1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 02 (Class XI, Semester - 2)
Module Name/Title	Straight Lines - Part 2
Module I	Kemh_21002
Pre-requisites	Basic knowledge of Straight Lines
Objectives	After going through this lesson, the learners will be able to do the following: • Lines parallel to coordinate axes • Point Slope form • Two point form • Slope-intercept form • Intercept form
Keywords	Lines parallel to coordinate axes, Point slope form, Two point form, Slope-intercept form, Intercept form

2. Development Team

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Table of Contents :

1. Introduction:

In our previous module on straight lines we have discussed "slope of a line" and learnt to calculate angle between two straight lines. You have also learnt conditions of parallelism and perpendicularity of two lines.

In analytical geometry, a line in the plane is defined as set of points whose coordinates satisfy a given linear equation.

Thus the equation of a straight line is a linear relation between two variables x and y, which is satisfied by the coordinates (x,y) of each and every point of the line and not by any other point in the Cartesian plane which does not lie on the line.

Suppose, P(x, y) is an arbitrary point in the XY-plane and L is the given line. Then to get an equation of line L, we construct a statement or condition for the point P, which is true for all those points P which lie on line L and not by any other point which does not on L.

The statement is merely an algebraic equation involving the variables x and y.

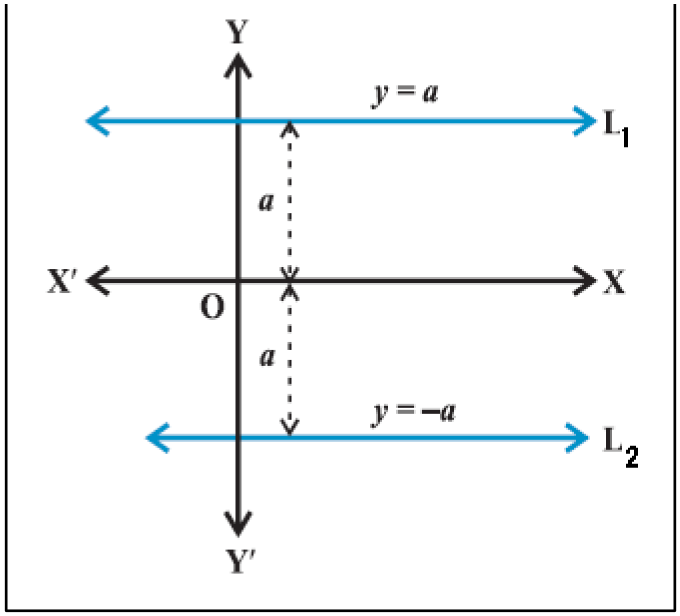
Equation of a line:

The equation of a line is the linear relation between two variables x and y, which is satisfied by the coordinates of each and every point on the line and not by coordinates of any other point.

Let us now find the equation of a line under different conditions to get various forms of equation of line;

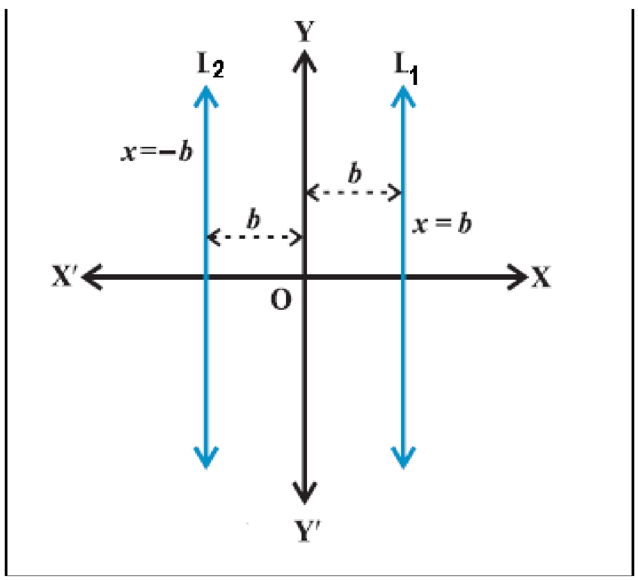
2. Lines parallel to the coordinate axes (Horizontal and Vertical lines):

