1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 02 (Class XI, Semester - 2)
Module Name/Title	Probability: Part-2
Module Id	kemh_21602
Pre-requisites	Knowledge about total outcomes, favourable outcomes.
Objectives	After going through this lesson, the learners will be able to understand the following: 1. Introduction 2. Axiomatic Approach to Probability 3. Probability of an event 4. Probabilities of equally likely outcomes 5. Probability of the event 'A or B' 6. Probability of the event 'not A' 7. Summary
Keywords	Mutually exclusive and exhaustive events, probability of an event, equally likely events, Probability of (A or B), (A and B), odds in favour/against.

2. Development Team

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1. Introduction

In this module we shall study about the axiomatic approach of probability, which was developed by A.N. Kolmogorov, a Russian mathematician. He laid down some axioms to interpret probability. To understand this approach we must know about few basic terms like random experiment, sample space, events etc., which we have studied in the earlier module.

2. Axiomatic Approach to Probability

In our day to day life we use many words about the chances of occurrence of events. Probability theory attempts to quantify these chances of occurrence or non occurrence of events.

We have already studied, in earlier classes, some methods of assigning probability to an event associated with an experiment having known the number of total outcomes.

Axiomatic approach is another way of describing probability of an event. In this approach some axioms or rules are depicted to assign probabilities.

Let S be the sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S and range is the interval [0,1] satisfying the following axioms:

(i) For any event E, $P(E) \geq 0$ (ii) $P(S) = 1$

(iii) If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$.

It follows from (iii) that $P(\phi) = 0$. To prove this, we take $F = \phi$ and note that E and ϕ are disjoint events. Therefore, from axiom (iii), we get

$P(E \cup \phi) = P(E) + P(\phi)$ or $P(E) = P(E) + P(\phi)$ i.e. $P(\phi) = 0$.

Let S be a sample space containing outcomes $\omega_1, \omega_2, ..., \omega_n$, i.e.,

$$S = \{\omega_1, \omega_2, ..., \omega_n\}$$

It follows from the axiomatic definition of probability that

(i) $0 \leq P(\omega_i) \leq 1$ for each $\omega_i \in S$

(ii) $P(\omega_1) + P(\omega_2) + ... + P(\omega_n) = 1$

(iii) For any event A, $P(A) = \sum P(\omega_i)$, $\omega_i \in A$.

For example, in 'a coin tossing' experiment we can assign the number $\frac{1}{2}$ to each of the outcomes H and T.

i.e. $$P(H) = \frac{1}{2} \text{ and } P(T) = \frac{1}{2} \qquad (1)$$

Clearly this assignment satisfies both the conditions i.e., each number is neither less than zero nor greater than 1 and

$$P(H) + P(T) = \frac{1}{2} + \frac{1}{2} = 1$$

Therefore, in this case we can say that probability of H $= \frac{1}{2}$, and probability of T $= \frac{1}{2}$

If we take $$P(H) = \frac{1}{4} \text{ and } P(T) = \frac{3}{4} \qquad ... (2)$$

Does this assignment satisfy the conditions of axiomatic approach?

Yes, in this case, probability of H = $\frac{1}{4}$ and probability of T = $\frac{3}{4}$.

We find that both the assignments (1) and (2) are valid for probability of H and T.

In fact, we can assign the numbers p and $(1 - p)$ to both the outcomes such that $0 \leq p \leq 1$ and $P(H) + P(T) = p + (1 - p) = 1$

This assignment, too, satisfies both conditions of the axiomatic approach of probability. Hence, we can say that there are many ways (rather infinite) to assign probabilities to outcomes of an experiment. We now consider some examples.

Example1:

Let a sample space be $S = \{\omega_1, \omega_2, ..., \omega_6\}$.Which of the following assignments of probabilities to each outcome are valid?

Outcomes	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
(a)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
(b)	1	0	0	0	0	0
(c)	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{4}$	$-\frac{1}{3}$
(d)	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{3}{2}$
(e)	0.1	0.2	0.3	0.4	0.5	0.6

Solution (a) Condition (i): Each of the number $p(\omega_i)$ is positive and less than one.

