1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 02 (Class XI, Semester - 2)
Module Name/Title	Conic Sections - Part 2
Module I	kemh_21102
Pre-requisites	Basic knowledge of Conic Sections
Objectives	After going through this lesson, the learners will be able to do the following: • Parabola • Types of Parabola • Latus Rectum • Standard Equation of Parabola
Keywords	Parabola, Types of Parabola, Latus Rectum, Standard equation of Parasbola

2. Development Team

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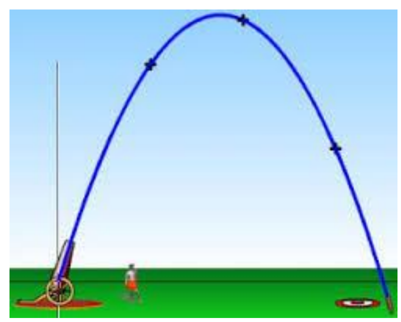
1. Introduction:

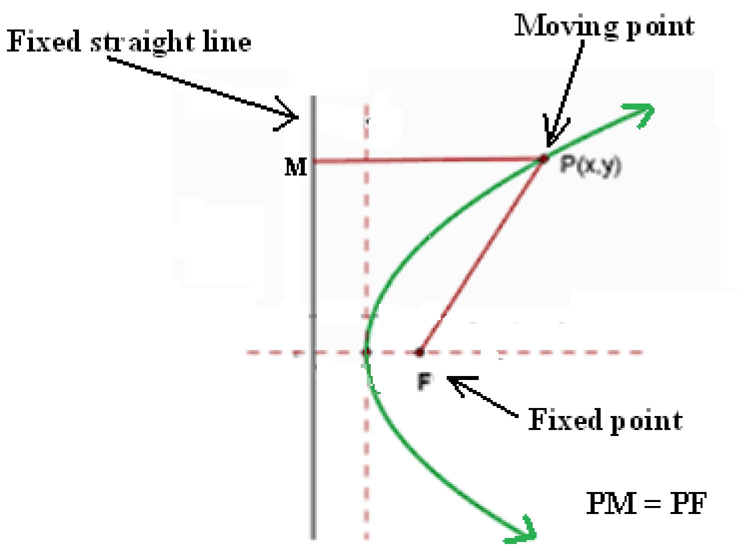
You have already learnt in earlier classes to plot the graph a quadratic equation, if you remember the shape of the graph up to some extent resembles with the following shapes;



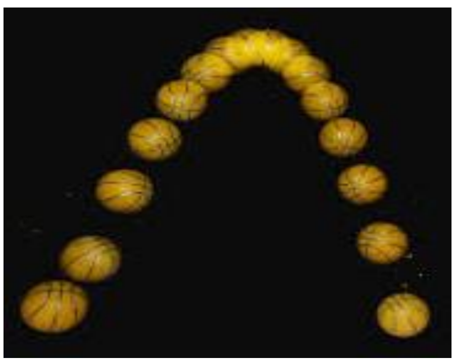


In previous module, you have learnt that we get such shapes when a cone is cut by a plane parallel to its generator and hence it is known as a conic section, called parabola.

Again, if we find locus of a point which moves in a plane such that its distance from a fixed point in the plane is always equal to its distance from a fixed straight line in the same plane, the curve so obtained is parabola.



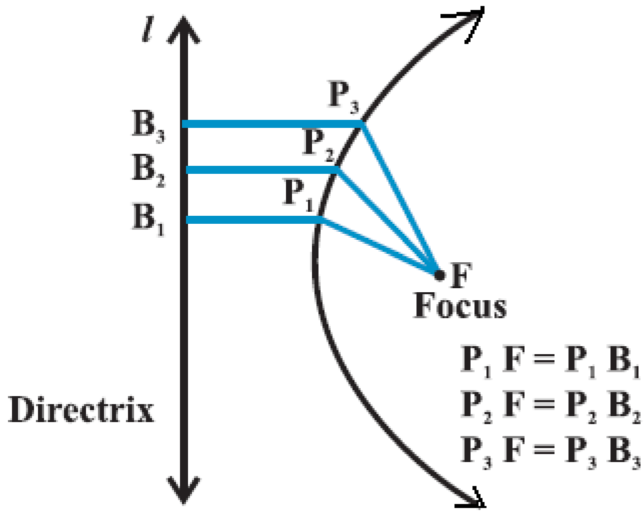
The name parabola has come from two words, ***'Para'*** and ***'bola'***; 'Para' means 'for' and 'bola' means 'throwing', i.e., the shape described when we throw a ball in the air.