If a line is at a distance *a*, parallel to x-axis then ordinate of every point lying on the line is either *a* or – *a*, as shown in the figure below;



Therefore, equation of the line L_1 (above x-axis) will be $y = a$ and equation of the line L_2 (below x-axis) will be $y = -a$, because y-coordinate of each and every point on line L_1 is a and y-coordinate of each and every point of line L_2 is $-a$;

Similarly if a line parallel to y-axis at a distance *b* from it, then abscissa of every point lying on the line is either *b* or – *b,* as shown in the figure below;



In this case, equation of the line L_1 is $x = b$ and equation of the line L_2 is $x = -b$, because x-coordinate of each and every point on line L_1 is b and that on line L_2 is $-b$; In general, we consider a horizontal line as x-axis and a vertical line is taken as y-axis so equations of horizontal lines are of the form $y = a$ or $y = -a$ and equations of vertical lines are of the form $x = b$ or $x = -b$.

Equation of x-axis:

We know that ordinate of every point on x-axis is 0,

If P(x, y) is any point on x-axis, then y = 0.

Hence, the equation of x-axis is, y = 0.

Equation of y-axis:

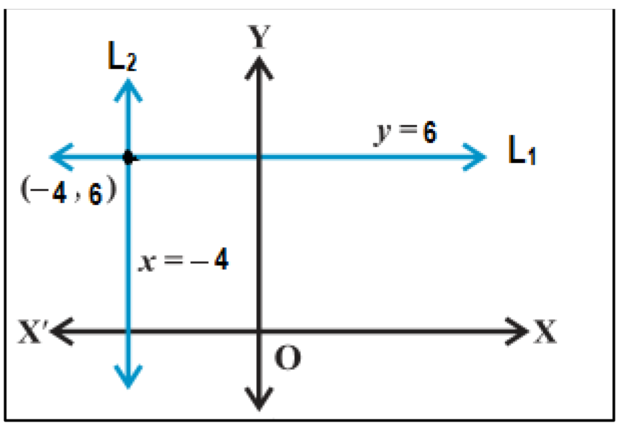
Again we know that abscissa of every point on y-axis is 0.

If P(x, y) is any point on y-axis, then x = 0.

Hence, the equation of y-axis is; x = 0.

Example:

Suppose we want to find the equations of the lines parallel to axes and passing through point P (–4, 6).

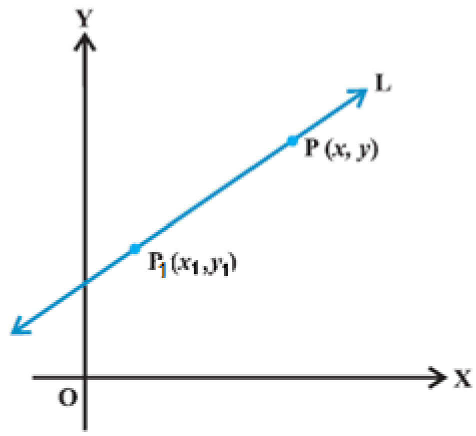


Line L_1 is parallel to x-axis, y-coordinate of every point on the line parallel to x-axis is 6, therefore, equation of the line parallel to x-axis and passing through point(– 4, 6) is y = 6.

Similarly, line L_2 is parallel to y-axis and x-coordinate of every point on the line parallel to y-axis is – 4, therefore, equation of the line parallel to y-axis and passing through point(– 4, 6) is x = – 4.

3. Point slope form:

Let us take a fixed point P_1 (x_1, y_1) on a line L and slope of the line be m. Let P (x, y) be any arbitrary point on L as shown in figure below,



Then, by the definition, slope of line L is given by,

$$m = \frac{y - y_1}{x - x_1}$$

All the points P(x, y) on line L will satisfy the above equation and no point of the plane other than the points on the line will satisfy this equation, hence, equation of the line passing through point $P_1(x_1, y_1)$ and with slope m is given by;

$$y - y_1 = m(x - x_1)$$

Note: Equation of line passing through origin with slope m will be,

$$y - 0 = m(x - 0) \quad \text{i.e.} \quad y = mx$$

Example:

Suppose we have to find equation of a line which passes through point (– 2, 3) and slope of the line is – 4.

then, m = – 4 and given point is (– 2, 3).