Condition (ii): Sum of probabilities

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$$

Therefore, the assignment of probabilities is valid

(b) Condition (i): Each of the number $p(\omega_i)$ is either 0 or 1.

Condition (ii): Sum of the probabilities = 1+0+0+0+0+1=1

Therefore, the assignment is valid.

(c) Condition (i): Two of the probabilities $p(\omega_5)$ and $p(\omega_6)$ are negative, the assignment is not valid.

(d)Since $p(\omega_6) = \frac{3}{2} > 1$, the assignment is not valid.

(e) Since, sum of probabilities = 0.1+0.2+0.3+0.4+0.5+0.6=2.1, the assignment is not valid.

Example 2:

In a simultaneous toss of two coins, find the probability of getting:

(i) 2 heads (ii) exactly one head (iii) exactly 2 tails (iv) exactly one tail (v) no tails.

Solution: Here, the sample space is S={HH,HT,TH,TT}. Therefore

(i) P(2 heads) = P(HH) = $\frac{1}{4}$

(ii)P(exactly one head) = P(HT,TH) = $\frac{1}{2}$

(iii)P(exactly 2 tails) = P(TT) = $\frac{1}{4}$

(iv)P(exactly one tail) = P(HT,TH) = $\frac{1}{2}$

(v)P(no tails) = P(HH) = $\frac{1}{4}$.

Example 3:

A die is thrown. Find the probability of getting

(i) an even number (ii) a prime number (iii) a number greater than or equal to 3

(iv) a number less than or equal to 4 (v) a number more than 6

(vi) a number less than or equal to 6.

Solution: Here, the sample space is S={1,2,3,4,5,6}.

(i) P(an even number) = $\frac{1}{2}$

(ii) P(a prime number)= $\frac{1}{2}$

(iii) P(a number greater than or equal to 3)= $\frac{2}{3}$

(iv) P(a number less than or equal to 4)= $\frac{2}{3}$

(v) P(a number more than 6)= 0

(vi) P(a number less than or equal to 6)= 1

Example4:

A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is (i) a vowel (ii) a consonant.

Solution: Here, S={A ,S, I, N, T, O}, therefore

$$P(A) = \frac{3}{13},\ P(S) = \frac{4}{13},\ P(I) = \frac{2}{13},\ P(N) = \frac{2}{13},\ P(T) = \frac{1}{13},P(O) = \frac{1}{13},$$

(i)P(a vowel)= P(A) + P(I) + P(O) = $\frac{3}{13}+\frac{2}{13}+\frac{1}{13}=\frac{6}{13}$

(ii)P(a consonant)= P(S) + P(N) + P(T) = $\frac{4}{13}+\frac{2}{13}+\frac{1}{13}=\frac{7}{13}$.

Example5:

A, B, C are three mutually exclusive and exhaustive events associated with a random experiment. Find P(A), it being given that P(B) = $\frac{3}{2}$ P(A) and P(C) = $\frac{1}{2}$ P(B).

Solution: Here,

$$P(A)+ P(B)+ P(C)=1$$

Therefore, $P(A)+ \frac{3}{2}P(A)+ \frac{3}{4}P(A)=1$

Or $P(A)= \frac{4}{13}$.

3. Probability of an event

Let S be the sample space associated with the experiment 'selecting three consecutive pens produced by a machine and classified as Good(non-defective)and bad(defective)'. We may get 0,1,2 or 3 defective pens as result of this checking .

A sample space associated with this experiment is

$$S = \{BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG\},$$

where B stands for a defective or bad pen and G for a non – defective or good pen.

Let the probabilities assigned to the outcomes be as follows

Sample point:	BBB	BBG	BGB	GBB	BGG	GBG	GGB	GGG
Probability:	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Let event A: there is exactly one defective pen and event B: there are atleast two defective pens.

Hence $A = \{BGG, GBG, GGB\}$ and $B = \{BBG, BGB, GBB, BBB\}$

Now $P(A) = \sum P(\omega_i), \forall \omega_i \in A$

$$= P(BGG) + P(GBG) + P(GGB) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

and $P(B) = \sum P(\omega_i), \forall \omega_i \in B$

$$= P(BBG) + P(BGB) + P(GBB) + P(BBB) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

4. Probabilities of equally likely outcomes

Let a sample space of an experiment be S = {□1, □2,..., □*n*}.

