1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 02 (Class XI, Semester - 2)
Module Name/Title	Probability: Part-1
Module Id	kemh_21601
Pre-requisites	Knowledge about total outcomes, favourable outcomes
Objectives	After going through this lesson, the learners will be able to understand the following: 1. Introduction 2. Random experiments 3. Event 4. Types of Events 5. Algebra of Events 6. Mutually Exclusive Events 7. Exhaustive Events 8. Summary
Keywords	Outcomes, random experiments ,sample space, sample point, simple and compound events.

2. Development Team

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1. Introduction

In this chapter we shall study about the axiomatic approach of probability, which was developed by A.N. Kolmogorov, a Russian mathematician. He laid down some axioms to interpret probability. To understand this approach we must know about few basic terms like random experiment, sample space, events etc. So in this module, we shall study about random experiment, sample space, events and their types, etc.

2. Random experiments

If we are given a quadrilateral and its different angles are not given, we can still say with surety that their sum is 360^0. So if an experiment is to find the sum of all angles of a quadrilateral, the outcome is known to us in advance.

We also perform many experimental activities, where the result may not be the same, when they are repeated under identical conditions. For example, when a die is tossed it may turn up 1,2,3,4,5 or 6, but we are not sure which of these results will be actually obtained. Such experiments are called <u>random experiments</u>.

An experiment is called random experiment if it satisfies the following two conditions:

(i) It has more than one possible outcome.
(ii) It is not possible to predict the outcome in advance.

Outcomes and sample space:

A possible result of a random experiment is called its outcome.

Consider an experiment of rolling a die. The outcomes of this experiment are 1,2,3,4,5 or 6.

The set of outcomes {1,2,3,4,5 or 6} is called the sample space of the experiment.

Thus the set of all possible outcomes of a random experiment is called the sample space associated with the experiment. Sample space is generally, denoted by the symbol S.

Each outcome of the random experiment is also called sample point.

Example 1: An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice. Write the sample space for this experiment.

Solution: Sample space is given by

S = {(2, H), (4, H), (6, H), (2, T), (4, T), (6, T), (1, HH), (1, HT), (1, TH), (1, TT), (3, HH), (3, HT), (3, TH), (3, TT), (5, HH), (5, HT), (5, TH), (5, TT)}

Example 2: The numbers 1, 2, 3 and 4 are written separately on four slips of paper. The slips are then put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement and without looking into the box. Describe the sample space for the experiment.

Solution: Sample space is given by

S={ (1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2) and (4, 3)}

Example 3: A bag contains one white and one red ball. A ball is drawn from the bag. If the ball drawn is white it is replaced in the bag and again a ball is drawn. Otherwise, a die is tossed. Write the sample space for this experiment.

Solution: Sample space is given by

S = {(W, W), (W, R), (R, 1), (R, 2), (R, 3), (R, 4), (R, 5), (R, 6)}.

3.Event

Consider an experiment of tossing a coin two times. An associated sample space is

S = {(H, H), (H, T), (T, H), (T, T)}.

Now suppose that we are interested in those outcomes which correspond to the occurence of exactly one head. We find that HT and TH are the only elements of S corresponding to the occurrence of this happening (event). These two elements form the set E = { (H, T), (T, H) }.

We know that the set E is a subset of the sample space S. Similarly, we find the following correspondence between events and subsets of S.

Description of events	Corresponding subset of 'S'
Number of tails is exactly 2	A = {TT}
Number of tails is atleast one	B = {HT, TH, TT}
Number of heads is atmost one	C = {HT, TH, TT}
Second toss is not head	D = { HT, TT}
Number of tails is atmost two	S = {HH, HT, TH, TT}
Number of tails is more than two	ϕ

The above table suggests that a subset of sample space is associated with an event and an event is associated with a subset of sample space. Thus

Any subset E of a sample space S is called an event.

Occurrence of an event:

Consider the experiment of throwing a die. Let E denotes the event "a number less than 4 appears". If actually '1' had appeared on the die then we say that event E has occurred. If outcomes are 2 or 3, we say that event E has occurred.

Thus, the event E of a sample space S is said to have occurred if the outcome ω of the experiment is such that $\omega \in$ E. If the outcome ω is such that $\omega \notin$ E, we say that the event E has not occurred.

4. Types of events

Events can be classified into various types on the basis of the elements they have.