$\therefore \quad x_1 = -2$ and $y_1 = 3$

Using, slope-intercept form,

$$y - y_1 = m\,(x - x_1)$$

the equation of the line is,

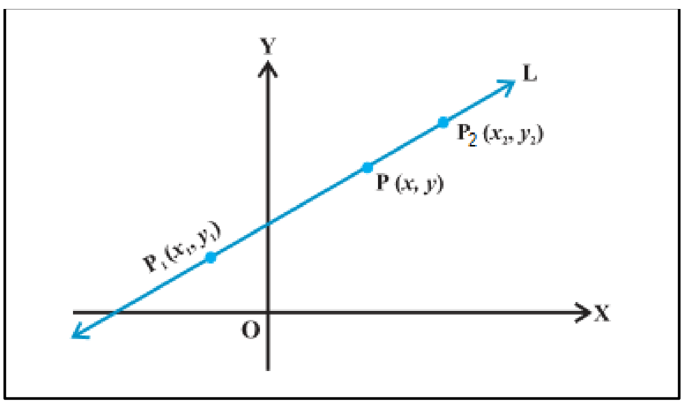
$$y - 3 = -4\,(x + 2)$$

hence the required equation is

$$4x + y + 5 = 0$$

4. Two point form:

Consider a line L passing through two given points $P_1\,(x_1, y_1)$ and $P_2\,(x_2, y_2)$ and let $P\,(x, y)$ be a general point on L, as shown in figure below,



Three points P_1, P_2 and P are collinear, therefore, we have

slope of P_1P = slope of P_1P_2

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1},$$

$$\text{or} \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

Thus, equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

Example:

Find equation of the line which passes through points (1, –1) and (3, 5).

Since required line passes through two points (1, –1) and (3, 5)

∴ $x_1 = 1$, $y_1 = -1$ and $x_2 = 3$ and $y_2 = 5$

Using two-point form

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

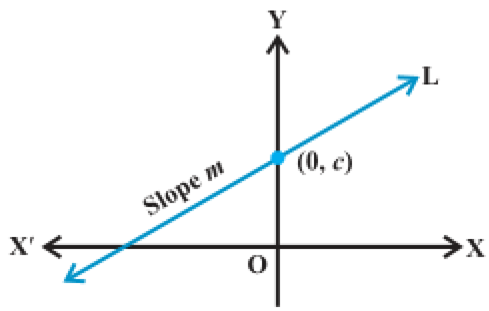
the required equation of the line is,

$$y - (-1) = \frac{5 - (-1)}{3 - 1}(x - 1)$$

Solving it the required equation is,

$$-3x + y + 4 = 0$$

Slope-intercept form:



coordinates of the point where the line meets the y-axis are (0, c). Thus, line L has slope m and passes through a fixed point (0, c). Therefore, by point-slope form, the equation of the line is;

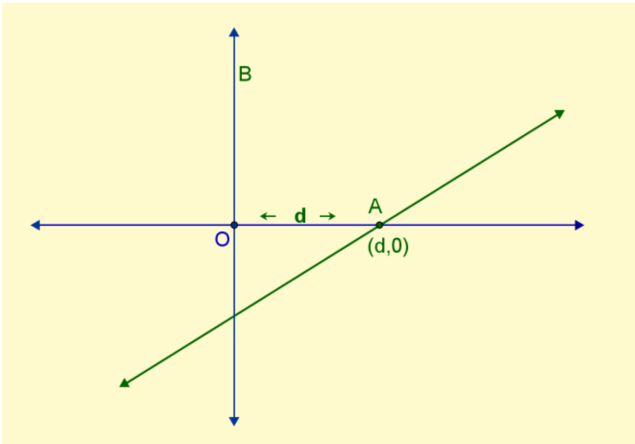
$$y - c = m(x - 0)$$

or $$\mathbf{y = mx + c}$$

The value of c will be positive or negative according as the intercept is made on the positive side or negative side of the y-axis.

