1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 02 (Class XI, Semester - 2)
Module Name/Title	Straight Lines-Part 1
Module I	kemh_21001
Pre-requisites	Basic knowledge of Straight Lines
Objectives	After going through this lesson, the learners will be able to do the following: • Meaning of slope of a line • Slope of line with two points on it • Condition for parallelism and perpendicularity • Angle between two lines • Collinearity of three points
Keywords	Slope of a line, Parallelism, Perpendicularity, Angle, Collinearity

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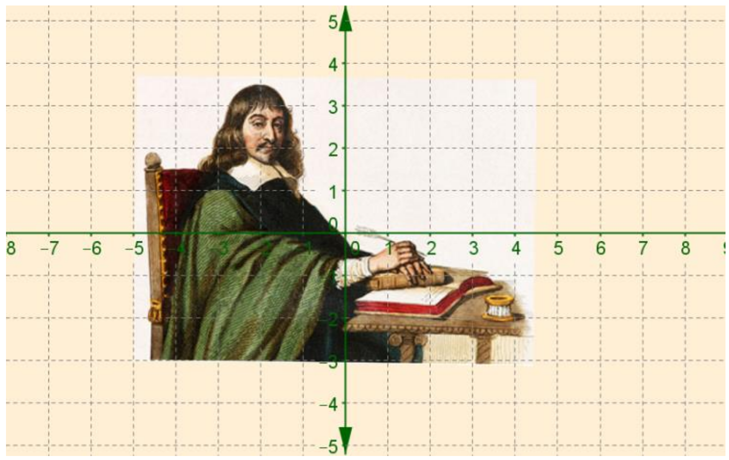
1. Introduction:

Straight line is the simplest geometrical figure, you have already learnt many theorems and properties related with straight lines in the earlier classes. If we look around we find many lines in our surroundings, both curved as well as straight lines;

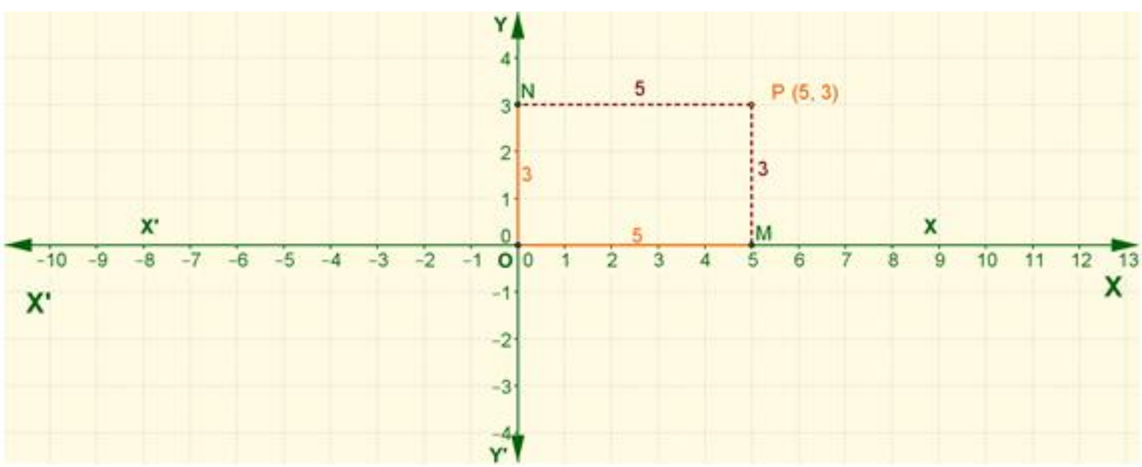





You are also familiar with two dimensional coordinate geometry, let us have a brief recall of coordinate geometry done in earlier classes and continue its study to discuss simplest geometric figure "**straight line**" and study its properties. The fusion of algebra and geometry is named as coordinate geometry. A systematic study of Geometry using Algebra was first carried out by French philosopher and mathematician Rene Descartes, who is known as father of modern geometry.



A plane with a pair of two perpendicular number lines, which divide the plane in four parts, is known as Cartesian-coordinate plane. He provided the method of describing the location of points in a plane in the form of ordered pairs.



