A1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 02 (Class XI, Semester - 2)
Module Name/Title	Derivatives
Module Id	Kemh_21303
Pre-requisites	Knowledge about Functions, concept of limits
Objectives	After going through this lesson, the learners will be able to understand the following: 1. Concept, notation and different interpretation of derivatives 2. Derivative through first principle 3. Algebra of derivative of functions 4. Derivatives of Polynomials and trigonometric functions.
Keywords	Derivatives, first principle, algebra of derivatives.

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Introduction

Have you ever observe such waves when drop a stone in quiet lake /pond or even a bucket full of water? Are you curious how fast the enclosed area changes when the radius of the circular wave changes?

In real life there are various such situations in which a particular parameter changes with respect to some other parameter. We always desire to know how such changes takes place at various instant of time and try to find the rate at which it is changing. For instances: the rate of change of the area of a circle per second with respect to radius; the change in the value of a particular stock knowing its present value; when will a reservoir overflow knowing the depth of the water at several instances of time; behavior of the function whether increasing or decreasing and many more. This rate of change of one thing due to occurrence of change in other is referred as 'Derivative' and the process of finding the derivative is known as 'Differentiation'.

There are various notation used to describe the derivatives. Some are as follows:

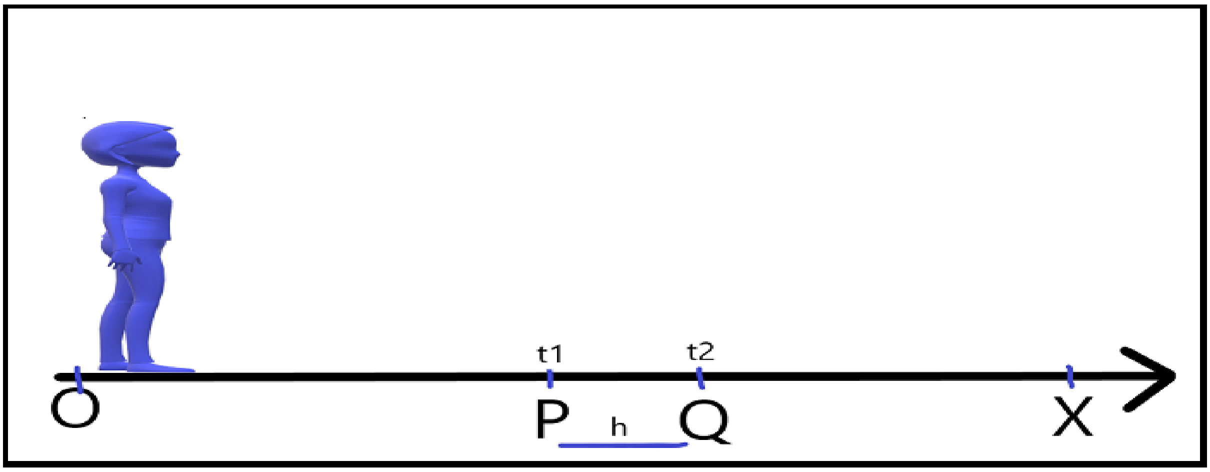
- $\frac{d}{dx}(f(x))$
- $f'(x)$
- $if\ y = f(x), then\ derivative\ is\ denoted\ as\ \frac{dy}{dx}$ or read as 'derivative of y with respect to x' or sometimes as y'
- $D(f(x))$
- Derivative of f at $x = a$ is also denoted by $\frac{d}{dx}f(x)\Big|_a\ or\ \frac{df}{dx}\Big|_a\ or\ even\ \left(\frac{df}{dx}\right)_a$

There are various different interpretation of derivatives as well. Let us discuss that:

Interpretation of derivatives as

- **Physical interpretation of derivative at a point (As a rate of measure)**

Let a particle (say a person) be moving in a straight line OX starting from a point O towards point X as shown,



Clearly, the position of a person at any time t depends on the time elapsed, that is, the distance walked by the person from O depends upon the time. So, it is the function of time taken by the person. Let at any time t_1 the person be at point P and after a further time h i.e., at time