Case II:

Suppose line L, makes x-intercept equal to d and its slope is m then the line will cut x-axis at the point (d, 0) as shown below,



Thus the line passes through the point (d, 0) and has he slope m,

Using point-slope form, the equation of the line is,

$$y - 0 = m(x - d)$$

or $$\mathbf{y = m(x - d)}$$

Note:

Hence equation of the straight line passing through origin is y = mx.

Example:

Suppose we are to find equation of the line whose slope is $\frac{1}{2}$ and it makes y-intercept equal to -3 and x-intercept is 4.

Since slope of the line is $\frac{1}{2}$, $\therefore$ $m = \frac{1}{2}$,

Case (i) When y-intercept is equal to -3, equation to be used is

$y = mx + c$ where $c = -3$

hence required equation is, $y = \frac{1}{2}x + (-3)$

or $2y - x + 6 = 0$

Case (ii) When x-intercept is equal to 4, equation to be used is

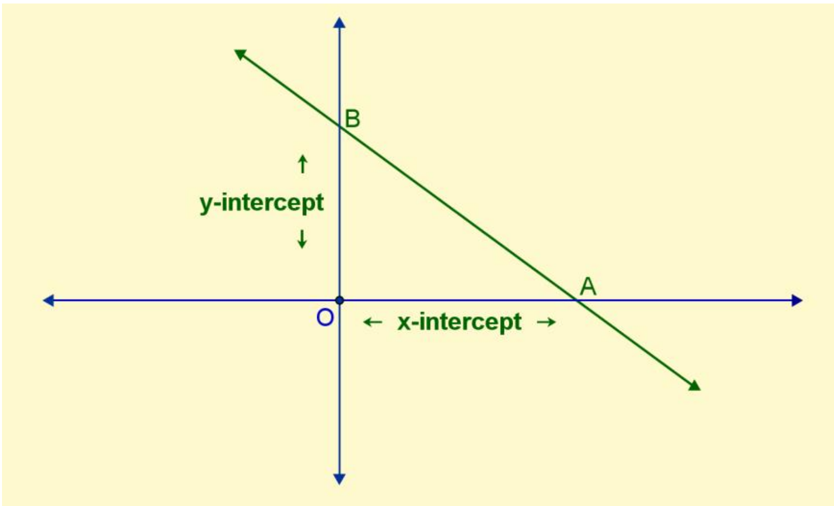
$y = m(x - d)$ where $m = \frac{1}{2}$ and $d = 4$

Substituting values we get, $y = \frac{1}{2}(x - 4)$

$\therefore$the required equation is $2y - x + 4 = 0$

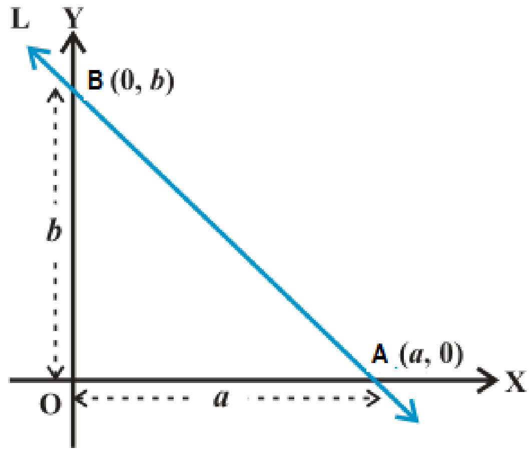
5. Intercept – form:

In the figure below, straight line cuts x-axis at A and the y-axis at B, then OA and OB are known as the intercepts of the line on x-axis and y-axis respectively.



OA = x-intercept, OB = y-intercept

Suppose a line L makes x-intercept equal to a and y-intercept equal to b on the axes. Then obviously line L cuts x-axis at point (a, 0) and y-axis at the point (0, b) as shown below;



By two-point form of the equation of line, we get,

$$y - 0 = \frac{b-0}{0-a}(x-a)$$

or $$ay = -bx + ab$$,

simplifying we get, $\frac{x}{a} + \frac{y}{b} = 1$, this is the required equation of the line making intercepts a and b on x-axis and y-axis respectively.