The lengths of directed line segments OM and ON are respectively x-coordinate and y-coordinate of the Point P, here OM = 5units and ON = 3 units, the ordered pair (5,3) represents the coordinates of point P.

To honour the work done by great philosopher and mathematician Rene Descartes, the coordinates of a point are referred as Cartesian coordinates and the coordinate plane as the Cartesian Coordinate Plane. You have also learnt plotting of points in a plane, distance between two points, section formula, etc. All these concepts are the basics of coordinate geometry. Let us have a brief recall of these formulae;

1) Distance between the points P (x_1, y_1) and Q (x_2, y_2) is,

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

For example, distance between the points A (6, – 4) and B (3, 0) is

$PQ = \sqrt{(3-6)^2 + (0+4)^2} = \sqrt{(3-6)^2 + (0+4)^2} = \sqrt{9+16} = 5$ Units.

2) The coordinates of a point dividing the line segment joining the points (x_1, y_1) and (x_2, y_2) internally, in the ratio m : n are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

For example, the coordinates of the point which divides the line segment

joining points A (1, –3) and B (–3, 9) internally, in the ratio 1: 3, are given by

$$\left(\frac{1.(-3)+3.1}{1+3}, \frac{1.9+3.(-3)}{1+3}\right) = (0,0)$$

3) The coordinates of a point dividing the line segment joining the points (x_1, y_1) and (x_2, y_2) externally, in the ratio m : n are

$$\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$$

4) In particular, if m = n, the coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

5) Area of the triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

For example, the area of the triangle, whose vertices are (4, 4), (3, – 2) and

(– 3, 16) is, $\Delta = \frac{1}{2}[4(-2-16) + 3(16-4) + (-3)(4+2)] = \frac{|-54|}{2} = 27$Units.

A straight line in a plane is the set points, such that Coordinates of all the points of the line satisfy a given linear equation and this linear equation is not satisfied by coordinates of any other point of the plane, which does not lie on the line.

Since a line is made up of points, therefore, using coordinate geometry, we can represent a line algebraically. Before doing so, let us learn "what do we mean by slope of a line".

Look at following figures and try to recall what do we mean by slope in our day to day life and try to observe that some slopes are very steep and some are not that steep. Slopes which are steeper, make greater angle with the horizontal line.



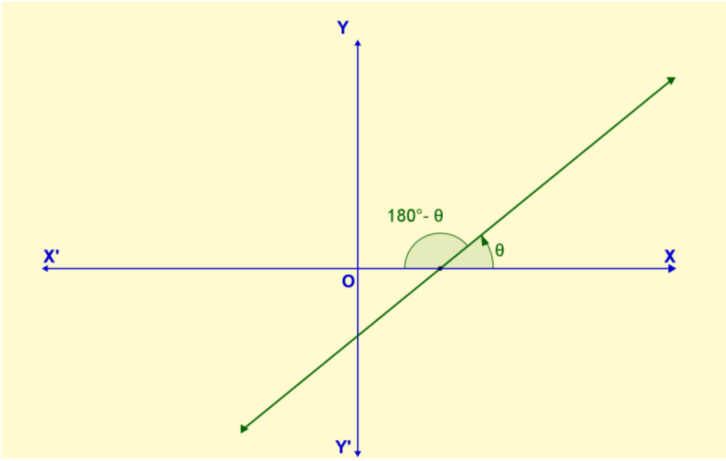



Similarly slope of a straight line shows '**how steep'** a straight line is.

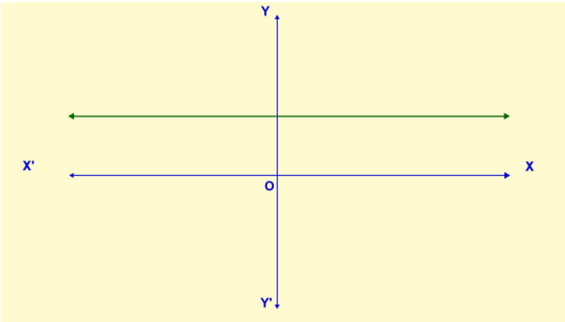
Slope of a line:

A line in a coordinate plane forms two angles with the x-axis, see the figure below, which are supplementary. The angle 'θ' made by the line with positive direction of x-axis, measured in anti-clockwise direction is called the inclination of the line,

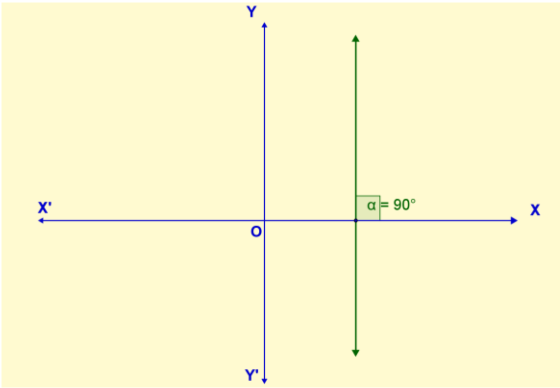
$0° \leq \theta \leq 180°$.



The angle of inclination of a line with positive direction of x-axis in anticlockwise direction always lies between 0° and 180°.The inclination of lines parallel to x-axis or coinciding with x-axis is 0°.



and inclination of a vertical line, i.e., line parallel to or coinciding with y-axis is 90°.



Definition:

If inclination of any line is θ, then tan θ is defined as slope or gradient of the line. It is denoted by m, thus slope of a line is

$$m = \tan\theta, \qquad \theta \neq 90°$$

The slope of any line whose inclination is 90° is not defined because tan 90° is not defined. Therefore, slope of y-axis or slope of lines parallel to y-axis is not defined. Slope of x-axis or slope of all lines parallel to x-axis is zero, since their inclination is zero.

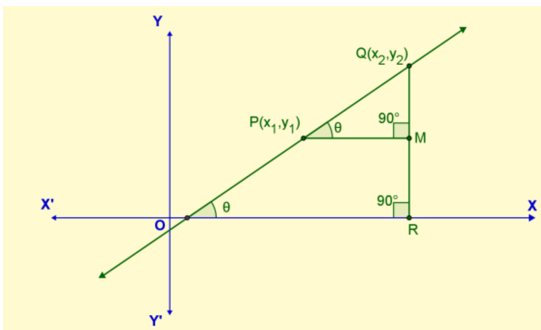
Slope of a line when coordinates of any two points on the line are given:

We have already discussed that a line is completely determined when two points on it are given. Now we are going to find slope of a line, when coordinates of two points on it are given.

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two distinct points on a non-vertical line whose inclination is θ. The inclination of the line may be acute or obtuse. Let us consider both the cases.

Case 1: When angle θ is acute:

Draw perpendicular QR to x-axis and PM perpendicular to QR as shown below,



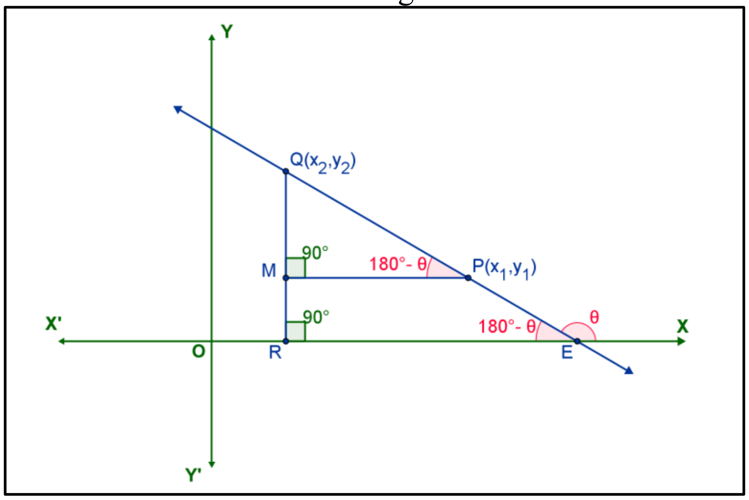
Inclination of the line is θ, so $\angle MPQ = \theta$.

Therefore, slope of line = m = tan θ.