$t_2 = t_1$ +h , the person is at point Q.

therefore, $OP = f(t_1)\ and\ OQ = f(t_1 + h)$

Distance travelled in time $h = PQ = OQ - OP = f(t_1 + h) - f(t_1)$

Average speed of the person during the walk from P to Q is $\frac{PQ}{h} = \frac{f(t_1 + h) - f(t_1)}{h}$

Now, As $h \to 0$, we may observe that $Q \to P$

$$\therefore (instantaneous\ change\ in\ speed\ at\ time\ t_1) = \lim_{h \to 0} \frac{f(t_1 + h) - f(t_1)}{h} = f'(t_1)$$

Thus, if $f(t)$ gives the distance of a moving particle (here a person) at time t then the derivative of f that is $f'(t)$ at $t = t1$ or at the point P.

Example: The distance $f(t)$ in kilometers moved by a train travelling in a straight line in t seconds is given by $f(t) = t^2 + 3t + 5$. Find the speed of the train at the end of 10 seconds.

Solution: We have, $f(t) = t^2 + 3t + 5$

The speed of the train at the end of 10 seconds is given by $f'(10)$ (the derivative of the function at 10)

Now,

$$f'(10) = \lim_{h\to 0} \frac{f(10+h) - f(10)}{h}$$

$$f'(10) = \lim_{h\to 0} \frac{(10+h)^2 + 3(10+h) + 5 - (100+30+5)}{h}$$

$$f'(10) = \lim_{h\to 0} \frac{(h^2 + 23h + 135) - (135)}{h}$$

$$f'(10) = \lim_{h\to 0} \frac{h^2 + 23h}{h} = 23$$

Hence, the speed of the train at the end of 10seconds is 23km/s.

Example: A balloon which is always remain spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the latter is 10cm

Solution: The volume of a spherical balloon with variable radius is given by $V = \frac{4}{3}\pi r^3$. Therefore, the rate of change in its volume at radius 10 is given by

$$f'(10) = \lim_{h\to 0} \frac{f(10+h) - f(10)}{h}$$

$$f'(10) = \lim_{h\to 0} \frac{\frac{4}{3}\pi(10+h)^3 - \frac{4}{3}\pi 10^3}{h}$$

$$f'(10) = \lim_{h\to 0} \frac{\frac{4}{3}\pi(1000 + h^3 + 300h + 30h^2 - 1000)}{h}$$

$$f'(10) = \lim_{h\to 0} \frac{\frac{4}{3}\pi(h^3 + 300h + 30h^2)}{h} = \lim_{h\to 0} \frac{\frac{4}{3}\pi(h^2 + 300 + 30h^1)}{1}$$

$$f'(10) = \frac{1200\pi}{3} = 400\pi cm^3/s$$

Hence, the rate of change in the volume of a spherical balloon is $400\pi cm^3/s$

Example: The radius of a circle is increasing at the rate of 0.7cm/s. What is the rate of increase of its circumference?

Solution: The circumference of a circle with radius r is given by $C = 2\pi r$. Therefore, the rate of change of circumference C with respect to time t is

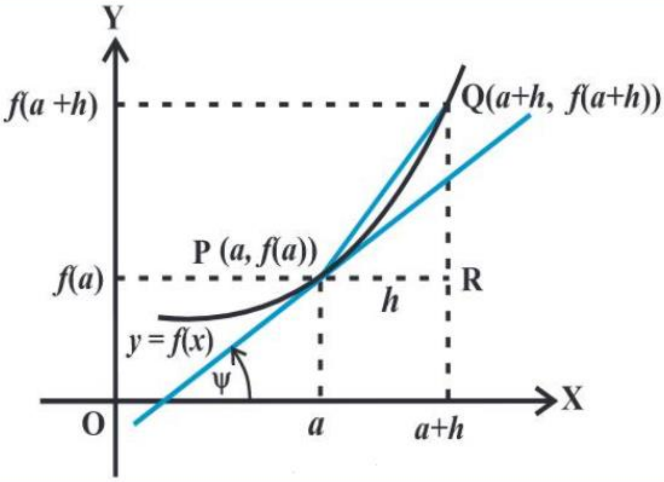
$$\frac{dc}{dt} = \frac{d}{dt}(2\pi r) = 2\pi\frac{dr}{dt} = 2\pi(0.7) = 1.4\pi cm/s$$

