1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 02 (Class XI, Semester - 2)
Module Name/Title	Arithmetic Progression (A.P.): Part – 2
Module I	kemh_20902
Pre-requisites	Patterns, Sequence, Arithmetic Progression
Objectives	After going through this lesson, the learners will be able to do the following: • Identify A.P. • Determine general term of A.P. • Determine sum to n terms of A.P. • Apply A.P. to solve real life problems
Keywords	Arithmetic progression, Common Difference, nth term, Sum to n terms

2. Development Team

Role	Name	Affiliation
National MOOC Coordinator (NMC)	Prof. Amarendra P. Behera	CIET, NCERT, New Delhi
Program Coordinator	Dr. Mohd. Mamur Ali	CIET, NCERT, New Delhi
Course Coordinator (CC) / PI	Prof. Til Prasad Sarma	DESM, NCERT, New Delhi
Course Co-Coordinator / Co-PI	Dr. Mohd. Mamur Ali	CIET, NCERT, New Delhi
Subject Matter Expert (SME)	Ms. Neenu Gupta	Ahlcon International School, Mayur Vihar, Phase-I
Review Team	Prof. Suresh Kumar Gautam (Retd.)	DESM, NCERT, New Delhi

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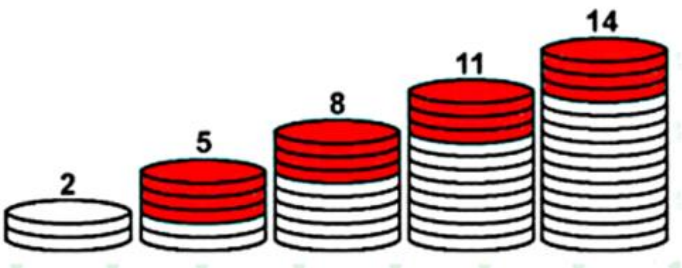
1. Introduction

Have you ever tried this? Take 2 coins of Rs. 10 and arrange them vertically. Then keep on adding fixed number of coins and check the height of the cylinder so formed.



You can easily observe constant growth in height of the cylinder. Which sequence is made due to addition of number of coins at every step? Let us study about such sequences in this module. These are Arithmetic progressions.

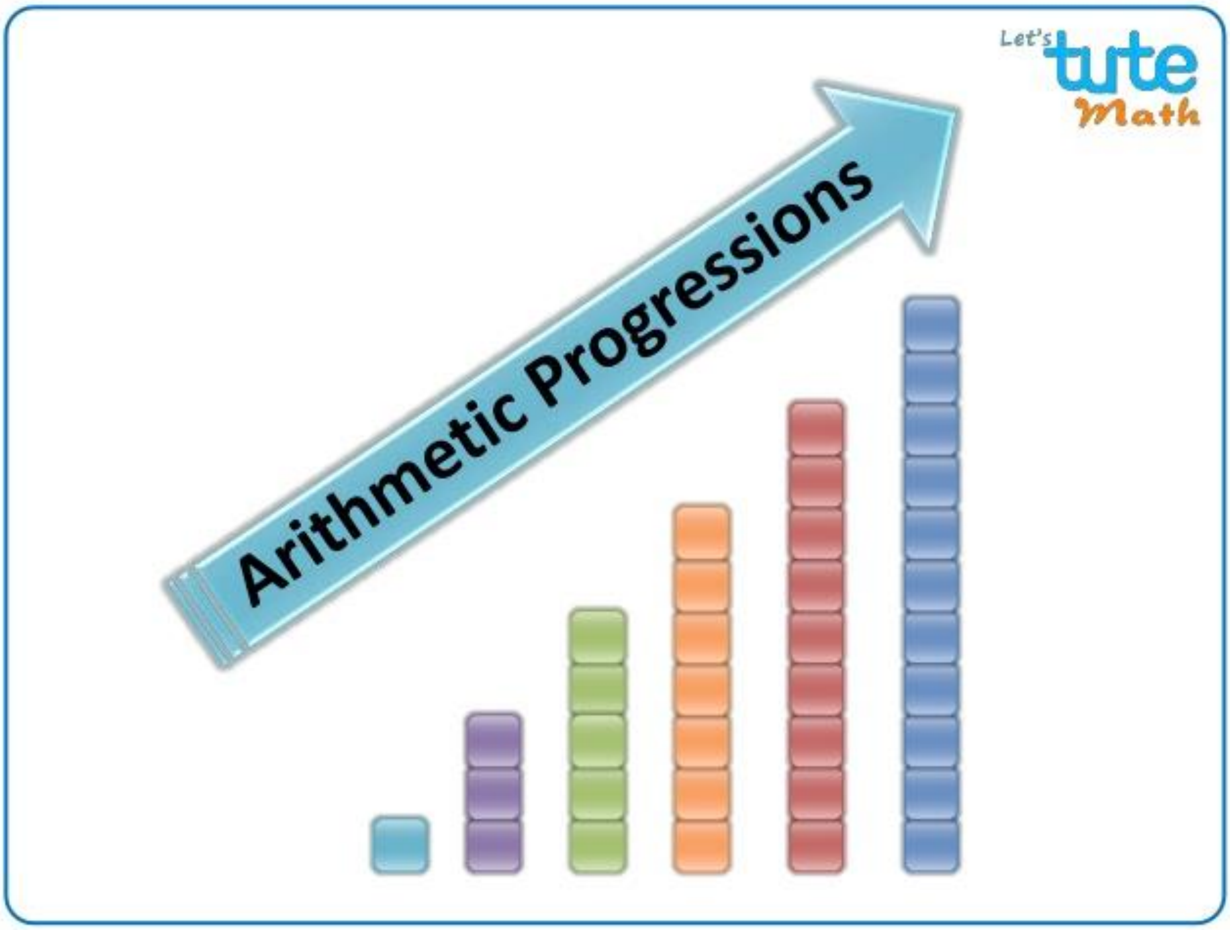
By now, you are well aware about sequences. The difference between sequences, progressions and series has also been illustrated in module 1.

