1. **Details of Module and its structure**

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 02 (Class XI, Semester - 2)
Module Name/Title	Mathematical Reasoning- Statements and its types, Words and Phrases- Part 1
Module Id	kemh_21401
Pre-requisites	Understanding of simple sentences of English Knowledge of simple mathematical concepts
Objectives	After going through this module, the learners will be able to understand the followings: 1. Mathematically acceptable statements, 2. Compound statement and their components, 3. Special words/phrases (connectives and quantifiers) 4. Inclusive ‘or’, exclusive ‘or’
Keywords	Inductive Reasoning, Deductive Reasoning, Words, Phrases, Negation, Simple and Compound Statement, Quantifiers

2. **Development Team**

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Introduction:

In this Module, we shall discuss about some basic ideas of Mathematical Reasoning. All of us know that human beings evolved from the lower species over a very long period of time. The main asset that makes humans far more superior than the other species is the ability to reason. The ability of reasoning varies from person to person. Also it is the ability of reasoning which makes one person superior than the other. How to develop this power? Here, we shall discuss the process of reasoning especially in the context of mathematics.

In mathematics, there are two kinds of reasoning.

(i) Inductive reasoning
(ii) Deductive reasoning

We have already discussed the inductive reasoning in the context of mathematical induction. In this Chapter, we shall discuss some fundamentals of deductive reasoning.

Statements:

The basic concept involved in mathematical reasoning is a mathematical statement. Humans have a special power to express their views, emotions *etc*., by producing voice using a collection of words. This collection of words is known as sentence. In our day life we communicate our ideas or thoughts using different types of sentences. Sometimes these sentences are true, sometimes false and sometimes we are unable to decide whether the sentence is true or false. Let us consider some examples of sentences;

Example: I

1) Every rectangle is a parallelogram.
2) Moon revolves around earth.

3) Sum of two even numbers is even.
4) Delhi is capital of India.

All these sentences are always true, irrespective of the person and the place.

Let us consider some more sentences.

Example: II

1) Every relation is a function.
2) Sun revolves around earth.
3) Three plus four is five.
4) Bombay is capital of India.

Observe all these sentences are always false, irrespective of the person and place.

Now consider third example of sentences,

Example: III

1) Close the door.
2) Switch off the light.
3) Please bring some water for me.
4) Go to the market.

What can you say about these sentences? Are they true or false? Look at these sentences cannot be true or false. Each of them either expresses a command or a request.

Example: IV

1) Where are you going?
2) Are you ready to go?
3) When are you planning to visit my place?
4) Is '2' a prime number?

Note, in each of these sentences, a question has been asked so we cannot assign a true or false with any of these sentences.

Similarly the following sentences also cannot be decided whether true or false.

Example: V

1) What a beautiful flower!
2) Such a beautiful waterfall!
3) May God bless you!
4) May you get grand success in exams!

Thus we have seen that to some sentences we can associate true or false and to some sentences we cannot.

Let us now focus on these three sentences:

(i) In 2003, the president of India was a woman.

(ii) The sun rises in the east and sets in west.

(iii) Women are more intelligent than men.

The first sentence is false and the second is true. There is no confusion regarding true or false for these sentences. Such sentences in Mathematics are called statements.

On the other hand, when we consider third sentence, some people may say that the sentence is true while others may disagree. So true or false of this sentence will vary from person to person.

Similarly consider the sentence;

"The sum of x and y is greater than 0".

Here also we are not in a position to tell whether the sentence is true or false, unless we know the values of x and y. For example, it is false when $x = 1$, $y = -3$ and true when $x = 1$ and $y = 0$.

Hence this sentence is also vague or ambiguous. Such sentences are not regarded as statements.

Now consider the sentence;

"For any natural numbers x and y, the sum of x and y is greater than 0". Can this sentence be considered to be a statement?

Yes, this sentence is a statement because it is always true for any two natural numbers.

As a result of this discussion we define a mathematically acceptable statement as below.