Example:

Suppose a line makes intercepts -3 and 2 on the x-axis and y-axis respectively, then equation of the line will be,

$$\frac{x}{-3} + \frac{y}{2} = 1$$

Because, a = –3 and b = 2, solving above equation,

the required equation of the line is,

$$2x - 3y + 6 = 0 .$$

6. Normal form:

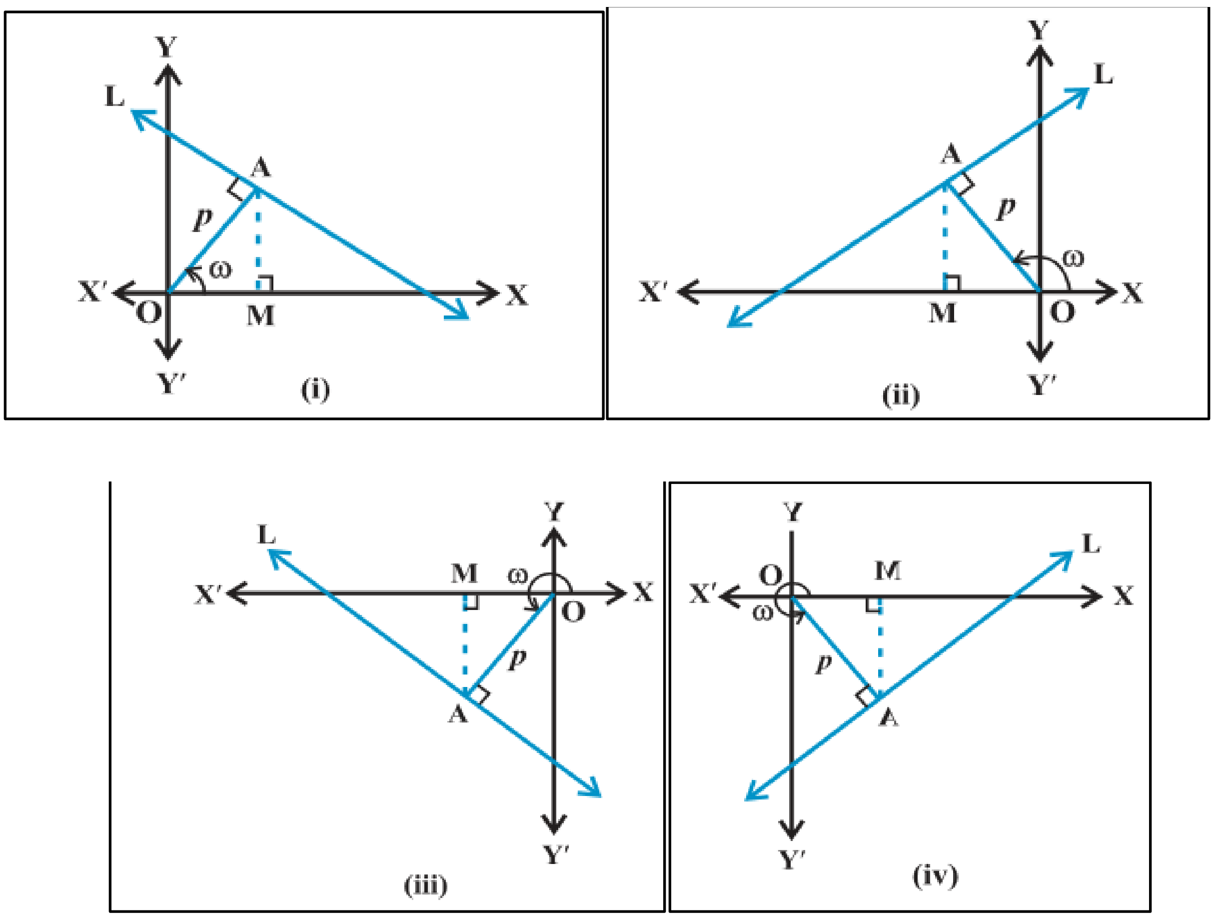
Suppose a non-vertical line i.e. line not parallel to y-axis is given to us with following data:

(i)Length of the perpendicular (normal) from origin to the line.

(ii) Angle made by normal with the positive direction of x-axis.

Let L be such line, whose perpendicular distance from origin O be OA = p and the angle between the positive x-axis and OA be ∠XOA = ω.

