1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 02 (Class XI, Semester - 2)
Module Name/Title	Sequence: Part – 1
Module I	kemh_20901
Pre-requisites	Patterns, Sequence, Arithmetic Progression
Objectives	After going through this lesson, the learners will be able to do the following: 1. Understand what is a sequence 2. Understand difference between sequence and series 3. Evaluate terms of sequence 4. Appreciate sequences to observe in real life situations
Keywords	Sequences, Series, Progressions, Fibonacci sequence, Recursive

2. Development Team

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1. Introduction

In our day to day life, we come across many patterns such as prints on the cloth, alphabets, flowers, leaves, grills on window, sweets arranged in the sweet shop, hand-movement of clock etc. Amount of simple or compound interest from a sum or depreciation of your car are all nothing but types of sequences.

Since mathematics is also embedded in our real life, it must also have some patterns. These patterns if observed carefully are nothing but sequences. So you think you have not observed patterns in mathematics so far. Then I must remind you that you are using patterns throughout your life without even knowing about them. Where? You remember you used to learn tables of 2, 3, 4, … in your childhood. What are these? Are not these patterns and hence sequences. Yes, they are the easiest sequences.

In this module, we are going to study about sequences, their types and their real life uses.

2. Sequences

Aditi and Aditya were playing a game. If Aditi climbed 1 step of staircase then Aditya climbed 2 steps of similar staircase. When Aditi completed 2 steps then Aditya completed 4 steps and this process continued for Aditi's 10 steps. Can you think of how many steps have been completed by Aditya? Undoubtedly, Aditya completed 20 steps of stairs. If we just write their number of step for comparison at any time then it would look like:

Aditi	1	2	3	4	5	6	7	8	9	10
Aditya	2	4	6	8	10	12	14	16	18	20

It can easily be observed that these two sets of numbers are patterns which are called sequences.

A sequence is an arrangement of numbers or terms in a specific pattern. It may have repeated elements. E.g. 1, 2, 3, 4, 5 and a, e, i, o, u are examples of sequences. These do not have any element or term repeating. But at the same time, 1, 1, 2, 3, 5 is also a sequence although it has first and second terms alike.

This may remind about the sets. In sets, elements are listed but order does not matter and moreover, elements cannot be repeated. But in the case of sequences, order does matter and repetition is also allowed. E.g. {S, I, L, E, N, T} is set consisting of letters of the word SILENT. At the same time, S, I, L, E, N, T and L, I, S, T, E, N are two different sequences consisting of letters of the words SILENT and LISTEN. It can easily be compared that S, I, L, E, N, T and L, I, S, T, E, N are elements of the same set as they have same elements but these are two different sequences.

The terms of the sequence are denoted by a_1, a_2, a_3, a_4, a_5, ……a_n, … where a is the term and subscripts 1, 2, 3, …, n denote the number of term. Sequences can be defined in general way by representing the pattern in the form of general term or n^{th} term, i.e., a_n. by using this, any term of the sequence can be defined easily.

e.g. the sequence 2, 4, 6, 8, 10, 12,….. can be generalized as $a_n = 2n$, $n \in N$

Similarly, if we are interested in evaluating 5^{th} term of the sequence whose nth term is given by $a_n = 2^n + 1$.

We can calculate 5^{th} term by putting n = 5 in a_n as

$a_5 = 2^5 + 1 = 32 + 1 = 33$

Definition: A sequence can be defined as a function whose domain is the set of natural numbers or some subset of natural numbers. Sometimes, we use the functional notation a(n) for a_n.

Example 1 – Find next three terms of the sequence 4, 16, 64,…..

Solution – Since sequence is $4^1, 4^2, 4^3$ so

$a_4 = 4^4 = 256$

$a_5 = 4^5 = 1024$

$a_6 = 4^6 = 4096$

Example 2 – Find the missing terms of the sequence: 1, 8, 27, 64, -- , --, 343, 512

Solution – Sequence consists of cubes of first eight natural numbers. So missing terms are 125 and 216

Example 3 – Find a_4 if $a_n = \frac{2n-3}{2}$, $n > 1$ and $n \in N$

Solution – Putting n = 4 in a_n, we get $a_4 = \frac{2\times 4-3}{2} = \frac{5}{2}$

3. Difference between Sequences & Series

Let $a_1, a_2, a_3, a_4, a_5, \ldots\ldots a_n$ be any sequence the expression $a_1 + a_2 + a_3 + a_4 + a_5 + \ldots + a_n$ is nothing but the Series corresponding to the sequence. Sum of the series means the number obtained when all the terms of the series are being added up. For convenience sake, series is denoted by Greek letter *Sigma* as $\sum_{k=1}^{n} a_k$ which means summation of $a_1, a_2, a_3, a_4, a_5, \ldots\ldots a_n$

Thus, it can be observed that the series represent the sum of terms but not the sum itself.