(i) Impossible and Sure Events

An event which cannot occur during an experiment is called an impossible event and is represented by ϕ. Its probability is 0. On the other hand, when all possible events of the sample space occur while an experiment is performed, is called Sure event. Probability of sure event is 1.

Consider an experiment of rolling a die. The sample space of this experiment

is {1,2,3,4,5, 6}.

Let E be an event" the number appears on the die is a multiple of 8".

There are no elements in the sample space that satisfy the condition given in the event. So we say that the empty set only correspond to the event E. It is impossible to have a multiple of 8 on the upper face of the die. Thus, the event E = ϕ is an impossible event.

Now let us take up another event F " the number turns up is odd or even".

Here F={1,2,3,4,5, 6}=S, i.e. all outcomes of the experiment ensure the occurrence of the event F. Thus, the event F=S is a sure event.

(ii) Simple Event

If an event E has only one sample point of the sample space , it is called a simple or elementary event.

In a sample space containing n distinct elements, there exactly n simple events.

For example in the experiment of tossing two coins, a sample space is

S = {HH,HT,TH,TT}.

There are four simple events associated to this sample space . These are

$E_1 = \{HH\}, E_2 = \{HT\}, E_3 = \{TH\}, E_4 = \{TT\}$.

(iii) Compound Events

If an event has more than one sample point, it is called a compound event.

For example, in the experiment of "tossing a coin thrice" the events

E: 'Exactly one head appeared'

F: 'Atleast one head appeared'

G: 'Atmost one head appeared' etc.

are all compound events. The subsets of S associated with these events are

E={HTT,THT,TTH}

F={HTT,THT, TTH, HHT, HTH, THH, HHH}

G= {TTT, THT, HTT, TTH}

Each of the above subsets contain more than one sample point, hence they are all compound events.

5.Algebra of events

Let A, B, C be events associated with an experiment whose sample space is S.

(i) **Complementary Event:** For every event A, there corresponds another event A' called the complementary event to A. It is also called the event 'not A'.

For example, take the experiment 'of tossing three coins'. An associated sample space is

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Let A={HTH, HHT, THH} be the event 'only one tail appears'

Clearly for the outcome HTT, the event A has not occurred. But we may say that the event 'not A' has occurred. Thus, with every outcome which is not in A, we say that 'not A' occurs.

Thus the complementary event 'not A' to the event A is

$$A' = \{HHH, HTT, THT, TTH, TTT\}$$

or $$A' = \{\omega : \omega \in S \text{ and } \omega \notin A\} = S - A.$$

(ii) **The Event 'A or B':** The union of two sets A and B is denoted by $A \cup B$ and contains all those elements which are either in A or in B or in both.

When the sets A and B are two events associated with a sample space, then '$A \cup B$' is the event 'either A or B or both'. This event '$A \cup B$' is also called 'A or B'.

Therefore $$\text{Event 'A or B'} = A \cup B$$
$$= \{\omega : \omega \in A \text{ or } \omega \in B\}$$

(iii) **The Event 'A and B':** The intersection of two sets A and B is denoted by $A \cap B$ and contains all those elements which are common to both A and B .

If A and B are two events, then the set $A \cap B$ denotes the event 'A and B'.

Thus, $A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$

For example, in the experiment of 'throwing a die twice' Let A be the event 'score on the first throw is six' and B is the event 'sum of two scores is atleast 11' then

$$A = \{(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}, \text{ and } B = \{(5,6), (6,5), (6,6)\}$$

so $A \cap B = \{(6,5), (6,6)\}$

Note that the set $A \cap B = \{(6,5), (6,6)\}$ may represent the event 'the score on the first throw is six and the sum of the scores is atleast 11'.

(iv) **The Event 'A but not B':** The set $A-B$ is the set of all those elements which are in A but not in B. Therefore, the set $A-B$ denote the event 'A but not B'. $A-B$ can also be written as $A \cap B'$.

Example 4: Consider the experiment of rolling a die. Let A be an event 'getting a prime number', B be the event 'getting an odd number'. Write the sets representing the events (i) A or B (ii) A and B (iii) A but not B (iv) 'not A'.