2. Parabola:

Definition:

A parabola is the set of all those points in a plane which are equidistant from a fixed line and a fixed point (not on the fixed line) in the plane.

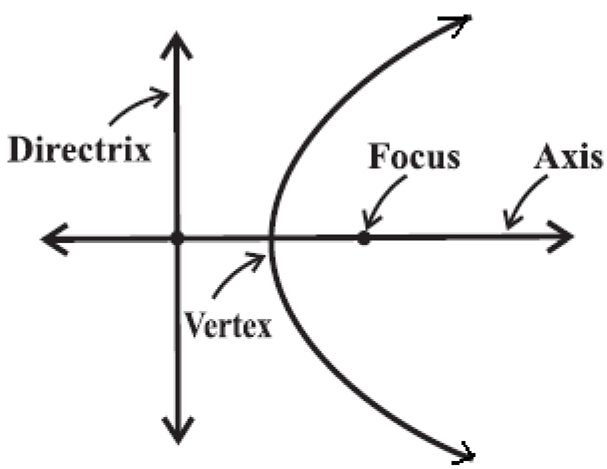


The fixed line is called the ***directrix*** of the parabola and the fixed point is called the ***focus.***

Note:

If the fixed point lies on the fixed line, then the set of points in the plane, which are equidistant from the fixed point and the fixed line is a straight line through the fixed point and perpendicular to the fixed line. We call this straight line as *degenerate case* of the parabola.

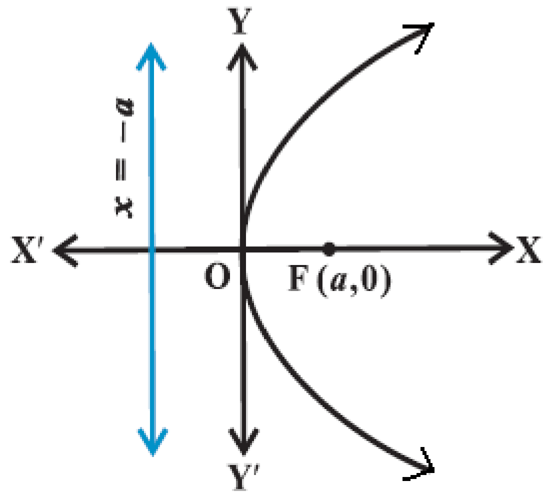
A line through the focus and perpendicular to the *directrix* is called the *axis* of the parabola. The point of intersection of parabola with the axis is called the *vertex* of the parabola.



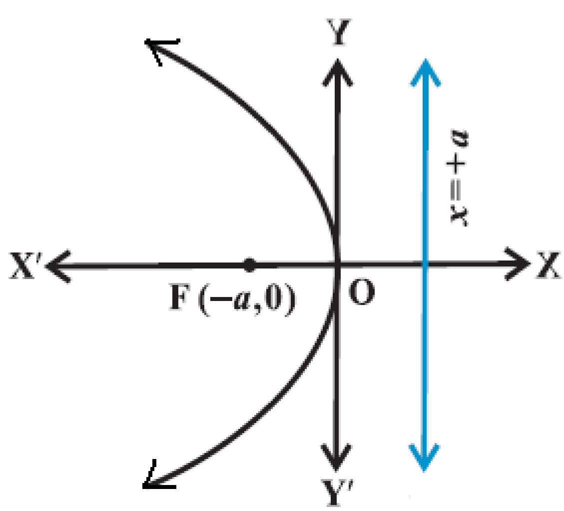
3. Standard equation of parabola:

The equation of a *parabola* is simplest if the vertex is at the origin and the axis of symmetry is along the x-axis or y-axis. There are four such possible orientations of parabola as shown below:

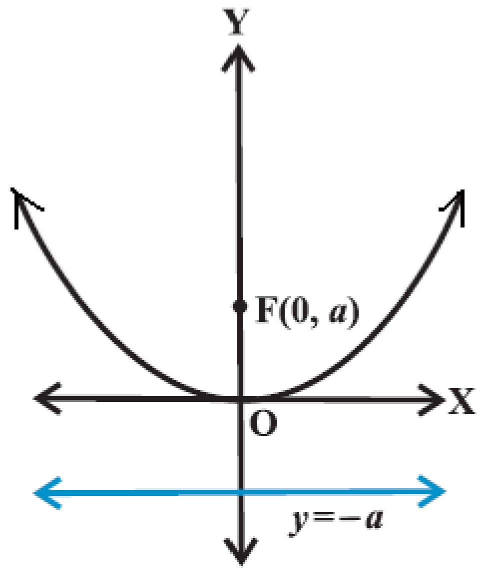
(i) focus at $(a, 0)$, $a > 0$ and directricx $x = -a$



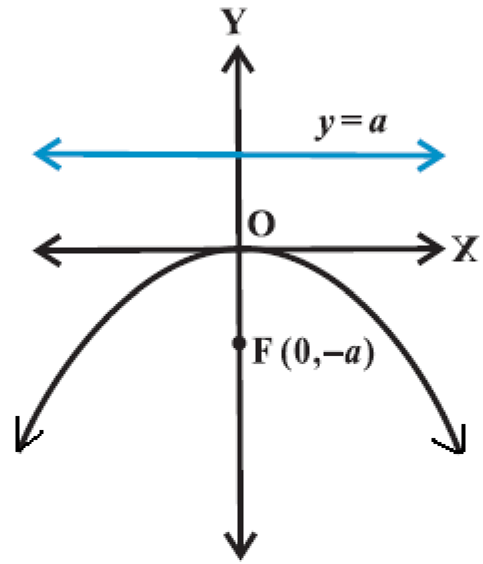
(ii) focus at $(-a, 0)$, $a > 0$ and directricx $x = a$



(iii) focus at $(0, a)$, $a > 0$ and directricx $y = -a$

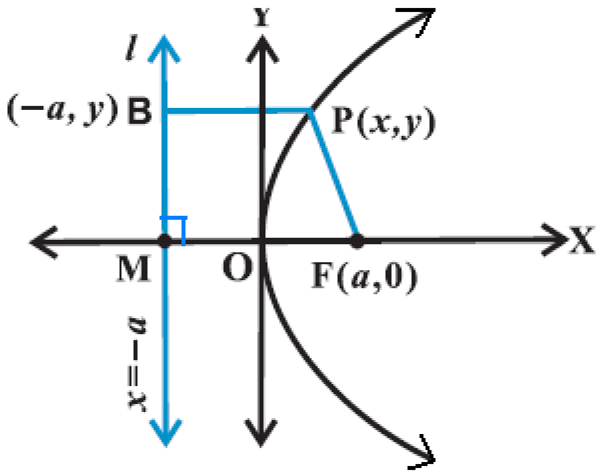


(iv) focus at $(0, -a)$, $a > 0$ and directricx $y = a$



Derivation of the equation for the parabola:

We will derive the equation for the parabola when, focus is at $(a, 0)$, $a > 0$ and directricx is $x = -a$



In the figure above, F is the focus and *l* the directrix.

Let FM be the perpendicular to the directrix and mid-point of FM be O. By the definition of parabola, the mid-point O is on the parabola and is called as the ***vertex*** of the parabola.

MO is extended to both the sides and is considered as *x*-axis. Let us take point O as origin. OY perpendicular to *x*-axis and passing through O is taken as the *y*-axis.

Let the distance from the directrix to the focus be $2a$. Then, the coordinates of the focus will be $(a, 0)$ and the equation of the directrix will be $x = -a$ and vertex is at $(0, 0)$. Let $P(x, y)$ be any point on the parabola such that PB is perpendicular to *l* then,

$$PF = PB, \qquad \ldots\ldots\ldots (1)$$

Distance of point P from focus = Distance of point P from directrix

The coordinates of point B are $(-a, y)$ as it lies on directrix,

Using the distance formula, we have,

$$\text{PF} = \sqrt{(x-a)^2 + y^2} \quad \text{and} \quad \text{PB} = \sqrt{(x+a)^2}$$