In ΔQPM, we have, $\tan\theta = \frac{\text{QM}}{\text{PM}} = \frac{y_2 - y_1}{x_2 - x_1}$,

Thus slope of the line, $m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$

Case II: When angle θ is obtuse:



Angle of inclination 'θ' of the line is obtuse, hence we have

$$\angle REQ = \angle MPQ = 180° - \theta$$

Therefore, $\theta = 180° - \angle MPQ$.

Now, slope of the given line

$$m = \tan\theta = \tan(180° - \angle MPQ)$$

$$= -\tan\angle MPQ$$

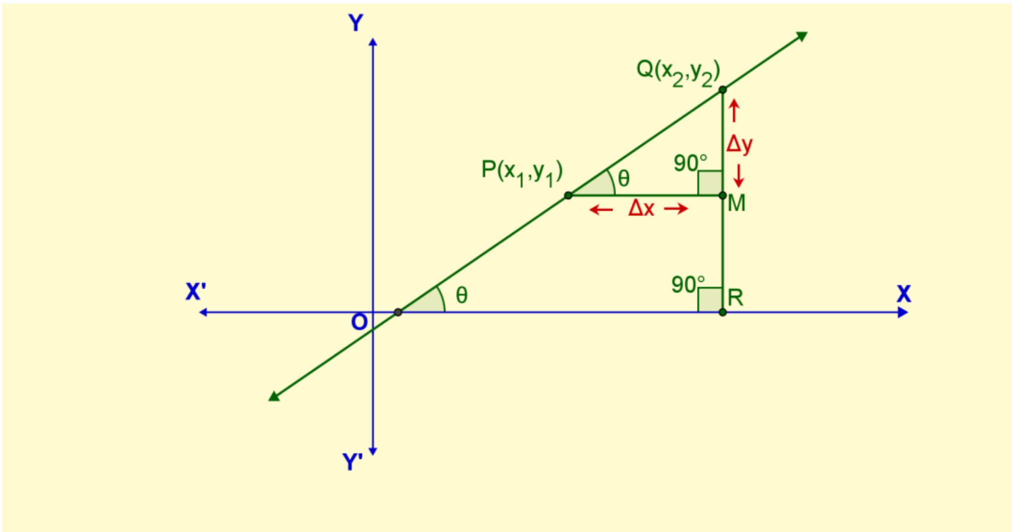
$$= -\frac{MQ}{MP} = -\frac{y_2 - y_1}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

Thus, we see that in both the cases the slope m of the line through the points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\Delta y}{\Delta x}$$

Where, $\Delta y = y_2 - y_1$ and $\Delta x = x_2 - x_1$

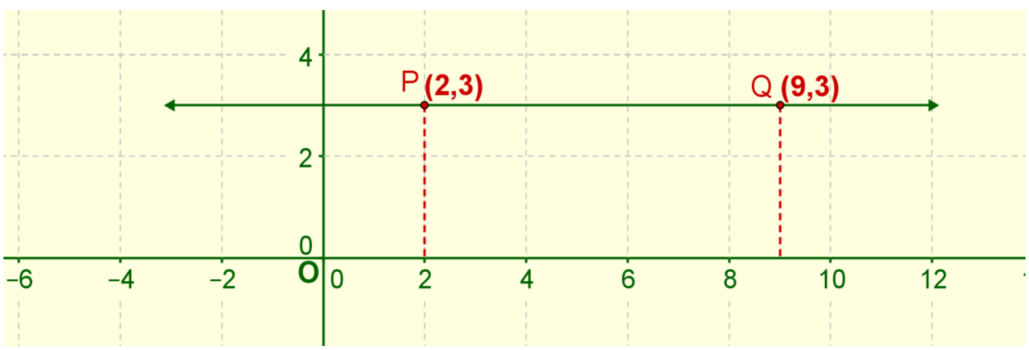


Slope of a line parallel to x-axis:

We have seen that slope of any line is given by,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

