1. **Details of Module and its structure**

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 02 (Class XI, Semester - 2)
Module Name/Title	Statistics: Mean Deviation about Mean / Median- Part 1
Module I	kemh_21501
Pre-requisites	Measures of Central Tendency
Objectives	After going through this module, the learner will be able to: 1. Calculate range of the data 2. Calculate mean deviation about mean 3. Calculate mean deviation about median
Keywords	Mean Deviation, Mean, Median, Range

2. Development Team

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1. Introduction

Statistics is a branch of mathematics dealing with collection, organization, analysis and interpretation of data. In earlier classes, you have already learnt to represent data graphically and in tabular form. You have also studied measures of central tendency, i.e., mean, median and mode, which are tools to calculate a representative value for a data. They give us the single value around which the observations are concentrated. If we are interested for better interpretation of data, we must know how data is scattered or dispersed around measure of central tendency.

2. Measures of Dispersion

Measures of central tendency give us a rough idea where the observations of the data are centered. But they are not sufficient to analyze the data completely as they can't give the extent of variability in any data. In order to get better interpretation of data, we must have an idea about its variability around the measure of central tendency.
Consider the following two data:

(i) 5, 10, 15, 20, 25
(ii) 3, 12, 15, 22, 23

Here, it can easily be observed that in both the data, number of observations (5), mean (15) and median (15) and range (20) are same, but if it is given that mean = 15 and then no one can tell that which data is being considered. Also, in first data has uniform distribution of observations while the second data has more variability. Thus, it can be observed that central value is not enough to analyze the sdata completely. So, other tools of statistics are also needed to check the consistency of data.

Measure of dispersion gives tools to determine the variability in a data about a given measure of central tendency. It gives us an idea that how well mean or median describes any given data. The data is considered to begood if little variation is calculated in it. It gives the extent to which a distribution is stretched or squeezed.

There can be many tools such as range, quartile deviation, mean deviation and standard deviation. In this module, we will study about mean deviations about mean and median.

3. Range

The range of a data is the difference between maximum and minimum values of the observations. It gives the difference between the two extreme observations of the data.

Range = Maximum value – Minimum value

e.g. consider the data 5, 10, 15, 20, 25

Here, range = 25 – 5 = 20

Range is the simplest measure of the dispersion, but it does not provide much details about the data. It hardly gives any analysis about the extent of variability of an observation about the measure of central tendency. So, more tools and techniques are required. One of such method is mean deviations.

4. Mean Deviation

Deviation of an observation from a fixed value is their difference. This means deviation of an observation ‘x’ from a central value ‘a’ is x – a.

An absolute measure of dispersion is the mean of all these deviations, i.e.,

Mean of Deviations = $\frac{\text{Sum of Deviations}}{\text{Number of Observations}}$

Sum of the deviations will be positive and negative, and, in many cases, they cancel out each other and the sum of deviations becomes zero. So, deviations are replaced byabsolute deviations.

Mean deviations may be calculated from any measure of central tendency but mean and median are more common in use for real life applications.

The following is the **step-by-step procedure to calculate mean deviation** about mean or median.

Step 1: Calculate the measure of central tendency (denoted by ‘a’) about which mean deviation is to be calculated.

Step 2: Calculate the absolute values of the deviations of each x_i from $\bar{x}$, i.e., $|x_i - a|$, i = 1, 2, 3, …, n

Step 3: Find the mean or average of the absolute values of the deviations, which is called mean deviation about ‘a’.

M.D. (a) = $\frac{\sum_{i=1}^{n}|x_i-a|}{n}$

Thus, M.D. ($\bar{x}$) = $\frac{\sum_{i=1}^{n}|x_i-\bar{x}|}{n}$, where $\bar{x}$ = Mean

and M.D. (M) = $\frac{\sum_{i=1}^{n}|x_i-M|}{n}$, where M = median

5. Mean Deviation about Mean

Mean deviation about mean gives the extent to which the given data is scattered when mean has been used as a measure of central tendency.

5.1 – Mean Deviation about Mean for Ungrouped Data

Let us learn how to calculate mean deviation about mean for an ungrouped or individual data.

Recall, the mean is calculated by using the formula: $\bar{x} = \frac{\text{Sum of Observations}}{\text{Number of Observations}} = \frac{\sum x_i}{n}$, i = 1, 2, 3…, n

Then mean deviation about mean = M.D. $(\bar{x})$ = $\frac{\sum_{i=1}^{n}|x_i - \bar{x}|}{n}$

Let us understand it through some examples.