Since PF = PB, we have

$$\sqrt{(x-a)^2 + y^2} = \sqrt{(x+a)^2}$$

i.e. $(x - a)^2 + y^2 = (x + a)^2$

or $x^2 - 2ax + a^2 + y^2 = x^2 + 2ax + a^2$

Simplifying we get,

$$y^2 = 4ax \; (a > 0)$$

Hence, any point on the parabola satisfies the equation,

$$y^2 = 4ax. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Conversely, let P(x, y) satisfy the equation (2), then by distance formula,

$$\text{PF} = \sqrt{(x-a)^2 + y^2}$$

Substituting for y^2 from equation (2), we get

$$\text{PF} = \sqrt{(x-a)^2 + 4ax}$$

$$= \sqrt{(x+a)^2} = \text{PB}$$

We get,

$$\text{PF} = \text{PB}$$

Therefore distance of point P from focus is equal to its distance from the dircetrix, thus the point P(x, y) lies on the parabola.

Hence the equation of the parabola with vertex at the origin,

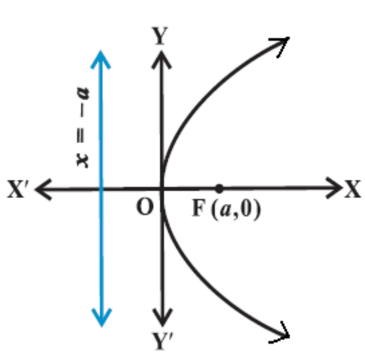
focus at (a, 0) and directrix $x = -a$ is:

$$\boldsymbol{y^2 = 4ax}$$

The parabola is symmetrical about x-axis, this line of symmetry of the parabola i.e., x-axis is called the axis of the parabola. Since $a > 0$, x can assume only positive values or zero but no negative value and the curve extends indefinitely far into the first and the fourth quadrants.

4. Types of parabola:

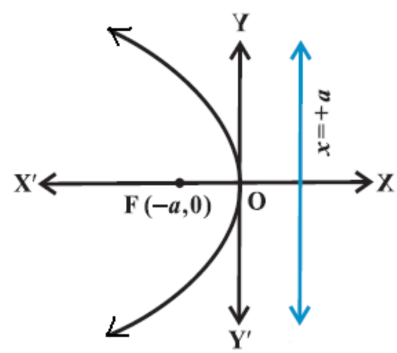
There are four standard equations of parabolas, depending upon the choice of axes. The shapes of these four forms are;



(i)

Parabola

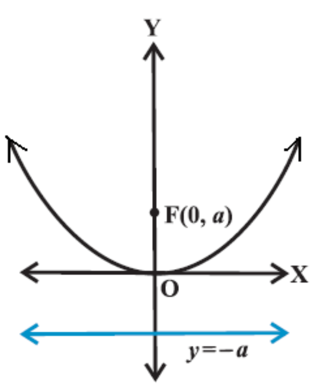
Opening towards right



(ii)

Parabola

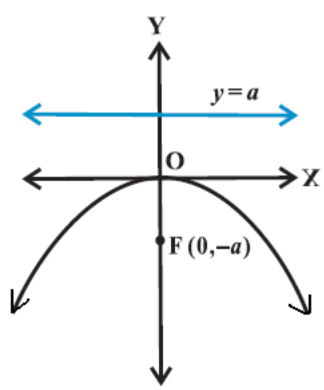
Opening towards left



(iii)

Parabola

Opening upwards



(iv)

Parabola

Opening downwards

We have already derived the equation of parabola of form (i) as, $y^2 = 4ax$ Similarly, we can derive the equations for other forms of the parabolas also, For fig (ii) the equation of parabola is $y^2 = -4ax$, this curve extends indefinitely far into the second and third quadrants; focus is at $(-a, 0)$ and directrix is $x = a$

For fig (iii) the equation of parabola is $x^2 = 4ay$, this curve extends indefinitely far into the first and second quadrants; focus is at $(0, a)$ and directrix is $y = -a$.

For fig (iv) the equation of parabola is $x^2 = -4ay$, this curve extends indefinitely far into the third and fourth quadrants; focus is at $(0, -a)$ and directrix is $y = a$. These four equations are known as standard equations of parabolas.

Note:

1) The standard equations of parabolas have focus on one of the coordinate axes; vertex at the *origin* and the directrix is parallel to the other coordinate axis.