A sequence is an arrangement of numbers according to some fixed pattern. A progression is a sequence having successive terms obtained by adding or subtracting some fixed number.

Arithmetic Progression (A.P.) is a type of progression and has already been introduced in class X. In this module, we are going to extend our knowledge about A.P.

2. **Arithmetic Progression (A.P.)**

Since Aditya is growing older, he is planning to be more responsible. Suppose he planned to save some amount of money from his daily pocket-money. On 1st of January, he saved Rs. 10, on 2nd Jan, he saved Rs. 12, on 3rd Jan, he saved Rs. 14, and subsequently he saved Rs. 2 more every day than the previous day's savings. Can you think of the amount that he will save on 15th Jan? Can you calculate the total amount of his savings on 31st January of the same year? Can you think of the pattern or the sequence of the amount that he is saving each day?



All of these questions have their answers hidden in the concept of Arithmetic progression, abbreviated as A.P.

Let's write the amounts in the form of a sequence:

10, 12, 14, 16, ……………. (till 31st term)

Can you just check out any pattern followed by the terms of this sequence?

You are correct !!!

Every term of this sequence can be obtained by adding 2 to the previous term.

This type of sequence is called Arithmetic Progression. The first term is the first number of the progression, which is10 here. This fixed number '2' to be added is called common difference.

In the generalized form we can denote the A.P. as $a_1, a_2, a_3, a_4, \ldots\ldots, a_n$

Here, a_1 is the first term, a_2 is the second term, and similarly a_n is the general term of the A.P. The common difference is denoted by 'd'.

We can define an A.P. as

A sequence $a_1, a_2, a_3, a_4, \ldots\ldots, a_n$ is called an arithmetic sequence or arithmetic progression if $a_2 = a_1 + d$, $a_3 = a_2 + d$, $a_4 = a_3 + d$, i.e., every term a_n can be obtained by adding some constant quantity 'd' to its previous term a_{n-1}, where n ∈ **N**

Let us consider an A.P. (in its standard form) with first term a and common difference d, i.e., a, a + d, a + 2d, a + 3d, ……….

So the above sequence 10, 12, 14, 16, …. can be rewritten as

10, 10 + 2, 10 + 2 x 2, 10 + 3 x 2, 10 + 4 x 2,……

Some of the questions may emerge at this time. Is common difference 'd' always positive? Is 'd' has to be an integer? Can 'd' be zero also?

So let's look at some examples of the sequences:

1, 3, 5, 7, 9, 11, 13, …..

10, 8, 6, 4, 2, 0, – 2, – 4, ….

$$1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}, \ldots.$$

5, 5, 5, 5, 5, 5, ……

Are these A.P.?

Let's calculate d in every case; a test to check if the given sequence an A.P.