Example 1: Find the mean deviation about mean for the following data:

3, 5, 8, 9, 10

Solution: Here, Mean = $\bar{x} = \frac{3+5+8+9+10}{5} = \frac{35}{5} = 7$

The absolute deviations about mean are $|x_i - \bar{x}|$,

i.e., $|3-7|, |5-7|, |8-7|, |9-7|, |10-7|$

i.e., 4, 2, 1, 2, 3

M.D. $(\bar{x})$ = $\frac{\sum_{i=1}^{n}|x_i - a|}{n}$

$$= \frac{4+2+1+2+3}{5}$$

$$= \frac{12}{5} = 2.4$$

Example 2: Find the mean deviation about mean for the following data:

5, 10, 15, 20, 25, 30, 35

Solution: Here, Mean =$\bar{x} = \frac{5+10+15+20+25+30+35}{7} = \frac{140}{7} = 20$

The absolute deviations about mean are $|x_i - \bar{x}|$,

i.e., $|5-20|, |10-20|, |15-20|, |20-20|, |25-20|, |30-20|, |35-20|$

i.e., 15, 10, 5, 0, 5, 10, 15

M.D. $(\bar{x})$ = $\frac{\sum_{i=1}^{n}|x_i - a|}{n}$

$$= \frac{15+10+5+0+5+10+15}{7} = \frac{60}{7} = 8.57$$

5.2 – Mean Deviation about Mean for Grouped Data (Discrete Frequency Distribution)

A discrete frequency distribution is a data which gives us the frequency for a specific observation; which means we can easily analyze that which particular observation is being repeated for how many number of times.

Let us learn how to calculate mean deviation about mean for discrete frequency distribution.

Consider the following data:

x:	x_1	x_2	x_3	…….	x_n
f:	f_1	f_2	f_3	…….	f_n

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{1}{N}\sum_{i=1}^{n} f_i x_i$$

In this case, Mean Deviation about Mean = $\mathrm{M.D.}\,(\bar{x}) = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{\sum_{i=1}^{n} f_i} = \frac{1}{N}\sum_{i=1}^{n} f_i |x_i - \bar{x}|$, where $N = \sum_{i=1}^{n} f_i$

Let us take a problem based on such data.

Example 3: Find the mean deviation about mean for the following data:

x	3	5	6	8	10
f	7	3	4	5	1

Solution: Consider the given data in the following tabular form with extended columns:

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
3	7	21	2.5	17.5
5	3	15	0.5	1.5
6	4	24	0.5	2.0
8	5	40	2.5	12.5
10	1	10	4.5	4.5
Total	20	110		38

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

$$= \frac{110}{20} = 5.5$$

Mean Deviation about Mean = $\mathrm{M.D.}\,(\bar{x}) = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{\sum_{i=1}^{n} f_i}$

$$= \frac{38}{20} = 1.9$$

5.3 – Mean Deviation about Mean for Grouped Data (Continuous Frequency Distribution)

A continuous data is a series in which data is being classified in the form of class – intervals without any gap along with their respective frequencies.

Let us learn how to calculate mean deviation about mean for a continuous frequency distribution.

Example 4: Find the mean deviation about mean for the following data:

Class – Intervals	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	4	3	2	3	2

Solution: Consider the given data as follows:

Class - Interval	**Frequency** f_i	**Mid – Points** x_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
0 – 10	2	5	10	17.5	35
10 – 20	3	15	45	7.5	22.5
20 – 30	4	25	100	2.5	10
30 – 40	2	35	70	12.5	25
40 – 50	1	45	45	22.5	22.5
Total	12		270		115

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

$$= \frac{270}{12} = 22.5$$

Mean Deviation about Mean = $\text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{n} f_i \lvert x_i - \bar{x} \rvert}{\sum_{i=1}^{n} f_i}$

$$= \frac{115}{12} = 9.58$$

Short – Cut Method for Calculating Mean Deviation about Mean

We can use step-deviation method for calculating mean of a continuous data. This method involves 'assumed mean' to make calculation work less tedious and deviations are taken from 'a'. Rest of the procedure and formaule are same. Recall, if assumed mean is denoted by 'a' then formula for calculation of mean is

$$\bar{x} = a + \frac{\sum_{i=1}^{n} f_i . d_i}{N} \times h$$

Let us take one example to understand this method.

