1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 03 (Class XII, Semester - 1)
Module Name/Title	Inverse Trigonometric Functions - Introduction ; Properties of Trigonometric Functions- Part 2
Module Id	lemh_10202
Pre-requisites	Basic knowloedge about Inverse of Cosine Function, Inverse of Secant Function
Objectives	After going through this lesson, the learners will be able to understand the following: • Understand the concept of cosine function is a function. • Understand the concept of Domain and Range of Inverse Trigonometric Functions
Keywords	Inverse of Cosine Function, Inverse of Secant Function

2. Development Team

Role	Name	Affiliation
National MOOC Coordinator (NMC)	Prof. Amarendra P. Behera	CIET, NCERT, New Delhi
Program Coordinator	Dr. Indu Kumar	CIET, NCERT, New Delhi
Course Coordinator (CC) / PI	Dr. Til Prasad Sarma	DESM, NCERT, New Delhi
Course Co-Coordinator/ Co-PI	Anjali Khurana	CIET, NCERT, New Delhi
Subject Matter Expert (SME)	Dr. Monika Sharma	Shiv Nadar University, Noida
Revised By	Manpreet Kaur Bhatia	IINTM College, GGSIP University
Review Team	Prof. V.P. Singh (Retd.) Prof. Ram Avtar (Retd.) Prof. Mahendra Shankar (Retd.)	DESM, NCERT, New Delhi DESM, NCERT, New Delhi DESM, NCERT, New Delhi

TABLE OF CONTENTS :

1. INVERSE OF COSINE FUNCTION

Consider the graph of cosine function given below.

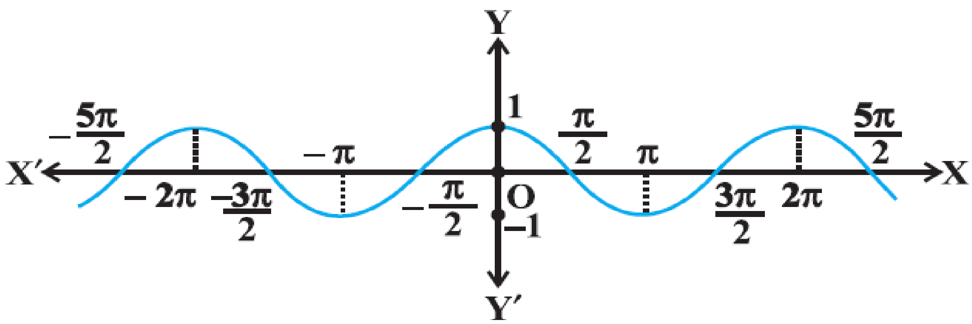


Fig.2.2 (i)

Like sine function, the cosine function is a function whose domain is the set of all real numbers and range is the set [-1,1]. If we restrict the domain of cosine function to [0, π], then it becomes one-one and onto with range [-1,1].

Actually, cosine function restricted to any of the intervals [-π, 0], [0, π], [π, 2π] etc., is bijective with range as [-1,1].

We plot the graph of inverse of cosine function.

Consider the graph of $\cos^{-1} : [-1,1] \rightarrow [0,\pi]$ given below.

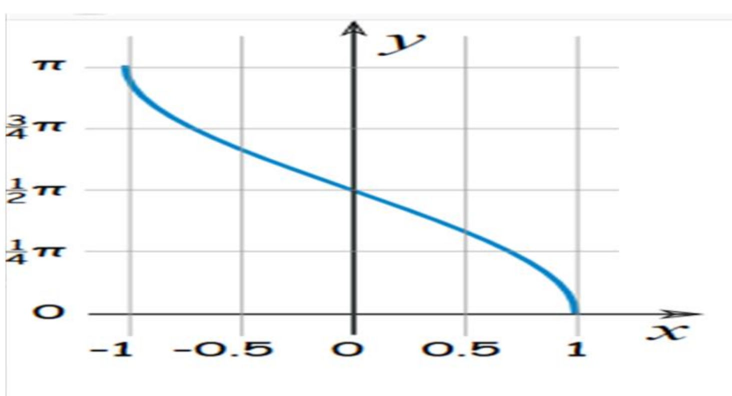


Fig 2.2 (ii)

From the above graph, it is clear that the branch with range [0, π] is called the principal value branch of the function $\cos^{-1}$ and the value of $\cos^{-1} x$ lying in

[0, π] for a given value of x ϵ [-1,1] is called the principal value. We write

$\cos^{-1} : [-1,1] \rightarrow [0,\pi]$

The curves y=cos x and y= $\cos^{-1}x$ are mirror images of each other in the line mirror y=x as shown in figure below.