2) Parabola is symmetrical with respect to the axis of the parabola. If the equation has a y^2 term, then the axis of symmetry is along the x-axis and if the equation has an x^2 term, then the axis of symmetry is along the y-axis.

3) When the axis of symmetry is along the x-axis the parabola opens to

   (a) the right if the coefficient of x is positive,

   (b) the left if the coefficient of x is negative.

4) When the axis of symmetry is along the y-axis the parabola opens

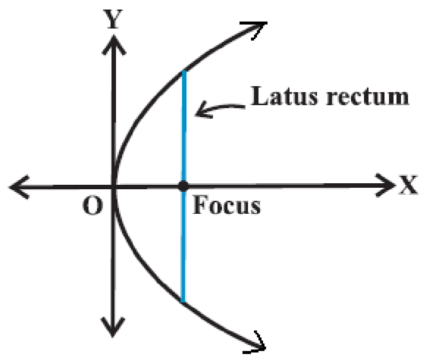
   (c) upwards if the coefficient of y is positive.

   (d) downwards if the coefficient of y is negative.

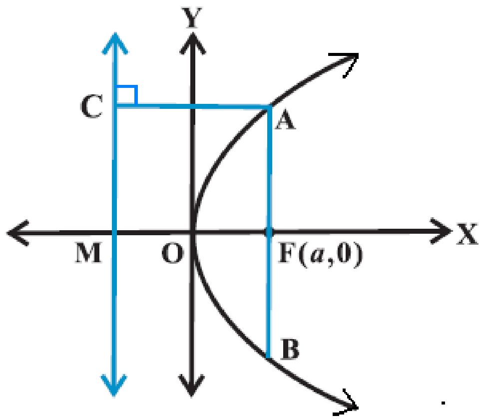
5. Latus-Rectum:

Definition:

Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola passing through its focus and whose end points lie on the parabola.



Let us now find the length of the latus rectum of the parabola $y^2 = 4ax$



Take a point A on the parabola, then according to the definition of parabola, distance of point A from its focus should be equal to its distance from directrix, hence AF = AC

But, AC = FM = $2a$

Therefore, AF = $2a$.

Now, since the parabola is symmetric with respect to x-axis

Therefore, AF = FB

So,

Length of the latus rectum = AB

= AF + FB

= 2 AF

= $4a$.

6. Examples

Example :

1) Find the coordinates of the focus, axis, the equation of the directrix, and latus rectum of the parabola $y^2 = 2\sqrt{3}x$.

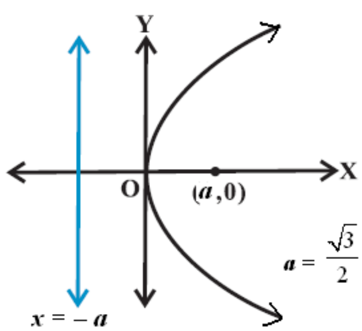
Solution:

The given equation involves y^2, so the axis of symmetry is along the x-axis. The coefficient of x is positive so the parabola opens to the right.

Comparing the given equation

$$y^2 = 2\sqrt{3}x.$$

with the standard equation $y^2 = 4ax$, we find,



$$4\,a = 2\sqrt{3}$$

$\sqrt{\frac{3}{2}}$ **Or** ***a* =**

Thus, the focus of the parabola is $(a, 0)$ i.e., (, 0) $\sqrt{\frac{3}{2}}$

Axis of parabola is along the x-axis.

The equation of the directrix of the parabola is $x = -a$

i.e., $x = -\sqrt{\frac{3}{2}}$

Latus rectum of the parabola = $4\,a = 2\sqrt{3}$

Example:

Find equation of the parabola with vertex at (0 ,0) and focus at (0 , 6).

Solution:

Vertex of the required parabola is at (0, 0) and the focus is at (0, 6). This shows that positive side of y-axis is the axis of parabola and it opens upwards, the equation of the parabola which opens upwards with vertex at (0 , 0) is given by

$$x^2 = 4ay$$

Vertex is at point (0,0) and the focus is at the point (0, 6), hence, distance between vertex and the focus = 6

therefore, $a = 6$

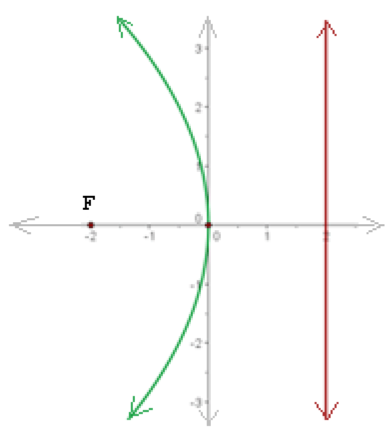
hence, the required equation of the parabola is $x^2 = 24\ y$

Example:

Determine the equation of the parabola, if thedirectrixisx = 2 andthefocusis(−2, 0).