Example 5: Find the mean deviation about mean for the following data:

Class – Intervals	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	5	6	7	12	10

Solution: Consider the given data as follows:

Class - Interval	Frequency f_i	Mid – Points x_i	$d_i = \frac{x_i - 50}{20}$	$f_i d_i$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
0 – 20	5	10	– 2	– 10	48	240
20 – 40	6	30	– 1	– 6	28	168
40 – 60	7	50 (= a)	0	0	8	56
60 – 80	12	70	1	12	12	144
80 – 100	10	90	2	20	32	320
Total	40			16		928

$$\bar{x} = a + \frac{\sum_{i=1}^{n} f_i . d_i}{N} \times h$$

$= 50 + \frac{16}{40} \times 20$

$= 50 + \frac{16}{2}$

$= 50 + 8$

$= 58$

Mean Deviation about Mean = $\text{M. D.}\,(\bar{x}) = \frac{\sum_{i=1}^{n} f_i\lvert x_i - \bar{x}\rvert}{\sum_{i=1}^{n} f_i}$

$= \frac{928}{40} = 23.2$

6. Mean Deviation about Median

Mean deviation about median gives the extent to which the given data is scattered when median has been used as a measure of central tendency.

6.1 – Mean Deviation about Median for Ungrouped Data

Let us learn how to calculate mean deviation about median for an ungrouped or individual data.

The data is arranged in ascending or descending order. Then number of observations is counted.

Case I: If the number of observations (n) is odd then Median = $\left(\frac{n+1}{2}\right)^{th}$ observation

Case II: If the number of observations (n) is even then Median = Mean of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations

Then mean deviation about median is calculated by using the formula, M.D. (M) = $\frac{\sum_{i=1}^{n}|x_i-M|}{n}$
Let us understand how to apply this concept to solve problems.

Example 6: Find the mean deviation about median for the following data:
11, 19, 23, 13, 15, 29, 17

Solution: The data can be re-written in ascending order as

11, 13, 15, 17, 19, 23, 29

Here, number of observations = 7 (odd)

Median (M) = 4^{th} term = 17

Deviations about Median are $|x_i - M|$,

$i.e., |11-17|, |13-17|, |15-17|, |17-17|, |19-17|, |23-17|, |29-17|$

i.e., 6, 4, 2, 0, 2, 6, 12

M.D. (M) = $\frac{\sum_{i=1}^{n}|x_i-M|}{n}$

= $\frac{6+4+2+0+2+6+12}{7}$

$$=\frac{32}{7}=4.57$$

Example 7: Find the mean deviation about median for the following data:
20, 8, 22, 12, 16, 14

Solution: Data in the ascending order is 8, 12, 14, 16, 20, 22

Here, number of observations = 6 (even)

Median (M) = $\frac{3^{rd}\text{term}+4^{th}\text{term}}{2}=\frac{14+16}{2}=\frac{30}{2}=15$

Deviations about Median are $|x_i - M|$,

$i.e., |8-15|, |12-15|, |14-15|, |16-15|, |20-15|, |22-15|$

i.e., 7, 3, 1, 1, 5, 7

M.D. (M) = $\frac{\sum_{i=1}^{n}|x_i-M|}{n}$

$= \frac{7+3+1+1+5+7}{6}$

$= \frac{24}{6} = 4$

6.2 – Mean Deviation about Median for Grouped Data (Discrete Frequency Distribution)

A discrete frequency distribution is a data which gives us the frequency for a specific observation; which means we can easily analyze that which particular observation is being repeated for how many number of times.

Let us learn how to calculate mean deviation about median for discrete frequency distribution.

Consider the following data:

x:	x_1	x_2	x_3	…….	x_n
f:	f_1	f_2	f_3	…….	f_n

Median is calculated by using c.f. (cumulative frequency) and rest of the formulas are same.

Mean Deviation about Median = $\text{M.D.}(M) = \frac{\sum_{i=1}^{n} f_i |x_i - M|}{\sum_{i=1}^{n} f_i} = \frac{1}{N} \sum_{i=1}^{n} f_i |x_i - M|$

Let us take a problem based on such data.