Definition:

A sentence is called a mathematically acceptable statement if it is either true or false but not both simultaneously.

Whenever we mention a statement in mathematics, we mean "mathematically acceptable" statement.

Now what can you say about the sentence; "Tomorrow is Friday"?

Is it a statement?

See, the sentence is correct (true) if it is spoken on a Thursday but false for all other days. Hence this sentence cannot be a mathematical statement.

On the basis of this discussion we see that the sentences of Examples I and II are statements and the sentences of Examples III, IV and V are not statements, they are simple sentences.

Consider the sentence; "There are 40 days in a month". Can you call this sentence a statement?

Note that this sentence is always false, irrespective of the month, because the maximum number of days in a month can never exceed 31. Therefore, this sentence is a statement. So, what makes a sentence a statement is the fact that the sentence should be either true or false.

We usually denote statements by small letters p, q, r,...

For example, if we denote the statement "Fire is always hot" by p.

We write as;

p: Fire is always hot.

Note:

On the basis of above discussion we find that, no sentence can be called a statement if;

(i) It is an exclamation

(ii) It is an order or request

(iii) It is a question

(iv) It involves variable time such as ‘today’, ‘tomorrow’,

‘yesterday’ etc.

(v) It involves variable places such as ‘here’, ‘there’, ‘everywhere’ etc. (vi) It involves pronouns such as ‘she’, ‘he’, ‘they’ etc.

Example: 1

Check whether the following sentences are statements. Give reasons for your answer.

(i) New Delhi is in India.
(ii) Every rectangle is a square.
(iii) Close the door.
(iv) How old are you?
(v) x is a natural number.
(vi) There is no rain without clouds.

Solution:

(i) This sentence is true because New Delhi is in India. Hence it is a statement

(ii) This sentence is false because every rectangle is not a square. Hence it is a statement.

(iii) The sentence “Close the door” cannot be assigned true or false, it is a command. So it is not a statement.

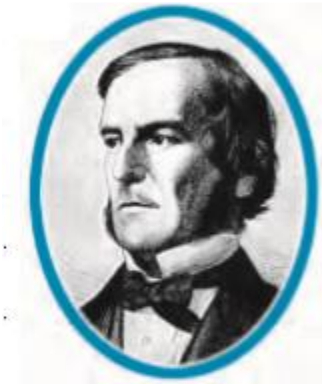
(iv)The sentence “How old are you?” is a question, hence cannot be assigned true or false. So it is not a statement.

(v) The truth or falsity of the sentence “x is a natural number” depends on the value of x. So it is not a statement.

(vi) It is a scientifically established natural phenomenon that cloud is formed before it rains. Therefore, this sentence is always true. Hence it is a statement.

New Statements from Old:

We now look into method for producing new statements from those that we already have. An English mathematician, “George Boole” discussed these methods in his book “The laws of Thought” in 1854.



George Boole
(1815 - 1864)

Here, we shall learn an important technique that we may use in order to deepen our understanding of mathematical statements. This technique will enable us to decide not only when a statement is true but to decide when the given statement is not true.

Negation of a statement:

The denial of a statement is called the negation of the statement. Let us consider the statement:

p: New Delhi is a city

The negation of this statement is;

It is not the case that New Delhi is a city.

This can also be written as,

It is false that New Delhi is a city.

This can simply be expressed as,

New Delhi is not a city.

Definition:

If p is a statement, then the negation of p is also a statement and is denoted by $\sim p$, and read as 'not p'.

Note:

While forming the negation of a statement, phrases like, "It is not the case" or "It is false that" are also used.

Let us understand it by looking at the negation of a statement, it will improve our understanding of it.

Let us consider the statement

p: Everyone in Germany speaks German.

The denial of this sentence tells us that not everyone in Germany speaks German. It does not mean that no person in Germany speaks German. It says that there is at least one person in Germany who does not speak German.