Solution:

Since, the directrix is, x = 2 and the focus is, (−2, 0), therefore, the parabola will open towards left and axis of the parabola will be x-axis, as shown below,



The vertex of the parabola will be half way between focus and the directrix, hence the coordinates of vertex are (0, 0) and equation of parabola will be y2 = − 4ax, where, a = 2 the required equation is,

$$y2 = -4(2)x \quad \text{or} \quad y2 = -8x$$

Example:

Find equation of the parabola with vertex at (0 , 0), the x-axis as its axis of symmetry and passing through the point (–2 , 4).

Solution:

Since the vertex of the parabola is at the origin and the x-axis is the axis of symmetry of the parabola,

Therefore, the equation of the parabola should be of the form,

$$y^2 = 4a\,x \quad \text{or} \quad y^2 = -4a\,x$$

the sign depends on whether the parabola opens towards right or left. But the parabola passes through (–2 , 4) which lies in the second quadrant, therefore it must open towards left, hence the equation is of the form, $y^2 = -4a\,x$.

Since the parabola passes through (–2 , 4), we have

$$4^2 = -4a\,(-2)$$

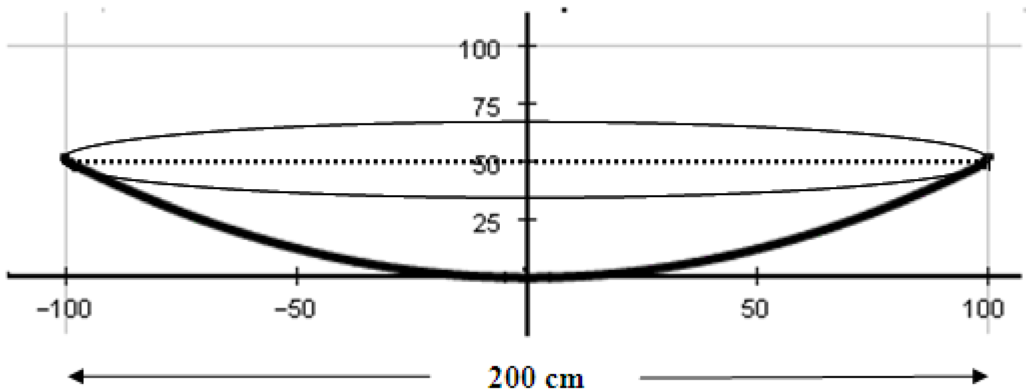
Solving we get, $a = 2$,

Substituting value of a, the required equation of the parabola is,

$$y^2 = -8\,x\,.$$

Example:

A parabolic dish with a diameter of 200 cm and a maximum depth of 50 cm is shown below. Find the focus of the dish.



Solution:

The parabolic dish is opening upwards hence its equation should be of the form: $x^2 = 4ay$ …………(i)

Maximum depth is 50cm and diameter is 200cm, therefore Point (100 , 50) lies on the curve of the parabolic dish,

Substituting in the equation we get,

$100^2 = 4a \times 50$ solving it we get, $a = 50$

Focus of the standard form of parabola (i) is; (0, a), hence focus of the parabolic dish is; (0, 50)

7. Summary

1) A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point in the plane.
2) Fixed point is called focus and fixed line is called directrix.
3) The equation of the parabola with focus at $(a, 0)$, $a > 0$ and directrix $x = -a$, is $y^2 = 4ax$.
4) Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the parabola
5) Length of the latus rectum of the parabola $y^2 = 4ax$ is $4a$.
6) There are four standard equations of parabolas, depending upon the choice of axes i.e.,

(i) $y^2 = 4ax$, (ii) $y^2 = -4ax$ (iii) $x^2 = 4ay$ (iv) $x^2 = -4ay$