For 1, 3, 5, 7, 9, 11, 13, ….., d = 3 – 1 = 2; d = 5 – 3 = 2 ⇒ It is an A.P. and d = 2

For 10, 8, 6, 4, 2, 0, – 2, – 4, …. d = 8 – 10 = – 2; d = 6 – 8 = – 2 ⇒ It is an A.P. and d = – 2

For $1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}, \ldots.$ d = $1\frac{1}{2} - 1 = \frac{1}{2}$; d = $2 - 1\frac{1}{2} = \frac{1}{2}$ ⇒ It is an A.P. and d = $\frac{1}{2}$

For 5, 5, 5,…. d = 5 – 5 = 0, ⇒ It is an A.P. and d = 0

Here, we can easily see that d is a constant quantity which is being added or subtracted in every term to get the next term. We have also seen that d can be positive or negative or zero or a fraction. This means 'd' can be any real number.

Selection of terms

Sometimes, for convenience sake, terms of A.P. are not taken as a , a + d, a + 2d, a + 3d,… But if product and sum of terms are given then terms are taken as follows:

Number of terms of A.P.	Terms	Common Difference
3	a – d, a, a + d	d
4	a – 3d, a – d, a + d, a + 3d	2d
5	a – 2d, a – d, a, a + d, a + 2d	d
6	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d	2d

Example 1: the sum and product of three numbers in A.P. are 9 and 15 respectively. Find these numbers.

Solution: let three numbers be a – d, a, a + d

Then, (a – d) + a + (a + d) = 9 ⇒ 3a = 9 ⇒ a = 3

$$(a-d)\times a\times(a+d)=15$$

$$\Rightarrow a(a^2-d^2)=15$$

$$\Rightarrow 3(9-d^2)=15$$

$$\Rightarrow (9-d^2)=\frac{15}{3}=5$$

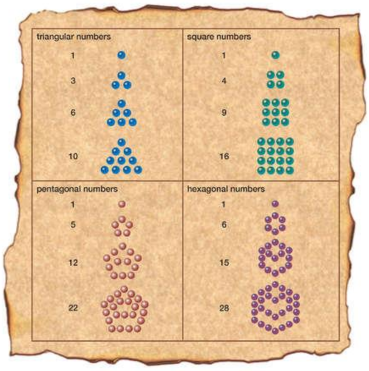
$$\Rightarrow d^2=9-5=4$$

$$\Rightarrow d=\pm 2$$

Hence, terms are 1, 3, 5 or 5, 3, 1

3. **History of A.P.**

No proof is there as to when and where did the arithmetic sequences were first used. But we do know that the Egyptians were the first to develop arithmetic math. In fact, they have used some numbers which are in A.P.



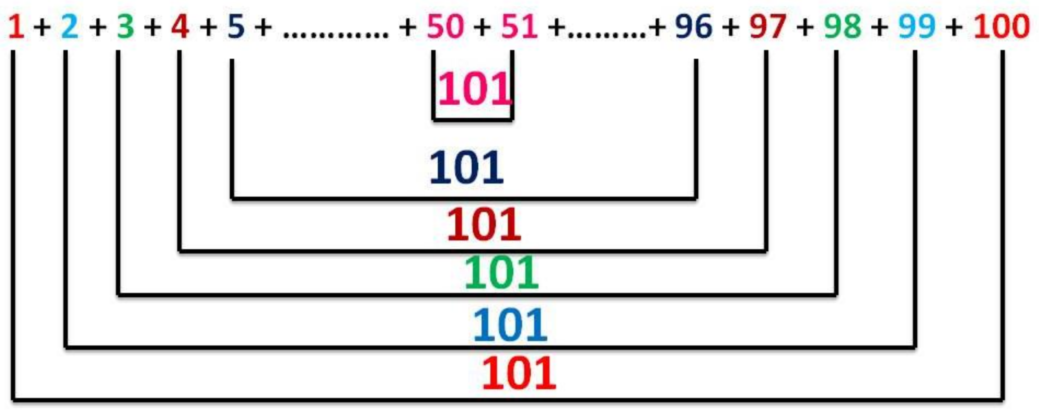
There is no specific history to when sequences were started although this story is popular that there was a young math student who created a formula to help solve for the sum of arithmetic sequences. His name was Carl Gauss, he was born in 1777 in a German Empire and at just ten years old he created this formula. His teacher asked him to solve the sum of the sequence (also known as a series) 1 + 2 + 3 +...+ 99 + 100 and he was the only one with the correct answer which was 5050. As Gauss grew older he became a very well known mathematician contributing to geometry, number theories, and many more.