Example 8: Find the mean deviation about median for the following data:

x	10	15	20	25	30	35
f	4	4	2	3	3	2

Solution: Consider the given data in the following tabular form with extended columns:

x_i	f_i	*c.f.*	$\|x_i - M\|$	$f_i\|x_i - M\|$
10	4	4	10	40
15	4	8	5	20
20	2	10	0	0
25	3	13	5	15
30	3	16	10	30
35	2	18	15	30
Total	18			135

Here, N = 18 ⇒ $\frac{N}{2} = 9$

c.f. just greater than 9 is 10, whose corresponding value is 20.

Thus, Median = M = 20

Mean Deviation about Median = $\text{M.D.}(M) = \frac{\sum_{i=1}^{n} f_i |x_i - M|}{\sum_{i=1}^{n} f_i} = \frac{1}{N} \sum_{i=1}^{n} f_i |x_i - M|$

M.D. (M) = $\frac{135}{18} = 7.5$

6.3 – Mean Deviation about Median for Grouped Data (Continuous Frequency Distribution)

Let us learn how to calculate mean deviation about median for a continuous frequency distribution.

Example 9: Find the mean deviation about mean for the following data:

Class – Intervals	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency	8	10	12	9	5

[NCERT Exemplar]

Solution: Consider the given data as follows:

Class	x_i	f_i	$c.f.$	$\lvert x_i - M\rvert$	$f_i\lvert x_i - M\rvert$
0 – 6	3	8	8	11	88
6 – 12	9	10	18	5	50
12 – 18	15	12	30	1	12
18 – 24	21	9	39	7	63
24 – 30	27	5	44	13	65
Total		N = 44			278

Here, N = 44, so, N / 2 = 22

c.f. greater than 22 is 30

Median class = 12 – 18

Median (M) = $l + \frac{\frac{N}{2} - c.f.}{f} \times h$

= $12 + \frac{22-18}{12} \times 6$

$$= 12 + \frac{4}{12} \times 6$$

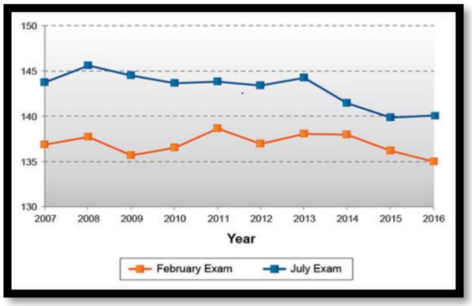
$$= 12 + 2 = 14$$

Mean Deviation about Median = $\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - M|$

$$= \frac{278}{44} = 6.318$$

7. Real-life Applications of Mean Deviations

- Average score by students of a class has how much deviations from the mean or median is calculated by using mean deviations about mean or median. This comparison may be for a single year or for many years.



Source: https://www.reddit.com/r/LawSchool/comments/6q0iul/how_your_bar_score_is_calculated_and_what_is/

- In architecture, while planning of a new town or renovating the old one, basic facilities like school, market, hospital etc. are located at a distance which is convenient for all the residents of that locality. For this mean deviation about the average distance from all the blocks of the locality is used so that the average distance covered by the residents can be minimized.

Source: https://www.designingbuildings.co.uk/wiki/Town_planning

- To check the consistency of any player to select him/her for the tournament of any sport can be checked by using the mean deviation as how a player shows consistency from the average score.

Source: https://www.noted.co.nz/archive/archive-listener-nz-2013/jane-clifton-always-the-bridesmaid

8. Summary

Let us summarize the concepts and formulas that we have learnt in this module:

- Statistics is a branch of mathematics dealing with collection, organization, analysis and interpretation of data.
- Measures of central tendency give us a rough idea where the observations of the data are centered.
- Measure of dispersion gives tools to determine the variability in a data about a given measure of central tendency.
- There can be many tools such as range, quartile deviation, mean deviation and standard deviation.
- The range of a data is the difference between maximum and minimum values of the observations. It gives the difference between the two extreme observations of the data.

Range = Maximum value – Minimum value

- Deviation of an observation from a fixed value is their difference. This means deviation of an observation 'x' from a central value 'a' is x – a.

An absolute measure of dispersion is the mean of all these deviations, i.e.,

Mean of Deviations = $\frac{\text{Sum of Deviations}}{\text{Number of Observations}}$

- M.D. (a) = $\frac{\sum_{i=1}^{n}|x_i-a|}{n}$

Thus, M.D. ($\bar{x}$) = $\frac{\sum_{i=1}^{n}|x_i-\bar{x}|}{n}$, where $\bar{x}$ = Mean

and M.D. (M) = $\frac{\sum_{i=1}^{n}|x_i-M|}{n}$, where M = median