Another difference lies in the representation that terms of the sequence are separated by commas and terms of the series are separated by the + sign.

4. Difference between Sequences & Progression

Let us take two examples of sequences: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 and 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. Can you observe any pattern or difference in both the sequences?

The first sequence 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 has all the terms progressing according to a definite rule. While in the second sequence 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 there is no specific rule to write next term. Why? Because the second sequence is sequence of prime numbers and the first sequence is sequence of even numbers. The first sequence is called progression while the second is just a sequence.

So a progression is a type of a sequence in which all the successive terms are increasing or decreasing according to a fixed rule.

Also, we can say that every progression is a sequence but vice-versa is not true.

Example 4 – Is the pattern 1, 11, 111, 1111, 11111, ….. a sequence or a progression?

Solution – Look at the terms carefully. Try to make a pattern.

$a_1 = 1 = \frac{10^1 - 1}{9}$

$a_2 = 11 = \frac{10^2 - 1}{9}$

$a_3 = 111 = \frac{10^3 - 1}{9}$

$a_4 = 1111 = \frac{10^4 - 1}{9}$

Similarly, $a_n = \frac{10^n - 1}{9}$

Therefore, the given sequence is a progression.

5. Types of Sequences

There can be many types of sequences, out of which some have been listed below:

Finite

If all the terms of the sequence are counted finitely then the sequence is finite. E.g. table of 2, 3, 4, etc.

Infinite

If the terms of the sequence continue infinitely then the resulting sequence is an infinite sequence. E.g. sequence of even numbers, i.e., 2, 4, 6, 8, 10, 12,....

Recursive

Sequence in which succeeding terms are related to previous terms by a given specific rule are called recursive sequences. Fibonacci sequence is the best example of this type. The Fibonacci sequence is 0, 1, 1, 2, 3, 5, 8, 13, 21,. which is obtained by adding the previous two terms. In this sequence, first two terms are 0 and 1 respectively and succeeding terms can be obtained by adding the previous two terms.

Arithmetic

If the difference between two consecutive terms of the sequence is constant then the resulting sequence is called arithmetic sequence. E.g. 2, 4, 6, 8,.......

Geometric

If ratio of two consecutive terms of the sequence is constant then the resulting sequence is called a geometric sequence. E.g. 2, 4, 8, 16, 32,.....

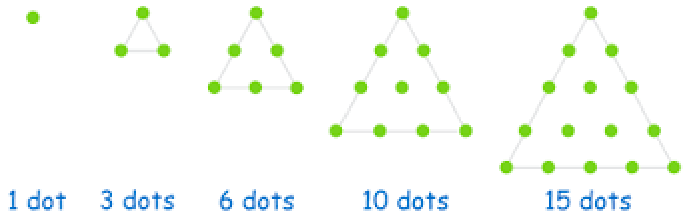
Harmonic

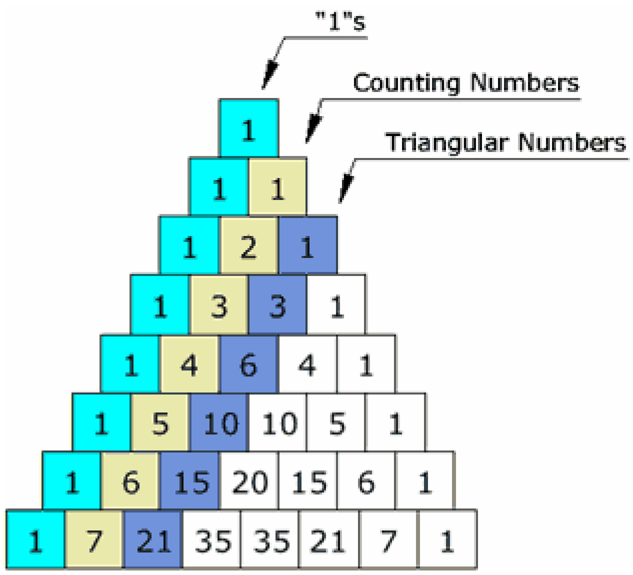
If reciprocal of the terms of a sequence makes an arithmetic sequence then it is called harmonic sequence. E.g. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, ...$