Are you interested in knowing the trick used by Gauss? Here it is….

Let’s write out the sum like this:

1 + 2 + 3 + 4 + 5 + ………+ 50 + 51 + ………+ 96 + 97 + 98 + 99 + 100

Let’s rewrite it as or observe the pattern:



It is easily observable that sum of two natural numbers on either side of the sequence is same. That means, there are 50 pairs having sum 101 so

1 + 2 + 3 + 4 + 5 + ………+ 50 + 51 + ………+ 96 + 97 + 98 + 99 + 100

= 50 x 101 = 5050

This made the basis of arithmetic progression and sum of its terms.

We will learn more about sum of n terms of the A.P. later in the same module.

4. $\mathbf{n^{th}}$ term a_n

Let's consider an A.P. 2, 4, 6, 8, 10, 12,…..

Here we can write a_1 = 2 and d = 2

a_2= 4 = 2 + 2 = $a_1 + d$

a_3= 6 = 4 + 2 = 2 + 2 x 2 = $a_1 + 2d$

a_4= 8 = 6 + 2 = 2 + 3 x 2 = $a_1 + 3d$

Here it can be observed that every term can be obtained by adding a multiple of 'd' to first term 'a'

Following the same pattern, we can write $a_{10} = a_1 + 9d$ or $a_{15} = a_1 + 14d$ or $a_{100} = a_1 + 99d$

If we have to write general term or nth term of A.P. then

$$a_n = a_1 + (n-1)d$$

Example 2: Find $a_n \wedge a_{20}$ for A.P. 1, 6, 11, 16, 21, 26,….

Solution: Here, a = 1

d = 6 – 1 = 5

$$a_n = a_1 + (n-1)d = 1 + (n-1)5 = 5n - 4$$

Putting n = 20, we get

$$a_{20} = 5 \times 20 - 4 = 96$$

Or, $a_{20} = a + 19d = 1 + 19 \times 5 = 96$

Example 3: For A.P. $3, 3+\sqrt{3}, 3+2\sqrt{3}, 3+3\sqrt{3}, \ldots \ldots$, which number term is $3 + 10\sqrt{3}$?

Solution: $a_1 = 3, d = \sqrt{3}$

$$a_n = 3 + 10\sqrt{3} \Rightarrow 3 + (n-1)\sqrt{3} = 3 + 10\sqrt{3} \Rightarrow n - 1 = 10 \Rightarrow n = 11$$

Example 4: If nth term of A.P. 3, 6, 9, 12, 15,… is same as nth term of A.P. 88, 86, 84, 82,…. Then find value of n.

Solution: 3 + (n – 1) 3 = 88 + (n – 1) (– 2)

⇒ 3 + 3n – 3 = 88 – 2n + 2

⇒ 5n = 90

⇒ n = 18

5. **Sum to n terms (S_n) of A.P. or Sum to n terms of Arithmetic Series**

We have already seen how Gauss calculated sum of first 100 natural numbers just by observing the pattern. The same fact can be used for summing terms of any A.P.

Let $a_1, a_2, a_3, a_4, \ldots\ldots, a_n$ be an A.P. with first term ‘a’ and common difference ‘d’.

Let $S_n = a_1 + a_2 + a_3 + a_4 + \cdots\ldots + a_n$

$$(a_1 + a_n) + (a_2 + a_{n-1}) + (a_3 + a_{n-2}) + \cdots$$

We can write S_n in reverse order also as

$$S_n = a_n + a_{n-1} + a_{n-2} + a_{n-3} + \cdots\ldots + a_1$$

$$(a_1 + a_n) + (a_2 + a_{n-1}) + (a_3 + a_{n-2}) + \cdots$$

$$Adding,$$

$$2S_n = 2[(a_1 + a_n) + (a_2 + a_{n-1}) + (a_3 + a_{n-2}) + \cdots]$$

$n(a_1 + a_n)$ [because every sum is equal]

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$Onputtingvalue a_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Example 5: Find sum of 25 terms of A.P. 5, 10, 15, 20, 25,….

Solution: Here, a = 5, d = 5, n = 25

$$S_n = \frac{25}{2}[2 \times 5 + 24 \times 5] = \frac{25}{2}[10 + 120] = \frac{25}{2} \times 130 = 25 \times 65 = 1625$$

Example 6: If for an A.P., $S_2 = 15$ and $S_1 = 10$ then find a and d.