Let us consider some examples:

Example: 2

Write the negation of the following statements;

(i) p: Australia is a continent.
(ii) q: Both the diagonals of every rectangle have same length.
(iii) r: 7 is rational number.
(iv) s: $5 > 11$

Solution:

(i)The negation of the statement is,

$\sim p$: It is false that Australia is a continent.

This can also be rewritten as,

$\sim p$: Australia is not a continent.

(ii) This statement says that in every rectangle both the diagonals have same length. This means that if you take any rectangle, then both the diagonals must have the same length. The negation of this statement is,

$\sim q$: It is false that both the diagonals of every rectangle have same length.

The negation of the statement in (ii) may also be written as

$\sim q$: There is at least one rectangle whose both diagonals do not have the same length.

(iii)The negation of the statement in (iii) is,

$\sim r$: It is not the case that 7 is rational number.

Or, $\sim r$: 7 is not rational number.

(iv)The negation of the statement in (iv) is,

$\sim s$: $5 \leq 11$

Or, $\sim s$: $5 \ngtr 11$

Compound statements:

Many mathematical statements are obtained by combining one or more statements using some connecting words like “and”, “or”, etc.

Consider the statement,
“There is something wrong with the bulb or with the wiring”.
This statement is actually made up of two smaller statements:

(i) There is something wrong with the bulb.
(ii) There is something wrong with the wiring.

And these smaller statements are connected by connecting word “or”.

Now, let us consider two statements as given below,

p: 7 is an odd number.

q: 7 is a prime number.

These two statements can be combined with connecting word “and” to get a combined statement,

r: Number 7 is both odd and prime.

Statement *r* is a compound statement.

Definition:

Compound Statement is a statement which is made up of two or more statements using some connecting words like “and”, “or”, etc., and each statement is called a component statement.

Simple statements:

A statement is called simple if it cannot be broken down into two or more statements. For example the statements,

p: 2 is an even number.

q: A square has all its sides equal” and

r: Chandigarh is the capital of Haryana.

are all simple statements.

Example: 3

Find the component statements of the compound statements.

“11 is both an odd and prime number”

Solution:

This statement can be broken into two statements,

p: 11 is an odd number.

and *q*: 11 is a prime number.

Here p and q are simple statements and the connecting word is ‘and’.

Note: The simple statements which constitutes a compound statement are called component statements.

Example: 4

Find the component statements of the following and check whether they are true or not.

(i) All prime numbers are either even or odd.
(ii) 24 is a multiple of 2, 4 and 8.
(iii) A square is a quadrilateral and its four sides equal.
(iv) $\sqrt{7}$ is a rational number or an irrational number.
(v) All integers are positive or negative.

Solution:

(i) The component statements are,

p: All prime numbers are odd numbers.

q: All prime numbers are even numbers.

Both these statements are false and the connecting word is ‘or’.

(ii) The component statements are,

p: 24 is a multiple of 2.

q: 24 is a multiple of 4.

r: 24 is a multiple of 8.

All the three statements are true. Here the connecting words are ‘and’.

(iii) The component statements are

p: A square is a quadrilateral.

q: A square has all its sides equal.

Both these statements are true. Here the connecting word is ‘and’.

(iv)The component statements are

p: $\sqrt{7}$ is a rational number .

q: $\sqrt{7}$ is an irrational number.

The first statement is false and second is true. Here the connecting word is ‘or’.

(v)The component statements are,

p: All integers are positive.

q: All integers are negative.

Both these statements are false and the connecting word is ‘or’.

Special Words/Phrases:

We have observed that the compound statements are actually made-up of two or more simple statements connected by the words like “and”, “or”, etc. These words are called connectives and have special meaning in mathematics.

When we use these compound statements, it is necessary to understand the role of these words. We will discuss it, in the following section.

The word “And” :

Let us take a compound statement with “And”.

p: A point occupies a position and its location can be determined.

The statement can be broken into two component statements,

q: A point occupies a position.

r: Its location can be determined.