Let all the outcomes are equally likely to occur, i.e., the chance of occurrence of each simple event must be same.

i.e. $P(\omega_i) = p$, for all $\omega_i \in S$ where $0 \leq p \leq 1$

Since $\sum_{i=1}^{n} P(\omega_i) = 1$ i.e., $p + p + ... + p$ (n times) $= 1$

or $np = 1$ i.e., $p = \frac{1}{n}$

Let S be a sample space and E be an event, such that $n(S) = n$ and $n(E) = m$. If each out come is equally likely, then it follows that

$$P(E) = \frac{m}{n} = \frac{\text{Number of outcomes favourable to E}}{\text{Total possible outcomes}}$$

5.Probability of the event 'A or B'

Let us now find the probability of event 'A or B', i.e., $P(A \cup B)$.

Let $A = \{HHT, HTH, THH\}$ and $B = \{HTH, THH, HHH\}$ be two events associated with 'tossing of a coin thrice'

Clearly $A \cup B = \{HHT, HTH, THH, HHH\}$

Now $P(A \cup B) = P(HHT) + P(HTH) + P(THH) + P(HHH)$

If all the outcomes are equally likely, then

$$P(A \cup B) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

Also $P(A) = P(HHT) + P(HTH) + P(THH) = \frac{3}{8}$

and $P(B) = P(HTH) + P(THH) + P(HHH) = \frac{3}{8}$

Therefore $P(A) + P(B) = \frac{3}{8} + \frac{3}{8} = \frac{6}{8}$

It is clear that $P(A \cup B) \neq P(A) + P(B)$

The points HTH and THH are common to both A and B. In the computation of $P(A) + P(B)$ the probabilities of points HTH and THH, i.e., the elements of $A \cap B$ are included twice. Thus to get the probability $P(A \cup B)$ we have to subtract the probabilities of the sample points in $A \cap B$ from $P(A) + P(B)$

i.e. $$P(A \cup B) = P(A) + P(B) - \sum P(\omega_i),\ \forall \omega_i \in A \cap B$$
$$= P(A) + P(B) - P(A \cap B)$$

Thus we observe that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

6. Probability of the event 'not A'

Consider the event A={2,4,6,8} associated with the experiment of drawing a card from a deck of ten cards numbered from 1 to 10. Here, the sample space is S={1,2,….,10}.

If all the outcomes 1,2,….,10 are considered to be equally likely ,then the probability of each outcome is $\frac{1}{10}$.

Now $P(A) = P(2) + P(4) + P(6) + P(8)$

$$= \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$$

Also event 'not A' = $A' = \{1, 3, 5, 7, 9, 10\}$

Now $P(A') = P(1) + P(3) + P(5) + P(7) + P(9) + P(10)$

$$= \frac{6}{10} = \frac{3}{5}$$

Thus, $P(A') = \frac{3}{5} = 1 - \frac{2}{5} = 1 - P(A)$

Also, we know that A' and A are mutually exclusive and exhaustive events i.e.,

$A \cap A' = \phi$ and $A \cup A' = S$

or $P(A \cup A') = P(S)$

Now $P(A) + P(A') = 1$,

or $P(A') = P(\text{not } A) = 1 - P(A)$

NOTE: An event of drawing at random means that from a bag or an urn containing undistinguishable items, one or more items are drawn without looking into it i.e. like picking up with closed eyes.

Example6:

A bag contains 9 discs of which 4 are red, 3 are blue and 2 are yellow. The discs are similar in shape and size. A disc is drawn at random from the bag. Calculate the probability that it will be (i) red, (ii) yellow, (iii) blue, (iv) not blue, (v) either red or blue.

Solution:

There are 9 discs in all. So the possible number of outcomes is 9.