Solution: Here S={1,2,3,4,5,6}, A={2,3,5}, B={1,3,5}. Hence,

(i) 'A or B' = $A \cup B = \{1, 2, 3, 5\}$

(ii) 'A and B' = $A \cap B = \{3,5\}$

(iii) 'A but not B' = $A - B = \{2\}$

(iv) 'not A' = $A' = \{1,4,6\}$

Example 5:

Two dice are thrown. The events A, B, C, D, E and F are described as follows:

A = Getting an even number on the first die.

B = Getting an odd number on the first die.

C = Getting at most 5 as sum of the numbers on the two dice.

D = Getting the sum of the numbers on the dice greater than 5 but less than 10.

E = Getting at least 10 as the sum of the numbers on the dice.

F = Getting an odd number on one of the dice.

Describe the following events:

(i)A and B (ii)B or C

(iii) B and C (iv) A and E

(v)A or F (vi)A and F

Solution: Here S={(1,1),(1,2)……………….,(6,5),(6,6)}

A={(2,1),(2,2)………,(4,1),(4,2),…….(6,1),(6,2),…..(6,6)}

B={(1,1),(1,2)………,(3,1),(3,2),…….(5,1),(5,2),…..(5,6)}

C={(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)}

D={(1,5),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6),(4,2)(4,3)(4,4),(4,5),(5,1),

(5,2),(5,3),(5,4),(6,1),(6,2),(6,3)}

E={((4,6),(5,5),(6,4),(5,6),(6,5),(6,6)}

F={(1,2),(1,4),(1,6),……….,(6,1),(6,3),(6,5)}

Hence, we get

(i) $A \cap B = \phi$

(ii) $B \cup C$ = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2),
(3, 3),(3, 4), (3, 5), (3, 6), (4, 1), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}

(iii)$B \cap C$ = {(1, 1), (1,2),(1,3), (1, 4) (3, 1), (3, 2)}

(iv)$A \cap E$ = {(4,6), (6,4),(6,5), (6,6)}

(v)$A \cup F$ = {(1, 2), (1, 4), (1, 6), (2, 1), (2,2), (2,3), (2,4), (2,5), (2, 6), (3, 2), (3,4), (3,6),
(4,1), (4,2), (4,3), (4,4), (4, 5), (4,6), (5, 2), (5,4), (5,6), (6,1), (6,2), (6,3), (6,4),
(6, 5), (6,6)}

(vi)$A \cap F$ = {(2,1), (2,3), (2,5), (4, l)/(4,3), (4,5), (6,1), (6,3), (6, 5),}

6.Mutually Exclusive Events

In the experiment of rolling a die, a sample space is S={1,2,3,4,5,6}. Consider events, A ‘an odd number appears’ and B ‘an even number appears’.

Clearly the event A excludes the event B and vice versa. In other words, there is no outcome which ensures the occurrence of events A and B simultaneously. Here

$A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$

Clearly $A \cap B = \phi$, i.e., A and B are disjoint sets.

In general, two events A and B are called *mutually exclusive* events if the occurrence of any one of them excludes the occurrence of the other event, i.e., if they can not occur simultaneously. In this case the sets A and B are disjoint.

Again in the experiment of rolling a die, consider the events A 'an odd number appears' and event B 'a number less than 4 appears'

Obviously $A = \{1, 3, 5\}$ and $B = \{1, 2, 3\}$

Now $3 \in A$ as well as $3 \in B$

Therefore, A and B are not mutually exclusive events.

Note: Simple events of a sample space are always mutually exclusive.

7.Exhaustive Events

In the experiment of rolling a die, a sample space is S={1,2,3,4,5,6}.

Let us define the following events:

A: 'a number less than 4 appears',
B: 'a number greater than 2 but less than 5 appears'
and C: 'a number greater than 4 appears'.

Then $A = \{1, 2, 3\}$, $B = \{3,4\}$ and $C = \{5, 6\}$. We observe that

$$A \cup B \cup C = \{1, 2, 3\} \cup \{3, 4\} \cup \{5, 6\} = S.$$

Such events A, B and C are called exhaustive events. In general, if $E_1, E_2, ..., E_n$ are n events of a sample space S and if

$$E_1 \cup E_2 \cup E_3 \cup ... \cup E_n = \bigcup_{i=1}^{n} E_i = S$$

then $E_1, E_2, ..., E_n$ are called *exhaustive events*. In other words, events $E_1, E_2, ..., E_n$ are said to be exhaustive if atleast one of them necessarily occurs whenever the experiment is performed.