See in the figure below, if any line is parallel to x-axis then,



Y-coordinate for all the points of the line will remain same and

$$y_2 - y_1 = 0$$

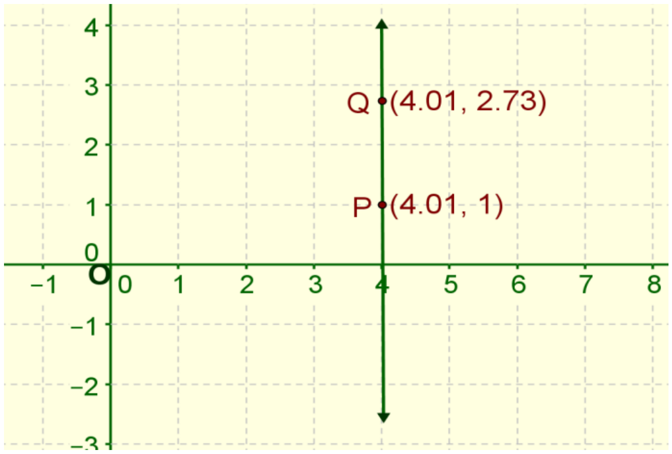
$$\Rightarrow \quad m = 0$$

Hence, slope of x-axis or any line parallel to x-axis will be zero.

Slope of a line perpendicular to x-axis:

Take a line parallel to y-axis or perpendicular to x-axis, observe in the figure

below,



x-coordinate for all the points on line parallel to y-axis will remain same,

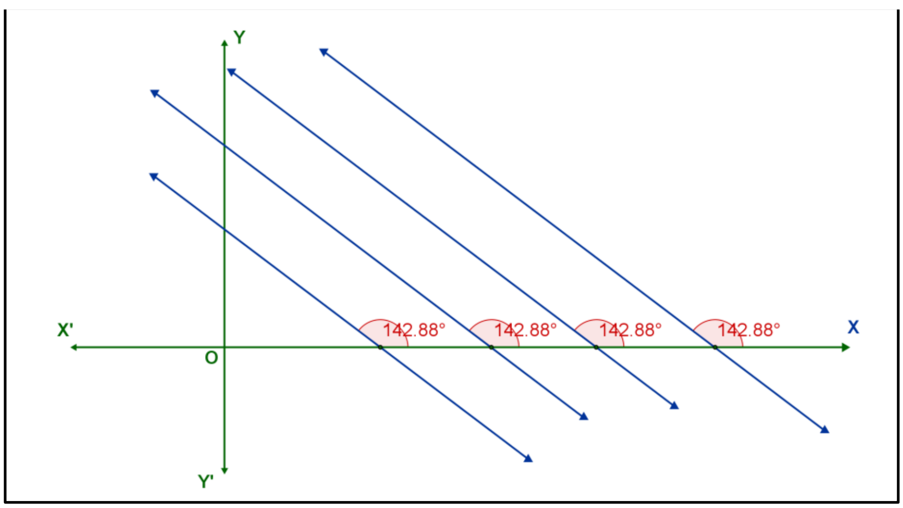
therefore, $x_2 - x_1 = 0$ for any two points of the line.

hence, slope of the line, $m = \frac{y_2 - y_1}{x_2 - x_1}$ is not defined.

So, slope of y-axis or any line parallel to y-axis is not defined. In other words we can say that slope of a vertical line is not defined.

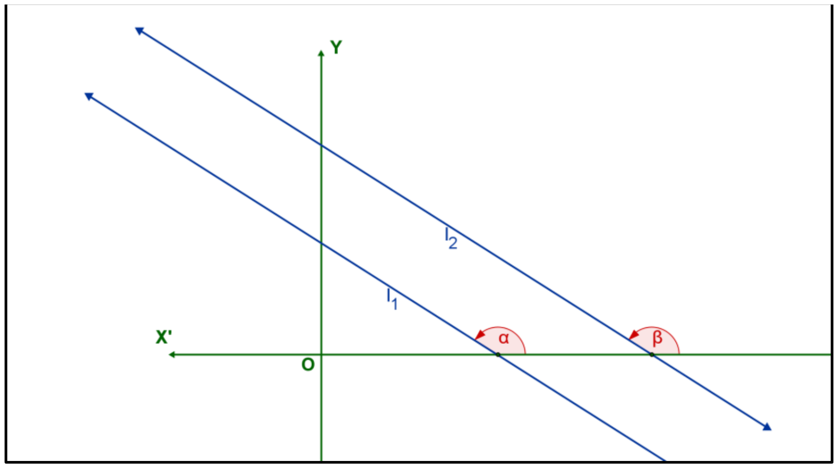
Conditions for parallelism and perpendicularity of lines in terms of their slopes:

In the figure below, we have parallel lines, what relation do you notice with these lines and x-axis?