Remark: The above such examples will be discussed more in XII class module

- **Graphical interpretation of a derivative at a point**

We now present a geometric interpretation of derivative of a function at a point. Let $y = f(x)$ be a function and let $P = (a, f(a))$ and $Q = (a+h, f(a+h)$ be two points close to each other on the graph of this function.

As shown in the figure



We know that $f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

From the triangle PQR, it is clear that the ratio,whose limit we are taking is precisely equal to tan(QPR), is the slope of the chord PQ. In the limiting process, as h tends to 0, the point Q tends to P and we have

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{Q\to P}\frac{QR}{PR}$$

This is equivalent to the fact that the chord PQ tends to the tangent at P of the curve $y = f(x)$. Thus the limit turns out to be equal to the slope of the tangent. Hence

$$f'(a) = \tan\psi .$$

For a given function f we can find the derivative at every point. If the derivative exists at every point, it defines a new function called the derivative of f.

Example: Find the slope of the tangent to the curve $y = x^2 + 2$ at (1, 3)

Solution: let $f(x) = x^2 + 2$. Clearly the slope of the tangent to the curve at (1, 3) is derivative of function $f(x)$ at $x = 1$.

Now,

$$f'(1) = \lim_{h\to 0}\frac{f(1+h) - f(1)}{h}$$

$$f'(1) = \lim_{h \to 0} \frac{(1+h)^2 + 3 - (1+3)}{h}$$

$$f'(1) = \lim_{h \to 0} \frac{(h^2 + 2h + 4) - (4)}{h}$$

$$f'(1) = \lim_{h \to 0} \frac{h^2 + 2h}{h} = 2$$

Hence, the slope of the tangent at (1, 3) is equal 2

Thus, in view of the discussions above, we define the derivative of a function at a point as:

Definition *Suppose f is a real valued function and a is a point in its domain of definition. The derivative of f at a is defined by*

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided the *limit exists. Derivative of $f(x)$ at a is denoted by $f'(a)$.*

Observe that $f'(a)$ quantifies the change in $f(x)$ at a with respect to x.

Differentiation from first principle

In all above interpretations we can find derivative of a function at a point. So, we can find derivative of a function at every point and if derivative exist at every point, it defines a new function called the derivative of function f. Therefore, formally we define derivative of the function as follows:

Definition *Suppose f is a real valued function, the function defined by*

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

wherever the limit exists is defined to be the derivative of f at x and is denoted by $f'(x)$ This definition of derivative is also known as the first principle of derivative.

Thus $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Clearly the domain of definition of $f'(x)$ is wherever the above limit exists.

Now, let us take some of the examples on it.

Example: Find the derivative of the $f(x) = 6x$

Solution: The derivative of the function at any point is $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$f'(x) = \lim_{h \to 0} \frac{6(x+h) - 6x}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{6x + 6h - 6x}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{6h}{h} = 6$$

Thus, $\frac{d}{dx}(6x) = 6$

Example: Find the derivative of the $f(x) = k \quad where\ k\ fixed\ real\ number$

Solution: The derivative of the function at any point is $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Since the function is a constant function. Therefore, $f(x+h) = f(x) = k$

$$f'(x) = \lim_{h\to 0}\frac{k-k}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{0}{h}$$

Thus, $\boldsymbol{\frac{d}{dx}(k) = 0}$

Remark: Observe that **derivative of any constant function is a Zero.**

Example: Find the derivative of the $f(x) = e^x$

Solution: The derivative of the function at any point is $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$f'(x) = \lim_{h\to 0}\frac{e^{x+h}-e^x}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{e^x.e^h-e^x}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{e^x(e^h-1)}{h} = e^x \qquad \ldots\ldots as\ we\ know\ \lim_{h\to 0}\frac{(e^h-1)}{h} = 1$$