Let the events A, B, C be defined as

A **:** 'the disc is red'

B **:** 'the disc is yellow'

C **:** 'the disc is blue'

(i) The number of red discs = 4, i.e., n (A) = 4

Hence $$P(A) = \frac{4}{9}$$

(ii) The number of yellow discs = 2, i.e., n (B) = 2

Therefore, $$P(B) = \frac{2}{9}$$

(iii) The number of blue discs = 3, i.e., n(C) = 3

Therefore, $$P(C) = \frac{3}{9} = \frac{1}{3}$$

(iv) Clearly the event 'not blue' is 'not C'. We know that P(not C) = 1 – P(C)

Therefore $$P(\text{not } C) = 1 - \frac{1}{3} = \frac{2}{3}$$

(v) The event 'either red or blue' may be described by the set 'A or C'

Since, A and C are mutually exclusive events, we have

$$P(A \text{ or } C) = P(A \cup C) = P(A) + P(C) = \frac{4}{9} + \frac{1}{3} = \frac{7}{9}$$

Example7:

Two students Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that

(a) Both Anil and Ashima will not qualify the examination.

(b) Atleast one of them will not qualify the examination and

(c) Only one of them will qualify the examination.

Solution:

Let E and F denote the events that Anil and Ashima will qualify the examination, respectively. Given that

$$P(E) = 0.05,\ P(F) = 0.10 \text{ and } P(E \cap F) = 0.02.$$

Then

(a) The event 'both Anil and Ashima will not qualify the examination' may be expressed as $E' \cap F'$.

Since, E' is 'not E', i.e., Anil will not qualify the examination and F' is 'not F', i.e., Ashima will not qualify the examination.

Also $E' \cap F' = (E \cup F)'$ (by Demorgan's Law)

Now $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

or $P(E \cup F) = 0.05 + 0.10 - 0.02 = 0.13$

Therefore $P(E' \cap F') = P(E \cup F)' = 1 - P(E \cup F) = 1 - 0.13 = 0.87$

(b) P (atleast one of them will not qualify)

= 1 – P(both of them will qualify)

= 1 – 0.02 = 0.98

(c)The event only one of them will qualify the examination is same as the event either (Anil will qualify and Ashima will not qualify) **or** (Anil will not qualify and Ashima will qualify) i.e., $E \cap F'$ or $E' \cap F$, where $E \cap F'$ or $E' \cap F$ are mutually exclusive .

Therefore, P(only one of them will qualify) = $P(E \cap F'$ or $E' \cap F)$

$= P(E \cap F') + P(E' \cap F)$

$= [P(E) - P(E \cap F)] + [P(F) - P(E \cap F)]$

$= (0.05 - 0.02) + (0.10 - 0.02)$

$= 0.11$

NOTE: (i) If odds in favour of an event A be m : n, then P(A)= $\frac{m}{m+n}$ **and**

$P(\bar{A}) = \frac{n}{m+n}$.

(ii) If odds against an event B be p : q, then $P(\bar{B}) = \frac{p}{p+q}$ **and P(B)=**$\frac{q}{p+q}$

Example8:

A box contains 100 bulbs, 20 of which are defective. 10 bulbs are selected for inspection. Find the probability that:

(i) all 10 are defective (ii) all 10 are good

(iii) none is defective (iv) at least one is defective

Solution:

(i) P(all 10 are defective)= $\frac{{}^{20}C_{10}}{{}^{100}C_{10}}$

(ii)P(all 10 are good)= $\frac{{}^{80}C_{10}}{{}^{100}C_{10}}$

(iii) P(none is defective)=$\frac{{}^{80}C_{10}}{{}^{100}C_{10}}$

(iv)P(at least one is defective)=1- P(none is defective)= $1-\frac{{}^{80}C_{10}}{{}^{100}C_{10}}$

Example9:

If odds in favour of an event be 2 :3, find the probability of occurrence of this event.

Solution: Let E be an event in consideration. Then odds in favour of E are 2 : 3,i.e., Chances of occurance of E are two times out of five times. Hence,

P(E)=$\frac{2}{5}$.

Summary

In this module we have learnt about;

- axiomatic approach to probability;
- how to find the probability of an event;
- probabilities of equally likely outcomes, of the event ‘A or B’ and of the event ‘not A’;
- using the above concepts, how to solve questions related to daily life problems.