Here, we observe that both statements are true.

Let us now take another statement.

p: 42 is divisible by 4, 8 and 7.

This statement has following component statements,

q: 42 is divisible by 4.

r: 42 is divisible by 8.

s: 42 is divisible by 7.

Here, we observe that the first two statements are false while the third statement is true.

Now question arises, how to decide true or false values for a compound statement with the connectives “And” or “Or”.

Rule regarding the truth value of compound statement with connective “And”:

1. The compound statement with ‘And’ is true if all its component statements are true.

2. The component statement with ‘And’ is false if any of its component

statements is false.

(Hence if some or all the component statements of a compound statement are false, the resulting compound statement will be false)

Example: 5

Write the component statements of the following compound statements and check whether the compound statement is true or false.

(i) 2 is an even number and a prime number.

(ii) All living things have two legs and two eyes.

(iii) A line has no thickness and does not extend indefinitely in both directions.

(iv) 0 is less than every negative integer and every positive integer.

Solution:

(i)The component statements are,

p: 2 is an even number.

q: 2 is a prime number.

Both of these statements are true, therefore, the compound statement is true.

(ii) The two component statements are,

p: All living things have two legs.

q: All living things have two eyes.

Both the statements are false. Therefore, the compound statement is false.

(iii)The two component statements are,

p: A line has no thickness.

q: A line does not extend indefinitely in both directions.

The first statement is true and the second statement is false. Therefore, the compound statement is false.

(iv)The component statements are,

p: 0 is less than every negative integer.

q: 0 is less than every positive integer.

The first statement is false and the second statement is true. The compound statement is false.

Now, look carefully at the following statement.

p: A mixture of alcohol and water can be separated by

chemical methods.

This sentence cannot be considered as a compound statement with “And”, because we cannot break it into simple statements. Here the word “And” refers to two things – alcohol and water. This leads us to an important note.

Note:

Do not think that a statement with "And" is always a compound statement as is explained in the above example. In the above statement, the word "And" is **not** used as a connective.

The word "Or":

Let us now consider the statements with the connect "or",

p: Two lines in a plane either intersect at one point or they are parallel.

What does this statement mean?

This means that if two lines in a plane intersect, then they are not parallel. And if the two lines are not parallel, then they intersect at a point. This statement is true in both the situations. Hence the statement "*p*" is a true statement.

In order to understand statements with "Or", let us first note that the word "Or" is used in two ways in English language. Look at the following statement,

p: An ice cream or cold drink is available with a Thali in a restaurant.

This means that if a person does not want ice cream can have a cold drink along with Thali or if a person does not want cold drink then he can have an ice cream along with Thali. According to this statement a person cannot have both ice cream and cold drink with the thali. This is called an exclusive "Or".

Now consider another statement,

p: A student who has taken biology or chemistry can apply for M.Sc. course with microbiology.

This statement means that the students who have taken both biology and chemistry can also apply for the M.Sc. course with microbiology along with the students who have taken only one of these subjects.

This is inclusive use of the word "Or".

It is important to note the difference between these two ways of using the word "Or", because we require this when we check whether the statement is true or not.

Example 6:

For each of the following statements, determine whether an inclusive "Or" or exclusive "Or" is used. Give reasons for your answer.

(i) Two lines intersect at a point or are parallel.
(ii) To enter a country, you need a passport or a voter registration card.
(iii) Students can opt French or German as their third language.
(iv) The school is closed if it is a holiday or a Sunday.

Solution:

(i) Here "Or" is exclusive because it is not possible for two lines to intersect and be parallel simultaneously.

(ii) In this case, a person can have both a passport and a voter registration card to enter a country, so "Or" is inclusive.

(iii) Since because a student cannot opt both French and German as the third language, so here use of "Or" is exclusive.

(iv) Since school remains closed on holidays as well as on Sundays, so the use of "Or" is inclusive.

