotangent Function
Keywords	Inverse of Tangent Function, Inverse of Cotangent Function, Domains and Ranges of Inverse Trigonometric Functions

2. Development Team

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1. INVERSE OF TANGENT FUNCTION

Consider the graph of y= tan x as shown below.

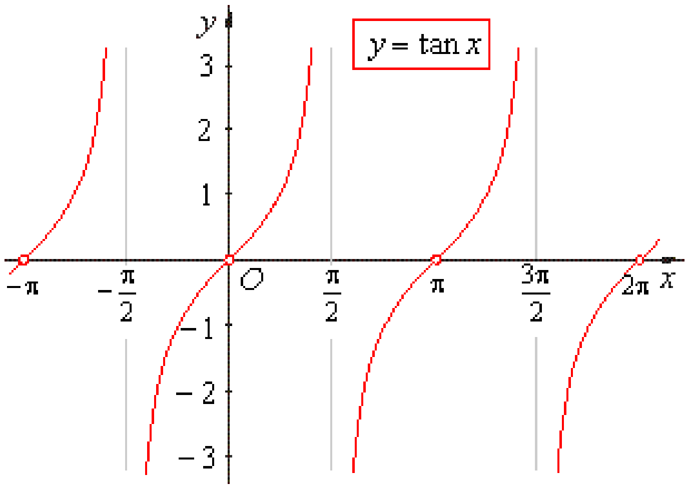
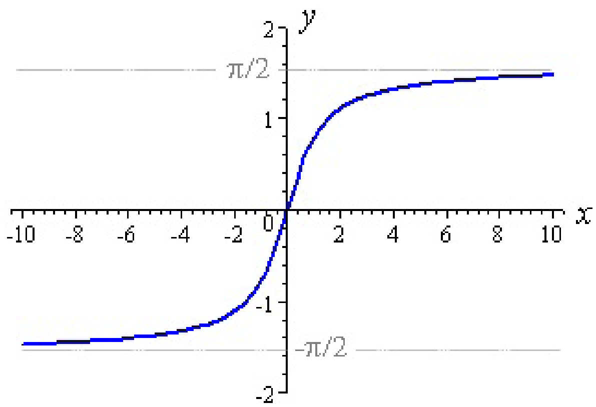


Fig.2.3 (i)

Consider the function f:R-{(2n+1)$\frac{\pi}{2}$:n ϵ Z}→R given by f(x)=tan x. It is evident that f(x)=tan x is a many-one onto function and hence it is not invertible. However, the function tan: [$\frac{-\pi}{2}$, $\frac{\pi}{2}$ →R associating each x ϵ [$\frac{-\pi}{2}$, $\frac{\pi}{2}$ to tan x ϵ R is bijection and so it is invertible.

Consider the function $\tan^{-1}$: $R \rightarrow \frac{-\pi}{2}, \frac{\pi C}{2}$given by ***y = tan^-1^x***



y= $\tan^{-1}x$

Fig.2.3 (ii)

Thus ,$\tan^{-1}$ can be defined as a function whose domain is R and range could be any of the intervals $\frac{-\pi}{2}$ $\left(\frac{-3\pi}{2},\frac{-\pi}{2}\right)$, $\frac{\pi}{2}$ $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$and so on. These intervals give different branches of the function $\tan^{-1}$.The branch with range $\frac{-\pi}{2}, \frac{\pi}{2}$ is called the principal value branch of the function $\tan^{-1}$ as shown in graph above. We thus have

$$\tan^{-1}:R \rightarrow \frac{-\pi}{2}, \frac{\pi}{2}$$

Consider the graph y= tan x and y= $\tan^{-1}x$ given below. We observe both curves are mirror images of each other in the line y=x.

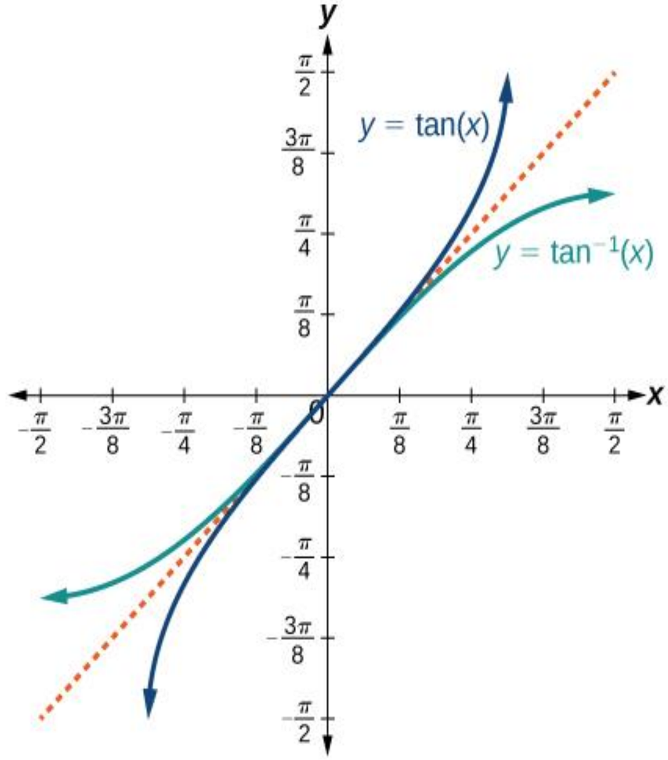


Fig.2.3 (iii)