Thus, $\boldsymbol{\frac{d}{dx}(e^x) = e^x}$

Example: Find the derivative of the $f(x) = logx$ provided $x > 0$

Solution: The derivative of the function at any point is $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$f'(x) = \lim_{h\to 0}\frac{\log(x+h)-\log(x)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\log\left(\frac{x+h}{x}\right)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\log\left(\frac{x+h}{x}\right)}{h/x}\frac{1}{x}$$

$$f'(x) = \lim_{h\to 0}\frac{\log\left(1+\frac{h}{x}\right)}{h/x}.\frac{1}{x} \qquad \left[as\ \lim_{h\to 0}\frac{\log(1+x)}{x} = 1\right]$$

Hence, $\frac{d}{dx}(logx) = \boldsymbol{\frac{1}{x}}$

(Note: Everywhere log function is taken with base e. otherwise stated)

Example: Find the derivative of $f(x) = 2x + 5$

Solution: The derivative of the function at any point is $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$$f'(x) = \lim_{h \to 0} \frac{(2(x+h)+5)-(2x+5)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2x+2h+5-2x-5}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2h}{h} = 2$$

Thus, $\boldsymbol{\frac{d}{dx}(2x+5) = 2}$

Observe the above example? Yes, derivative of any function of type $f(x) = ax + b$ is the coefficient of x ,that is:

$$\boldsymbol{\frac{d}{dx}(ax+b) = a}$$

Example: Find the derivative of the $f(x) = \frac{x+1}{x-1}$ $where\ x \neq 1$

Solution: The derivative of the function at any point is $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$$f'(x) = \lim_{h \to 0} \frac{\frac{(x+h+1)}{(x+h-1)} - \frac{(x+1)}{(x-1)}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{(x+h+1)(x-1)-(x+1)(x+h-1)}{(x+h-1)(x-1)}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2+xh-h-1-x^2-xh-h+1}{(x+h-1)(x-1)h}$$

$$f'(x) = \lim_{h \to 0} \frac{-2h}{(x+h-1)(x-1)h} = \frac{-2}{(x-1)^2}$$

Thus, $\boldsymbol{\frac{d}{dx}\left(\frac{x+1}{x-1}\right) = \frac{-2}{(x-1)^2}}$

Algebra of Derivatives

In above examples, notice that the functions taken are sum, product or subtraction of one or more functions. Observed that the derivatives respects addition, subtraction, multiplication and division. Is it a coincidence? No! In fact we can formalize these as theorems

Theorem Let f and g be two functions such that their derivatives are defined in a common domain. Then

(i) Derivative of sum of two functions is sum of the derivatives of the functions.

$$\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x).$$

(ii) Derivative of difference of two functions is difference of the derivatives of the functions.

$$\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x).$$

(iii) Derivative of product of two functions is given by the following *product rule*.

$$\frac{d}{dx}[f(x)\,.\,g(x)]=\frac{d}{dx}f(x).g(x)+f(x).\frac{d}{dx}g(x)$$

(iv) Derivative of quotient of two functions is given by the following *quotient rule* (whenever the denominator is non–zero).

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{\frac{d}{dx}f(x).g(x)-\ f(x)\ \frac{d}{dx}g(x)}{(g(x))^2}$$

Example: Find the derivative of the function $f(x) = 10x$

Solution: Now, see the function $f(x) = 10x$ can be written as $f(x) = uv$.

Where u is a constant function taking value 10 everywhere and $v(x) = x$.

On differentiating the function by product rule we have,

$f'(x) = (10x)' = 0.x + 10.1 = 10$

Observation: From the above example we can make following observation

- The derivative of a function $f(x) = kx$ where k is any constant will be k.

  $\boldsymbol{f'(x) = (kx)' = k \qquad where\ k\ is\ any\ constant}$

- The derivative of a function $f(x) =\ k\,u(x)$ where k is any constant will be $ku'(x)$

  $\boldsymbol{f'(x) =\ ku'(x)\ where\ k\ is\ any\ constant}$

On the similar lines it is easy to find that derivative of x^2 will be $2x$.