Observe they all are making same angle with the positive direction of x-axis, hence inclinations of all these lines are same.

So, if we have two parallel lines l_1 and l_2 with inclinations α and β, respectively, then we must have $\alpha = \beta$ or $\tan \alpha = \tan \beta$



If slopes of two parallel lines l_1 and l_2 be m_1 and m_2, then

$$m_1 = \tan \alpha \quad \text{and} \quad m_2 = \tan \beta$$

$$\alpha = \beta \Rightarrow \tan \alpha = \tan \beta \Rightarrow m_1 = m_2$$

∴**Slopes of parallel lines are equal.**

Conversely, if the slopes of two lines l_1 and l_2 are same,

i.e. if $m_1 = m_2$, then $\tan \alpha = \tan \beta$.

Where α and β are inclinations of the lines l_1 and l_2 .

Since inclinations α and β lie between 0° and 180° , hence by the property of tangent function (between 0° and 180°),

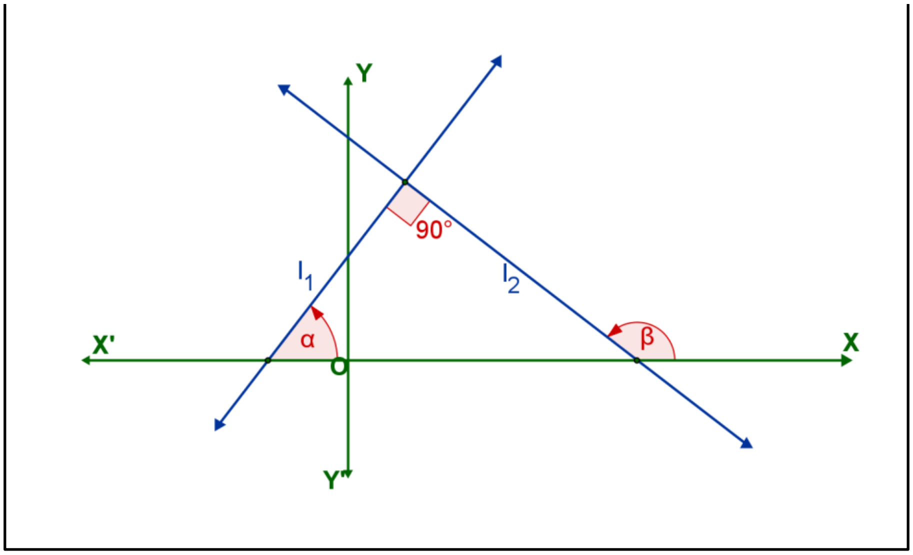
we have $\alpha = \beta$, therefore, thus the lines are parallel.

Hence, two non-vertical lines l_1 and l_2 are parallel if and only if their slopes are equal.

Perpendicular Lines:

Now consider two perpendicular lines l_1 and l_2 with inclinations α and β and slopes m_1 and m_2,

then, $\beta = \alpha + 90°$, and $m_1 = \tan \alpha$, $m_2 = \tan \beta$.



$\therefore$ $\tan\beta = \tan(\alpha + 90°) = -\cot\alpha = -\frac{1}{\tan\alpha}$

Or $m_2 = -\frac{1}{m_1}$

$\Rightarrow$ $m_2 . m_1 = -1$ or $m_1 . m_2 = -1$,

Conversely, if $m_1 . m_2 = -1$, then, $\tan\alpha \tan\beta = -1$.

Or $\tan\alpha = -\cot\beta = \tan(\beta + 90°)$ or $\tan(\beta - 90°)$

$\Rightarrow$ either $\alpha = \beta + 90°$ or $\alpha = \beta - 90°$

$\Rightarrow$ either $\alpha = \beta + 90°$ or $\beta = \alpha + 90°$

$\therefore$ α and β differ by 90°.

Thus, lines l_1 and l_2 are perpendicular to each other.