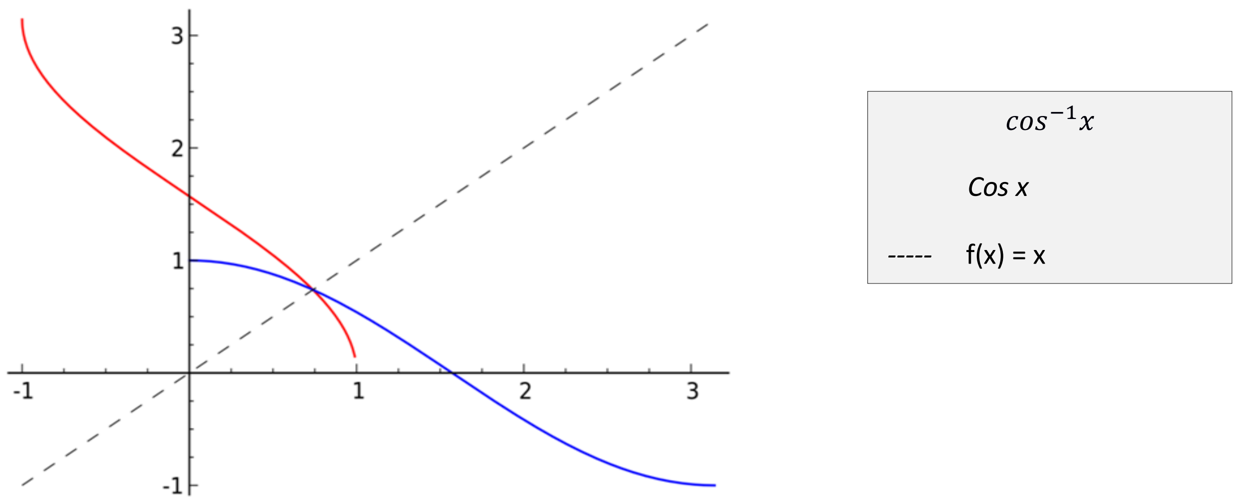


Fig.2.2 (iii)

Example: Find the principal value of (a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ and (b) $\cos^{-1}(-\frac{1}{2})$.

Solution: For any x ϵ [-1,1] , $\cos^{-1}x$ represents an angle in [0, π] whose cosine is x. Therefore,

a) $\cos^{-1}(\frac{\sqrt{3}}{2})$ = (An angle θ ϵ [0, π] such that cos θ $=\frac{\sqrt{3}}{2}$) $=\frac{\pi}{6}$

b) $\cos^{-1}(-\frac{1}{2})$ =(An angle θ ϵ [0, π] such that cos θ $=\frac{-1}{2}$) $=\frac{2\pi}{3}$

Example: Find the principal value of $\cos^{-1}\{\sin[\cos^{-1}\frac{1}{2}]\}$.

Solution: We know that $\cos^{-1}\frac{1}{2}) = \frac{\pi}{3}$.

$= \cos^{-1}\{\sin[\cos^{-1}\frac{1}{2}]\}$

$= \cos^{-1}(\sin\frac{\pi}{3})$ $[\because \sin\frac{\pi}{3} = (\frac{\sqrt{3}}{2})]$

$= \cos^{-1}(\frac{\sqrt{3}}{2})$

$= \frac{\pi}{6}$ $[\because \cos^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{6}]$

Example: Find the domain of $\cos^{-1}(2x-1)$.

Solution: The domain of $\cos^{-1}x$ is [-1,1], so the domain of $\cos^{-1}(2x-1)$ is the set of all values of x satisfying $-1 \leq 2x - 1 \leq 1$

$\Rightarrow \quad 0 \leq 2x \leq 2$

$\Rightarrow \quad 0 \leq x \leq 1$

Hence, the domain of $\cos^{-1}(2x-1)$ is [0,1].

2. INVERSE OF SECANT FUNCTION

Consider the graph of $y = \sec x$ as given below.