Example: Find the principal values of $\tan^{-1}(-\sqrt{3}$

Solution: We know that for any x ϵ R, $\tan^{-1}$ x represent an angle in $\frac{-\pi}{2}, \frac{\pi}{2}$ whose tangent is x.

Therefore,

a) $\tan^{-1}(-\sqrt{3}$= (An angle θ $\epsilon\frac{-\pi}{2}, \frac{\pi}{2}$ such that tan θ=$-\sqrt{3}$=$-\frac{\pi}{3}$

b) $\tan^{-1}(1)$= (An angle θ $\epsilon\frac{-\pi}{2}, \frac{\pi}{2}$ such that tan θ=1)=$\frac{\pi}{4}$

Example: Find the principal values of $\tan^{-1}\{\sin(-\frac{\pi}{2})\}$

Solution: We know that sin $(-\frac{\pi}{2})$=-1,

$\therefore \tan^{-1}\{\sin(-\frac{\pi}{2})\}= \tan^{-1}(-1)=-\frac{\pi}{4}$

Example: For the principal value, evaluate $\tan^{-1}\{2\cos(2\sin^{-1}\frac{1}{2})\}$

Solution: We know that $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

$\therefore \tan^{-1}\{2\cos(2\sin^{-1}\frac{1}{2})\} = \tan^{-1}\{2\cos(2\mathrm{x}\frac{\pi}{6})\} = \tan^{-1}\{2\cos\frac{\pi}{3}) = \tan^{-1}(2\mathrm{x}\frac{1}{2}) = \tan^{-1}1 = \frac{\pi}{4}$

2. INVERSE OF COTANGENT FUNCTION

Consider the graph of y=cotx given below.

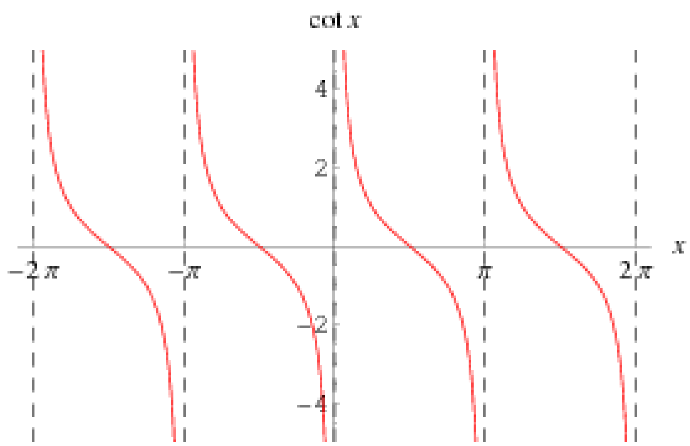


Fig. 2.3 (iv)

The function f(x) = cot x has domain = R-{nπ:n ϵ Z} and range R. Therefore,f:R-{nπ : n ϵ Z}→R is a many-one onto function .Now, If we consider cot:(0,π →R, then it is bijection and hence invertible.

In fact, cotangent function restricted to any of the intervals $(-\pi, 0)$,(0, π), $(\pi, 2\pi$ etc., is bijective and its range is R. These intervals give different branches of the function $\cot^{-1}$x.

Consider the graph of $\cot^{-1}$: R → (0, π) given below.

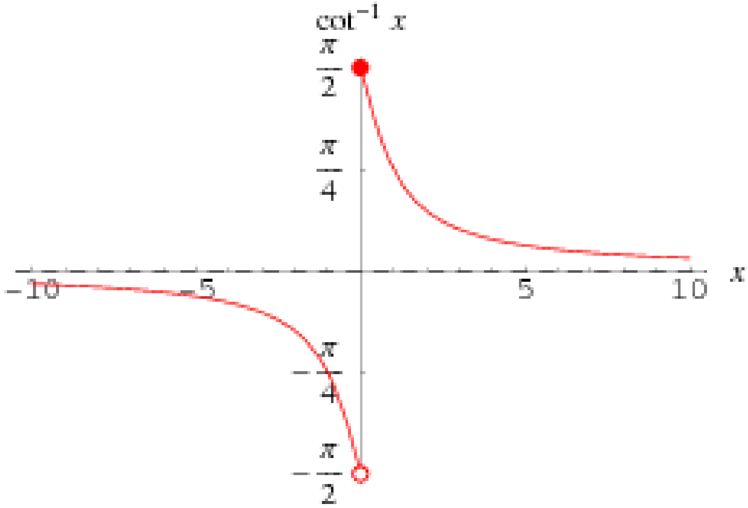


Fig. 2.3 (v)

The function with range (0, π) is called the principal value branch of the function $\cot^{-1}$ as give in the above graph. We thus have $\cot^{-1}$: R ⟶ (0, π) as principal value branch.