Hence, two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

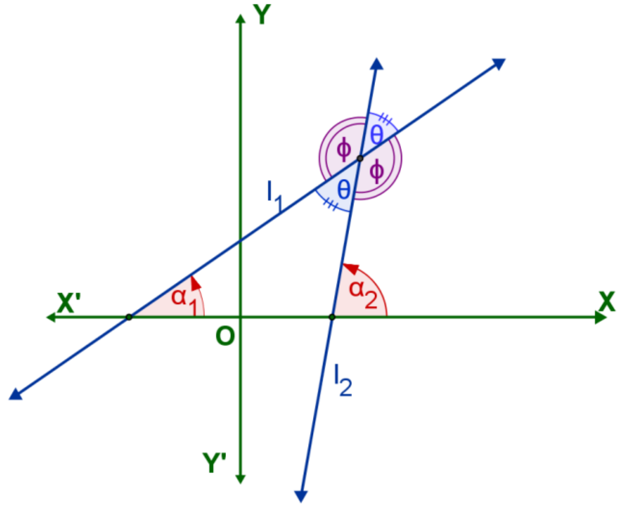
i.e. $m_2 = -\frac{1}{m_1}$ or $m_1 . m_2 = -1$

Angle between two lines:

When we have more than one line in a plane, then the lines are either intersecting or parallel. Let us now discuss the angle between two lines in terms of their slopes.

Let l_1 and l_2 be two non-vertical lines with slopes m_1 and m_2, respectively. If α_1 and α_2 be the inclinations of lines l_1 and l_2, respectively, then

$$m_1 = \tan\alpha_1 \qquad \text{and} \qquad m_2 = \tan\alpha_2$$



Observe, when lines l_1 and l_2 intersect each other, they make two pairs of vertically opposite angles such that sum of any two adjacent angles is 180°.Here θ and φ are the adjacent angles between the lines l_1 and l_2, then, $\theta = \alpha_2 - \alpha_1$ and $\alpha_1, \alpha_2 \neq 90°$, (∵ l_1 and l_2 are not vertical)

∴ $\tan\theta = \tan(\alpha_2 - \alpha_1)$

$$= \frac{\tan\alpha_2 - \tan\alpha_1}{1+\tan\alpha_1\tan\alpha_2} = \frac{m_2 - m_1}{1+m_1m_2} \quad (1 + m_1m_2 \neq 0)$$

See from figure, θ and φ are supplementary angles, hence,

$$\varphi = 180° - \theta \qquad \therefore \tan\varphi = \tan(180° - \theta)$$

$$= -\tan\theta = -\frac{m_2 - m_1}{1+m_1m_2}, \quad 1 + m_1m_2 \neq 0$$

Observe values of tan θ and tan φ are numerically same but opposite in

sign, now two cases arise;

Case I : if, $\frac{m_2 - m_1}{1+m_1m_2}$ is positive, then tan θ will be positive and tan φ will be negative, which means θ will be acute and φ will be obtuse.

Case II: if, $\frac{m_2 - m_1}{1+m_1m_2}$ is negative, then tan θ will be negative and tan φ will be positive, which means that θ will be obtuse and φ will be acute. Thus, the acute angle (say θ) between lines l_1 and l_2 with slopes m_1 and m_2, respectively, is given by

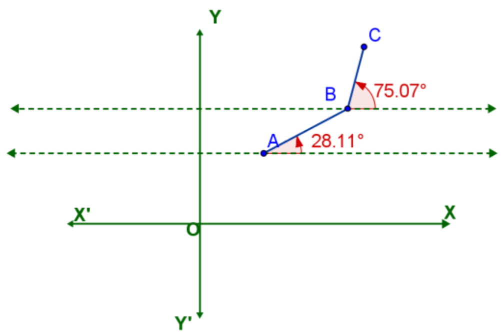
$$\tan\theta = \left|\frac{\tan\alpha_2 - \tan\alpha_1}{1+\tan\alpha_1\tan\alpha_2}\right| \qquad 1 + m_1m_2 \neq 0$$

The obtuse angle (say φ) can be found by using $\varphi = 180^0 - \theta$.

Collinearity of three points:

Three points are said to be collinear if they all lie on same straight line.