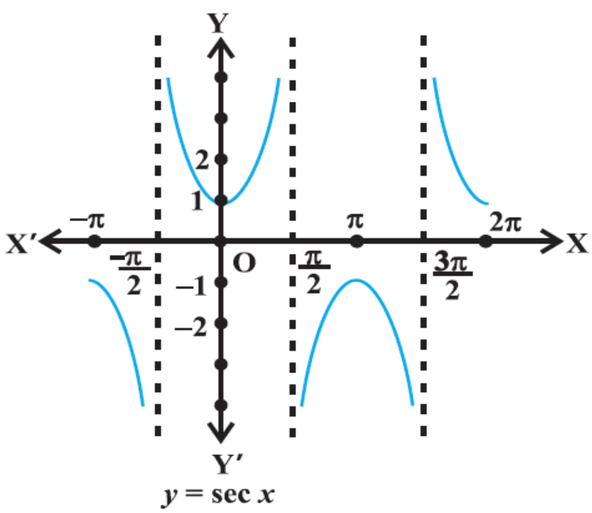
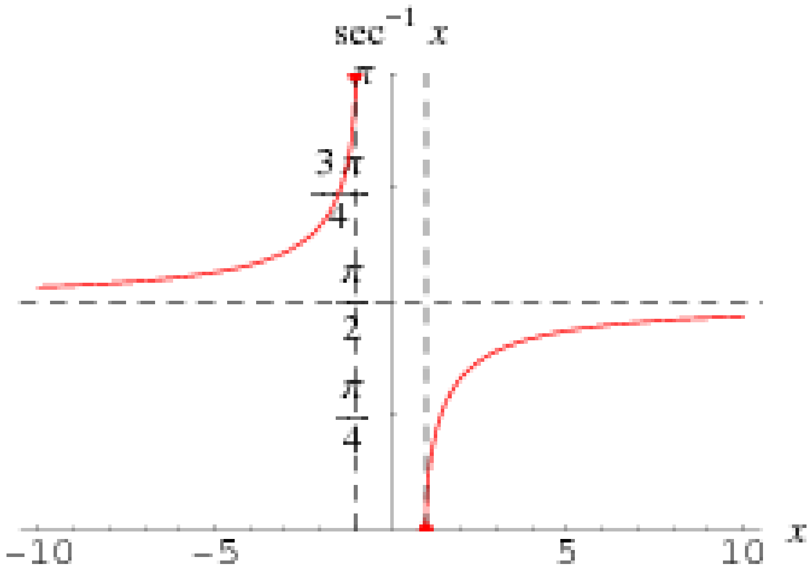


Fig.2.2 (iv)

Since, $\sec x = \frac{1}{cosx}$, the domain of the $y = \sec x$ function is the set R- $\{x:x = \frac{(2n+1)\pi}{2}, n \in Z\}$ and the range is the set $\{y : y \in R, y \geq 1 \text{ or } y \leq -1\}$ i.e., the set R-(-1,1). It means that y=sec x assumes all real values except -1<y<1 and is not defined for odd multiple of $\frac{\pi}{2}$. If we restrict the domain of secant function to $[0,\pi]$-$\{\frac{\pi}{2}\}$, then it is one to one and onto with its range as the set R-(-1,1). Actually, secant function restricted to any of the intervals $[-\pi,0]$-$\{\frac{-\pi}{2}\}$,$[0,\ \pi]-\left\{\frac{\pi}{2}\right\}$, $[\pi,2\pi]-\left\{\frac{3\pi}{2}\right\}$ etc., is bijective and its range is the set of all real numbers R-(-1,1).

Consider the function y= sec^{-1} : R-(-1,1) →$[\ 0,\pi]$–$\left\{\frac{\pi}{2}\right\}$ as shown in graph below.



S

Fig. 2.2(v)

Thus sec^{-1} can be defined as a function whose domain is R-(-1,1) and range could be any of the intervals $[-\pi,0]$-$\{\frac{-\pi}{2}\}$,$[0,\ \pi]-\left\{\frac{\pi}{2}\right\}$, $[\pi,2\pi]-\{\frac{3\pi}{2}\}$ etc. The function corresponding to the range $[0,\pi]$– $\left\{\frac{\pi}{2}\right\}$ is called the principal value branch of sec^{-1}. We thus have principal branch as

sec^{-1} : R-(-1,1) →$[\ 0,\pi]$–$\{\pi/2\}$.

Example: Find the principal values of $sec^{-1}(2)$.

Solution : For any x ϵ (-∞,-1]U[1,∞), i. e., R-(-1,1), $sec^{-1}x$ is an angle θ ϵ $[0,\frac{\pi}{2})\cup(\frac{\pi}{2},\pi]$ whose secant is x i. e. sec θ = x.

Therefore, $sec^{-1}(2)$ = (An angle θ ϵ $[0,\frac{\pi}{2})\cup(\frac{\pi}{2},\pi]$ such that sec θ=2) = $\frac{\pi}{3}$

Example: Find the domain of $sec^{-1}(2x+1)$.

Solution : The domain of $sec^{-1}x$ is $(-\infty,-1]U[1,\infty)$.Therefore, $sec^{-1}(2x+1)$ is meaningful, if $2x+1\geq 1$ or $2x+1\leq -1$.

$\Rightarrow 2x\geq 0$ or $2x\leq -2$

$\Rightarrow x\geq 0$ or $x\leq -1$

$\Rightarrow x \in (-\infty, -1]U[0,\infty)$

Hence, the domain of $sec^{-1}(2x+1)$ is $(-\infty, -1] \cup [0,\infty)$

3. Summary

- The domain and ranges of different trigonometric functions are given below:

Function	**Domain**	**Range(Principal value branches)**
$Y = Cos^{-1} x$	[-1,1]	$[0, \pi]$
$Y = Sec^{-1} x$	R-(-1,1)	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$