Let us take some more examples

Example: Find the derivative of the function $f(x) =\ xe^x$

Solution: We can see that the function $f(x)$ is a product of two functions $u(x) = x$ and $v(x) =\ e^x$. Therefore, on differentiating the function by product rule we have,

$f'(x) =\ (xe^x)' = x'e^x + x(e^x)' = 1.e^x + xe^x = e^x(1 + x) \qquad ((e^x)' = e^x)$

Thus, $(xe^x)' =\ e^x(1 + x)$

Now, let us make a slight change in the function. Instead of taking $(x) = xe^x$. Find the derivative of the function $h(x) = \frac{e^x}{x}$ $provided\ x \neq 0$

Here, we can see that $h(x)$ is a quotient of two functions $m(x) = e^x$ and $n(x) = x$.

Therefore by quotient rule the derivative of the function is

$h'(x) = \frac{x(e^x)'+x'e^x}{x^2} = \frac{xe^x+e^x}{x^2} = \frac{e^x(x+1)}{x^2}$

Example: Find the derivative of the function $f(x) = \frac{x^2+1}{3}$

Solution: We see that the above function can be written as $f(x) = \frac{1}{3}(x^2 + 1)$. Therefore, from the above observation. The derivative of the function is$f'(x) = \frac{1}{3}(x^2 + 1)'$. Further through algebra of derivatives (i) we get $f'(x) = \frac{1}{3}(x^2 + 1)' = \frac{1}{3}((x^2)' + (1)') = \frac{1}{3}\ (2x)$

So, $f'(x) = \frac{2x}{3}$

Now, before moving on to the next example let have the following theorem

Theorem Derivative of $f(x) = x^n$ is nx^{n-1} for any positive integer n.

This theorem can be easily verified through first principle which you can see in web links of NCERT book at page 310.

It is very interesting to note that this theorem is true for all powers of x that is n can be any real numbers

Example: Find the derivative of the function $f(x) = 2e^x - x^6$

Solution: Now, it is easy to find the derivative of the function$f(x) = 2e^x - x^6$.

$f'\ (x) = (2e^x - x^6)' = 2e^x - 6x^5$

Example: Find the derivative of the function $f(x) = 1 + x + x^2 + x^3 + x^4 + \cdots + x^{40}$

Solution: This function $f(x) = 1 + x + x^2 + x^3 + x^4 + \cdots + x^{40}$ can be written as sum of finite number of different functions. $f(x) = u(x) + v(x) + k(x) + \cdots + n(x)$ where $u(x) = 1, v(x) = x, k(x) = x^2$ and so on. Therefore, by algebra of derivatives part (i) and last theorem above we get, $f'(x) = 0 + 1 + 2x^1 + 3x^2 + 4x^3 + \cdots + 40x^{39}$

You may also notice that the above function is a polynomial function. So, derivative of such functions can be find out more generally by this theorem given below.

Differentiation of Polynomial and Trigonometric functions

Theorem Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ be a polynomial function, where $a_i s$ are all real numbers and $a_n \neq 0$. Then, the derivative function is given by

$$\frac{df(x)}{dx} = na_n x^{n-1} + (n-1)a_{n-1}x^{x-2} + ... + 2a_2 x + a_1.$$

Example: Find the derivative of the polynomial function $f(x) = 10x^5 + 12x^3 + 2x + 1$. Also, find $f'(-1) + 2f(0) + 1$.

Solution: By using above theorem directly we can see

$f'(x) = 50x^4 + 36x^2 + 2$

Therefore, $f'(-1) + 2f(3) + 1 = (50(-1)^4 + 36(-1)^2 + 2)$ +2+1 = (50+36+2+2+1) =91

Example: Let us find the derivative of the following trigonometric functions:

A. $\sin x$

B. $x\sin x$

C. $\cos x$

D. $\cot\ x$

Solution: **A.** The derivative of the function at any point is $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Here, $f(x+h) = \sin\,(x+h)$ and $f(x) = \sin\,(x)$