In the figure below, we have three non-collinear points, A, B and C, let us join them by two line segments AB and BC,



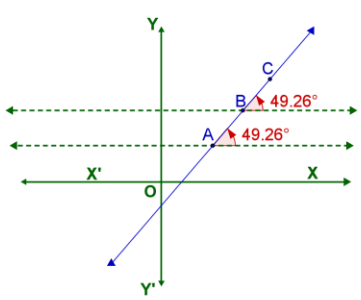
Obviously,

angle made by line segment AB with horizontal line

$\neq$ angle made by line segment BC with horizontal line

$\therefore$ slope of AB $\neq$ slope of BC

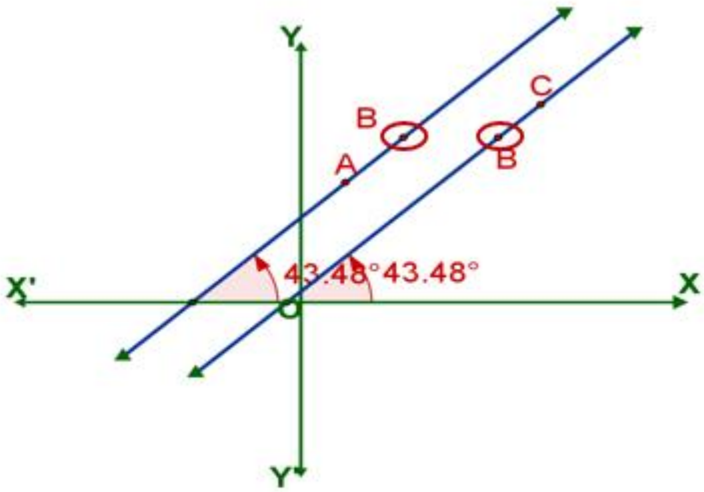
Now, move point B to make the three points, A, B and C collinear,



You will find that, line segments AB and BC, will fall on same straight line *l*, so line segments AB and BC will have same inclinations,

Hence, their slopes will be equal.

Let us understand it, in other way, we know that slopes of two parallel lines are equal.



If possible, let us take two distinct lines having the same slope and passing through a common point B,

then, it can be possible only when the two lines coincide.

So we can conclude that,

If A, B and C are three points in the XY-plane, then they will be collinear if and only if

slope of AB = slope of BC.

Summary:

1. Straight line is the simplest geometrical figure, it has length only, no breadth and no height.
2. Every point of the line segment joining any two points on a straight line, lies completely on it.
3. The angle made by a line with positive direction of x-axis measured in anti-clockwise direction is called the inclination of the line.
4. Angle of inclination of a straight line always lies between 0° and 180°.

5. The inclination of lines parallel to x-axis or coinciding with x-axis is 0° and the inclination of lines parallel to or coinciding with y-axis is 90°.

6. If inclination of any line is θ, then tan θ is defined as slope or gradient of the line. It is denoted by $m = \tan\theta$, $\theta \neq 90°$.

7. Slope of x-axis or slope of all lines parallel to x-axis is zero, the slope of any line whose inclination is 90° is not defined because tan 90° is not defined, hence slope of y-axis or any line parallel to y-axis is not defined.

8. Slope (m) of any non-vertical line passing through the two points P (x_1, y_1) and Q (x_2, y_2) is given

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}, \qquad x_1 \neq x_2$$

9. Two lines are parallel if and only if their slopes are equal.

$$m_1 = m_2$$

10. Two lines are perpendicular if and only if product of their Slopes is –1, i.e. $m_1 . m_2 = -1$

11. Three points A, B and C are collinear, if and only if,

Slope of AB = slope of BC.

12. Thus, the acute angle (say θ) between lines l_1 and l_2 with slopes m_1 and m_2, respectively, is given by

$$\tan\theta = \left|\frac{\tan\alpha_2 - \tan\alpha_1}{1 + \tan\alpha_1 \tan\alpha_2}\right| = \frac{m_2 - m_1}{1 + m_1 m_2} \qquad 1 + m_1 m_2 \neq 0$$

13. The obtuse angle (say φ) can be found by using $\varphi = 180^0 - \theta$.