Solution: $S_1 = a_1 = 10$

$S_2 = a_1 + a_2 = 15 \Rightarrow a_2 = 15 - 10 = 5$

$d = a_2 - a_1 = 5 - 10 = -5$

Hence, a = 5 and d = – 5

6. Applications of A.P.

Following are the properties of A.P

1) If a constant is added to or subtracted from an A.P. then the resulting sequence is also an A.P. with same constant difference.

   e.g. Let 1, 2, 3, 4, 5 be an A.P.

   Then add 2 to each and every term, we get 3, 4, 5, 6, 7

   If we subtract 2 from each and every term then resulting A.P. is – 1, 0, 1, 2, 3

2) If a non-zero constant 'm' multiply or divide the terms of A.P. then also resulting sequence is an A.P. with constant difference 'md' or 'd/m' respectively.

   Let 1, 2, 3, 4, 5 be an A.P.

   Then multiply every term by 2, we get 2, 4, 6, 8, 10; A.P. with common difference 2

3) In a finite A.P., the sum of terms equidistant from both the ends is same and equal to sum of first and last term.

   Check out the Gauss' method of summing first 100 natural numbers.

4) Three numbers a, b and c are in A.P. iff 2b = a + c

   Let a, b c form an A.P. then b – a = c – b , i.e., 2b = a + c

7. Real Life Applications of A.P.

Halley's Comet

Halley's Comet appears in the sky approximately every 76 years. The comet was first spotted in the year 1531. Appearance of this comet in the sky till current century will make an A.P.

Example 7: Find the nth term and the 10th term for the sequence represented by this situation of Halley's Comet.

Solution: From the information given, we can conclude that $a_1 = 1531$ and $d = 76$.

We now have what we need to plug into the rule:

$a_n = 1531+(n - 1)(76)$

$= 1531+76n-76$

Thus the nth term is an = 76n+1455

Now to find the 10th term we can use our rule and replace n with 10: $a_{10} = 76(10)+1455 = 760+1455 = 2215$

Everyday Situations

- Any quantity changing in equal amount at set time period then this is a situation of A.P. Any situation in which regular increase or decrease can be observed is an A.P.
- Any example in which you get a straight line graph is an A.P.
- If you are saving money in equal installments then cumulative savings at each saving's period form an arithmetic sequence.

- If you are travelling up or down a slope in a vehicle then the amount of petrol left in the tank, if measured every minute of the travel, forms an A.P.

- If you want to calculate maximum capacity of audience in the auditorium is case of A.P. , specially if seats are increasing in every subsequent row.

- A ladder with sloping sides is an example of A.P. in which each rung is uniformly increasing in length.

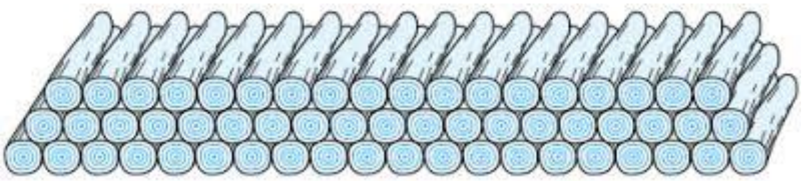
- The thickness of a roll of paper or cloth when its thickness is common difference and diameter of the core is the first term

- Estimated projected earnings of a company

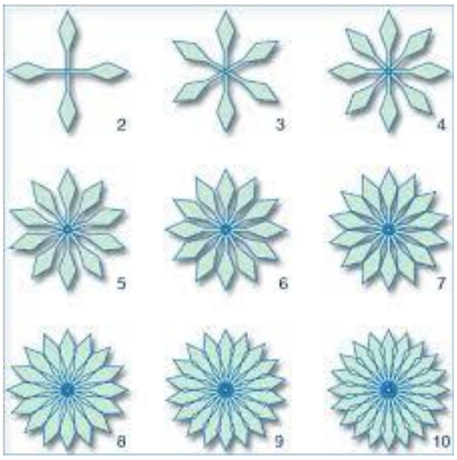
- Depreciation of any asset by a fixed rate per year.

- Amount generated from simple interest on any amount of money

- Arrangement of wood logs or anything else in decreasing order



- Creation of similar figures in ascending order



- Construction of buildings of same design and size but of different floors

Pascal's triangle