Therefore, we get $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$f'(x) = \lim_{h\to 0}\frac{\sin\,(x+h) - \sin\,(x)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{sinx\ cosh + \sinh cosx - sinx}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{sinx\ cosh - sinx}{h} + \lim_{h\to 0}\frac{\sinh cosx}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{sinx(\cos h - 1)}{h} + cosx\ \lim_{h\to 0}\frac{\sin h}{h}$$

$$f'(x) = 0 + cosx\,.1 \qquad .....as\ \lim_{x\to 0}\frac{(cosx-1)}{x} = 0\ \ and\ \ \lim_{x\to 0}\frac{sinx}{x} = 1$$

Thus, $f'(x) = cosx$

On the similar lines we can find the derivative of $cosx$ which is $-sinx$. See below!

C. The derivative of the function at any point is $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

Here, $f(x+h) = \cos(x+h)$ and $f(x) = \cos(x)$

Therefore, we get $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$f'(x) = \lim_{h\to 0} \frac{\cos(x+h) - \cos(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{cosxcosh - sinhsinx - cosx}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{cosxcosh - cosx}{h} - \lim_{h\to 0} \frac{sinhsinx}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{cosx(cosh-1)}{h} - sinx \lim_{h\to 0} \frac{sinh}{h}$$

$$f'(x) = 0 - sinx\,.1 \qquad \ldots\ldots as \lim_{x\to 0} \frac{(cosx-1)}{x} = 0 \;\; and \;\; \lim_{x\to 0} \frac{sinx}{x} = 1$$

Thus, $f'(x) = -sinx$

Similarly it is easy to check that

$$\boldsymbol{\frac{d}{dx}(tanx) = sec^2x}$$

$$\boldsymbol{\frac{d}{dx}(secx) = secxtanx}$$

Now, **see part B**. It is product of two functions x and $sinx$. So, using $\frac{d}{dx}(sinx) = cosx$

We can find the derivative of the function $f(x) = xsinx$ by using Product rule

$$f'(x) = (xsinx)'$$

$$f'(x) = x'sinx + x(sinx)'$$

$$f'(x) = sinx + xcosx$$

Thus, $\boldsymbol{\frac{d}{dx}(xsinx) = sinx + xcosx}$

Solution for part D.

In order to find the derivative of $cotx$ function.

We have $f(x) = cotx$ which can also be written as $f(x) = \frac{1}{tanx}$

By using quotient rule. We get,

$$f'(x) = \frac{1(tanx)' - 0.tanx}{tan^2x}$$

$$f'(x) = \frac{sec^2x}{tan^2x}$$

Thus, $\boldsymbol{\frac{d}{dx}(cotx) = -cosec^2x}$

It is easy to check that the $\boldsymbol{\frac{d}{dx}(cotx) = -cosec^2x}$ by using first principle.

Before ending, let us recall what we have studied in this module

Summary

- The derivative of a function f at a is defined by $f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$
- Interpretation of derivatives as
    - Physical interpretation of derivative at a point (as a rate measure)
    - Graphical interpretation of derivative
- The derivative of a function f at any point x is defined by $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ which is also known as first principle of derivative.
- Algebra of derivatives: For functions u and v the following holds:
    - $(u \pm v)' = u' \pm v'$
    - $(uv)' = u'v + uv'$
    - $\left(\frac{u}{v}\right)' = \frac{vu' - v'u}{v^2}$,provided all are defined
- Some of the standard derivatives
    - $\frac{d}{dx}(x^n) = nx^{n-1}$
    - $\frac{d}{dx}(k) = 0 \quad , where\ k\ is\ a\ constant$
    - $\frac{d}{dx}(ax + b) = a$
    - $\frac{d}{dx}(sinx) = cosx$
    - $\frac{d}{dx}(e^x) = e^x$
    - $\frac{d}{dx}(tanx) = sec^2x$
    - $\frac{d}{dx}(logx) = \frac{1}{x} \quad , provided\ x > 0$
    - $\frac{d}{dx}(cosx) = -sinx$
    - $\frac{d}{dx}(cotx) = -cosec^2x$