Example: Find the set of values of $\cot^{-1}(1)$ and $\cot^{-1}(-1)$.

Solution: For any x ϵ R, $\cot^{-1}x$ is an angle θ ϵ(0, π) such that cot θ=x

∴ $\cot^{-1}(1)$=(An angle θ ϵ (0, π) such that cot θ=1)=$\frac{\pi}{4}$

and $\cot^{-1}(-1)$= (An angle θ ϵ (0, π) whose cotangent is equal to -1)=$\frac{3\pi}{4}$

Hence, required set is $\{\frac{\pi}{4}, \frac{3\pi}{4}\}$.

Example: Find the principal values of $\cot^{-1}\sqrt{3}$ and $\cot^{-1}(-1)$.

Solution: We know that for any x ϵ R, $\cot^{-1}x$ denotes an angle in (0, π) whose cotangent is x.

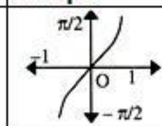
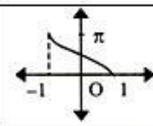
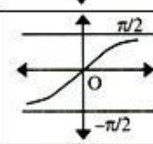
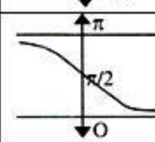
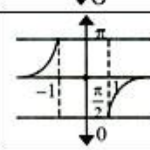
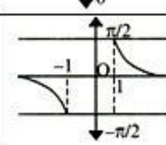
∴ $\cot^{-1}\sqrt{3}$=(An angle in (0, π) whose cotangent is$\sqrt{3}$)$\frac{\pi}{6}$

Similarly, $\cot^{-1}(-1)$= (An angle (0, π) whose cotangent is (-1))=$\frac{3\pi}{4}$.

What we have learnt In summary, The domains and ranges (principal value branches) of above six inverse trigonometric functions are given below:

Functions	**Domain**	**Range (Principal value branches)**
$y=\sin^{-1}x$	[-1,1]	$[\frac{-\pi}{2}\frac{\pi}{2}]$
$y=\cos^{-1}x$	[-1,1]	$[0,\pi]$
$y=\mathrm{cosec}^{-1}x$	R-(-1,1)	$\left[\frac{-\pi}{2}\frac{\pi}{2}\right]$ -{0}
$y=\sec^{-1}x$	R-(-1,1)	$[0,\pi]-\{\frac{\pi}{2}\}$
$y=\tan^{-1}x$	R	$(\frac{-\pi}{2}\frac{\pi}{2})$
$y=\cot^{-1}x$	R	$(0,\pi)$

OR

S.No.	Inverse Cir. Fn.	Domain	Range	Graph
1.	$\sin^{-1} x = \theta$ iff $\sin\theta = x, -\pi/2 \le \theta \le \pi/2$	$[-1, 1]$	$[-\pi/2,\pi/2]$	
2.	$\cos^{-1} x = \theta$ iff $\cos\theta = x, 0 \le \theta \le \pi$	$[-1, 1]$	$[0, \pi]$	
3.	$\tan^{-1} x = \theta$ iff $\tan\theta = x, \frac{\pi}{2} < \theta < \frac{\pi}{2}$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$	
4.	$\cot^{-1} x = \theta$ iff $\cot\theta = x, 0 \le \theta \le \pi$	$(-\infty, \infty)$	$(0, \pi)$	
5.	$\sec^{-1} x = \theta$ iff $\sec\theta = x, 0 \le \theta \le \pi$ and $\theta \ne \frac{\pi}{2}$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi]$ $\theta \ne \frac{\pi}{2}$	
6.	$\mathrm{cosec}^{-1} x = \theta$ iff $\mathrm{cosec}\,\theta = x$ $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}, \theta \ne 0$	$(-\infty, -1] \cup [1, \infty)$	$[-\pi/2, \pi/2]$ $\theta \ne 0$	

3. Summary

- The domain and ranges of different trigonometric functions are given below:

Function	**Domain**	**Range(Principal value branches)**
$Y = Tan^{-1} x$	R	$(\frac{-\pi}{2}\frac{\pi}{2})$
$Y = Cot^{-1} x$	R	$(0,\pi)$