There may be four possible positions of line L in the Cartesian plane, depending upon the position of the line, these are shown in the figure below,



If we know slope of the line L and a point on it, then equation of the line can be obtained. Let us draw perpendicular AM on the x-axis in each case.

From the figure it is clear that perpendicular distance of the line L from origin O is OA = p and the angle between OA and the positive x-axis is ∠XOA = ω.

In each case, we have OM = p cos ω and MA = p sin ω, Hence, coordinates of the point A are (p cos ω, p sin ω) and line L is perpendicular to OA. Therefore,

$$\text{The slope of the line L} = -\frac{1}{\text{slope of OA}} = -\frac{1}{\tan\omega} = -\frac{\cos\omega}{\sin\omega}$$

$\therefore$ slope of line L is $-\frac{\cos\omega}{\sin\omega}$

and it passes through the point A *(p cosω, p sinω)*.

Hence, by point-slope form, the equation of the line L, is

$$y - p\sin\omega = -\frac{\cos\omega}{\sin\omega}(x - p\cos\omega)$$

Solving we get,

$$x\cos\omega + y\sin\omega = p(\sin^2\omega + \cos^2\omega)$$

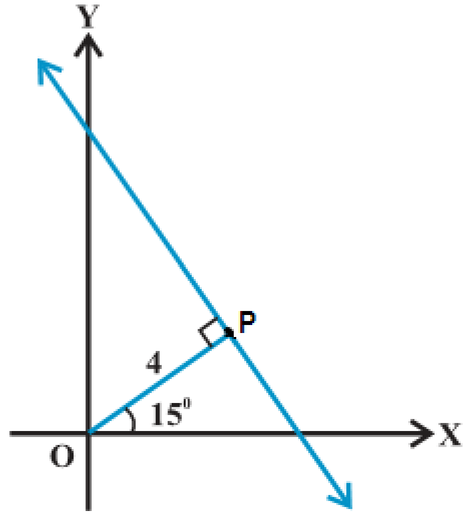
or $$x\cos\omega + y\sin\omega = p.$$

Hence, the equation of the line having normal distance p from the origin and angle ω which the normal makes with the positive direction of x-axis is given by

$$x\cos\omega + y\sin\omega = p.$$

Example:

Write equation of the line whose perpendicular distance from the origin is 4 units and the angle which the normal to the line makes with positive direction of x-axis is 15°



Here, perpendicular distance of the line from the origin is 4 units

∴ p = 4 and the angle made by the normal to the line with positive direction of x-axis is 15°, ∴ω = 15°

Normal form of the equation is,

$$x \cos \omega + y \sin \omega = p$$

Substituting the values we get,

$$x \cos 15^0 + y \sin 15^0 = 4$$

we know that,

$$\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Substituting these values we get

$$\frac{\sqrt{3}+1}{2\sqrt{2}}x + \frac{\sqrt{3}-1}{2\sqrt{2}}y = 4$$

Simplifying, the required equation is,

$$\left(\sqrt{3}+1\right)x+\left(\sqrt{3}-1\right)y=8\sqrt{2}$$

Note: In this module we have discussed equation of a line under different conditions to get various forms of equation of line. In all

these forms we have two constants. Hence we need two conditions to determine equation of a line.

7. **Summary:**

1) Equation of the horizontal line having distance a from the x-axis is either y = a or y = – a
2) Equation of the vertical line having distance b from the y-axis is either x = b or x = – b.
3) The point (x, y) lies on the line with slope m and through the fixed point (x_1, y_1), if and only if its coordinates satisfy the equation $y - y_1 = m(x - x_1)$.
4) Equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1).$$

5) If a line with slope m makes y-intercept equal to c then equation of the line is y = m x + c.
6) If a line with slope m makes x-intercept d, then equation of the line is y = m (x – d).
7) Equation of a line making intercepts a and b on the x-axis and y-axis respectively, is $\frac{x}{a}+\frac{y}{b}=1$
8) If length of the perpendicular (normal) from origin to a line is equal to p and angle made by the normal with the positive direction of x-axis is ω then equation of the line is

$$x\cos\omega+y\sin\omega=p.$$