Further, if $E_i \cap E_j = \phi$ for $i \neq j$ i.e., events E_i and E_j are pairwise disjoint and $\bigcup_{i=1}^{n} E_i = S$, then events $E_1, E_2, ..., E_n$ are called *mutually exclusive and exhaustive events*.

Example 6:

Two dice are thrown. The events A, B, C, D, E and F are described as follows:

A = Getting an even number on the first die.

B = Getting an odd number on the first die.

C = Getting at most 5 as sum of the numbers on the two dice.

D = Getting the sum of the numbers on the dice greater than 5 but less than 10.

E = Getting at least 10 as the sum of the numbers on the dice.

F = Getting an odd number on one of the dice.

State true or false:

(a) A and B are mutually exclusive.

(b) A and B are mutually exclusive and exhaustive events,

(c) A and C are mutually exclusive events.

(d) C and D are mutually exclusive and exhaustive events.

(e) C, D and E are mutually exclusive and exhaustive events.

(f) A' and B' are mutually exclusive events.

(g) A, B, F are mutually exclusive and exhaustive events.

Solution: Here S={(1,1),(1,2)……………….,(6,5),(6,6)}

A={(2,1),(2,2)………,(4,1),(4,2),…….(6,1),(6,2),…..(6,6)}

B={(1,1),(1,2)………,(3,1),(3,2),…….(5,1),(5,2),…..(5,6)}

C={(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)}

D={(1,5),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6),(4,2)(4,3)(4,4),(4,5),(5,1),

(5,2),(5,3),(5,4),(6,1),(6,2),(6,3)}

E={(4,6),(5,5),(6,4),(5,6),(6,5),(6,6)}

F={(1,2),(1,4),(1,6),……….,(6,1),(6,3),(6,5)}

Hence, we get

(a) A and B are mutually exclusive- True $\because A \cap B = \phi$

(b) A and B are mutually exclusive and exhaustive events-True

$\because A \cap B = \phi \text{ and } A \cup B = S.$

(c) A and C are mutually exclusive events-False $\because A \cap C = \{(2,1),(4,1)\}$

(d) C and D are mutually exclusive and exhaustive events-False

$\because C \cap D = \phi$ *but* $C \cup D \neq S.$

(e) C, D and E are mutually exclusive and exhaustive events-True

$\because C \cap D \cap E = \phi$ *and* $C \cup D \cup E = S.$

(f) A' and B' are mutually exclusive events:

Here, A'={(1,1),(1,2)………,(3,1),(3,2),…….(5,1),(5,2),…..(5,6)}

and B'={(2,1),(2,2)………,(4,1),(4,2),…….(6,1),(6,2),…..(6,6)}

Now, A' ∩ B' = ϕ, therefore A' and B' are mutually exclusive events. Hence true.

(g) A, B, F are mutually exclusive and exhaustive events-False

$\because A \cap B \cap F = \phi$ *but* $A \cup B \cup F \neq S.$

Example7: The numbers 1, 2, 3 and 4 are written separately on four slips of paper. The slips are then put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the following events:

A = The number on the first slip is larger than the one on the second slip.

B = The number on the second slip is greater than 2

C = The sum of the numbers on the rwo slips is 6 or 7

D = The number on the second slips is twice that on the first slip.

Which pair(s) of events is (are) mutually exclusive?

Solution: Here,

A = {(2, 1), (3,1), (3,2), (4,1), (4,2), (4,3),}

B = {(1, 3), (2,3), (1,4), (2,4), (3,4), (4,3)}

C = {(2, 4), (3, 4), (4, 2), (4, 3)}

D = {(1,2),(2,4)}

Now A ∩ B ={(4,3)}, therefore, A and B are not mutually exclusive events.

A ∩ C ={(4,3), (4,2)}, therefore, A and C are also not mutually exclusive events.

A ∩ D = ϕ, A and D form a pair of mutually exclusive events.

B ∩ C ={(4,3), (2, 4), (3, 4) }, therefore, A and B are not mutually exclusive events.

C ∩ D ={(2,4)}, therefore, A and B are not mutually exclusive events.

Summary:

In this module we have learnt about;

- random experiment and the events associated with it
- various types of events and algebra of these events
- mutually exclusive and exhaustive events