Rule regarding the truth value of compound statement with connective "Or":

Let us consider some compound statements to learn the rules to check whether the compound statement with connector "Or" is true or not.

Consider the compound statement,

p: Two lines intersect at a point or they are parallel.

The component statements are;

q: Two lines intersect at a point.
r: Two lines are parallel.

Look here, when *q* is true *r* is false and when *q* is false *r* is true, the compound statement *p* is true.

Consider another compound statement,

p: 125 is a multiple of 7 or 8.

Its component statements are,

q: 125 is a multiple of 7.
r: 125 is a multiple of 8.

Look both the component statements *q* and *r* are false. The compound statement *p* is false.

Again, consider a compound statement:

p: The school is closed, if there is a holiday or Sunday.

The component statements are,

q: School is closed if there is a holiday.
r: School is closed if there is a Sunday.

Both *q* and *r* are true, the compound statement is true.

Consider one more compound statement.

p: Mumbai is the capital of Kolkata or Karnataka.

The component statements are,

q: Mumbai is the capital of Kolkata.
r: Mumbai is the capital of Karnataka.

Both these component statements are false. The compound statement is also false.

By studying all these above examples you must have understood the rules to check when a compound statement with connector “or” is true or not.

Rule:

i) A compound statement with an ‘Or’ is true when one component statement is true or both the component statements are true.

ii) A compound statement with an ‘Or’ is false when both the component statements are false.

Example: 7

Identify the type of “Or” used in the following statements and check whether the statements are true or false:

(i) To apply for a driving licence, a person should have a ration card or a passport.
(ii) $\sqrt{17}$ is a rational number or an irrational number.

Solution:

(i)The component statements are,

p: To apply for a driving licence a person should have a ration card.
q: To apply for a driving licence a person should have a passport.

A person can apply for a driving licence if he has either of the two, a ration card or a passport, as well as when he has both.
Therefore, it is inclusive “Or” and the compound statement is also true.

(ii)The component statements are,

p: $\sqrt{17}$ is a rational number.
q: $\sqrt{17}$ is an irrational number.

Here, we know that the first statement is false and the second is true.
Therefore, it is exclusive “Or” and the compound statement is true.

Quantifiers:

Quite many times we use some phrases. Quantifiers are the phrases like, “There exists” and “For all” etc.

For example take the statement,

p: There exists a rectangle whose all sides are equal.

This means that there is at least one rectangle whose all sides are equal.

Quantifiers "For all" and "There exists" are closely related in the sense that while forming negation of the statement involving "For all" we observe that the phrase "There exists" occurs.

Similarly forming negation of the statement involving "There exists" we observe that the phrase "For all" occurs as is clear from the following example;

Consider the statement;

q: For every prime number k, $\sqrt{k}$ is an irrational number.

The negation of this statement is;

$\sim q$: There exists a prime number k, such that $\sqrt{k}$ is not an irrational number.

Consider another statement;

r: All squares are rectangles.

Its negation is,

$\sim r$: There exists a square which is not a rectangle.

Thus, we have seen that many mathematical statements contain some special words and it is important to know the meaning attached to them, especially when we have to check the validity of different statements.

Example 8:

Identify the Quantifiers in the following statements.

(i) There exists a triangle which is not equilateral.

(ii) For every natural number x, $x + 1$ is also a natural number.

(iii) For all negative integers x, x^3 is also a negative integers.

Solution:

(i) There exists

(ii) For every

(iii) For all

Summary:

(i) A mathematically acceptable statement is a sentence which is either true or false but not both simultaneously.

(ii) In this module, following concepts were discussed;

1. Negation of a statement p;

   If p denotes a statement, then the negation of p is denoted by $\sim p$.

2. Compound statements and their related component statements;

   A statement is a compound statement if it is made up of two or more simple statements. The simple statements are called component statements of the compound statement.

3. The role of "And", "Or", "There exists" and "For every" in compound statements.

   The words "And" and "Or" are called connectives and "There exists" and "For all" are called quantifiers.