Triangle

This sequence is generated from a pattern of dots which form a triangle. By adding another row of dots and counting all the dots we can find the next number of the sequence. An example of this type of number sequence could be the following:

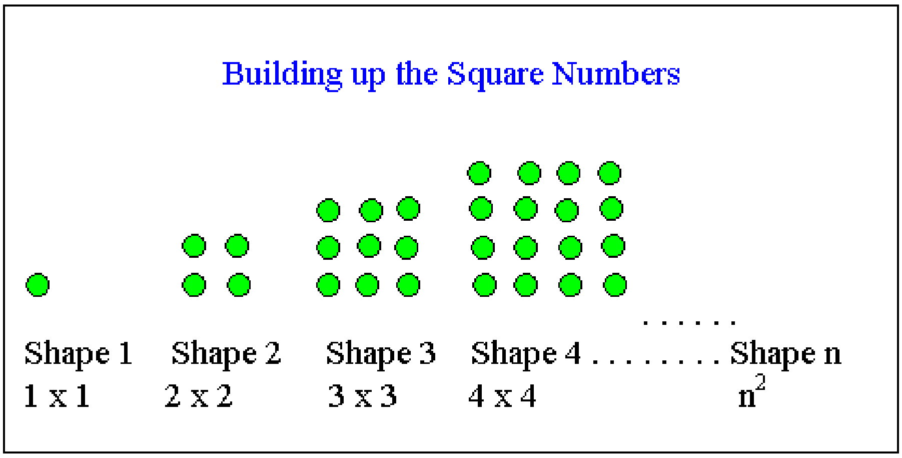
1, 3, 6, 10, 15, 21, 28, 36, 45, …





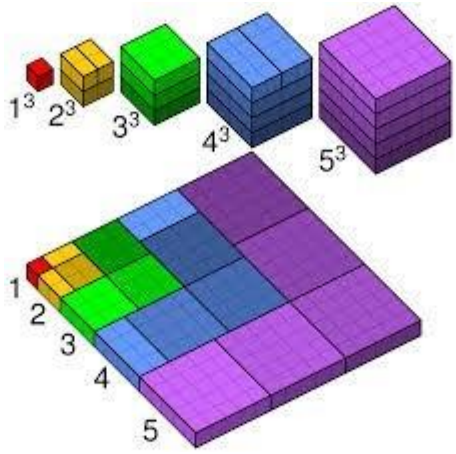
Squares

Sequence consisting of squares of natural numbers or integers resulting in sequence of squares. It consists of only positive terms. E.g. 1, 4, 9, 16, 25, 36,.....



Cubes

Sequence consisting of cubes of natural numbers is called sequence of cubes. E.g. 1, 8, 27, 64, 125,.....



Increasing and Decreasing Sequence

Sequence in which every term is greater than or equal to the preceding term is called monotonically increasing sequence. Similarly, if every term is less than or equal to the preceding term then sequence is called monotonically decreasing sequence.

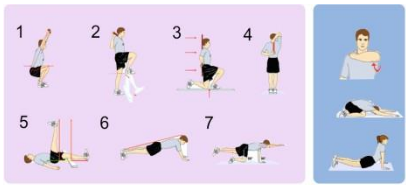
6. Real Life Applications of Sequences

Sequences are a part of our day to day life. These are embedded in our daily life so deeply that sometimes they are neglected also. Some of the applications are listed below:

Day to Day Life

In our day to day life, we come across many patterns such as prints on the cloth, alphabets, braids, embroidery patterns, handicrafts, architecture, grilled sandwich, flowers, leaves, grills on window, sweets arranged in the sweet shop, hand-movement of clock, Amount of simple or compound interest from a sum or depreciation of your car, etc. This list is endless. Once you will start observing these patterns, you will get fascinated to all these.


1
2
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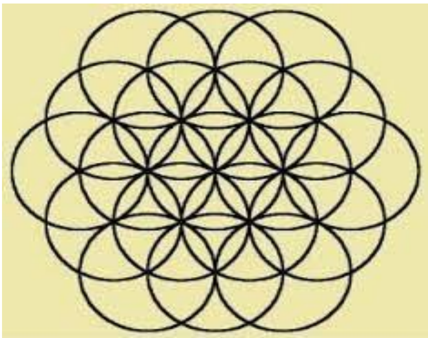
7

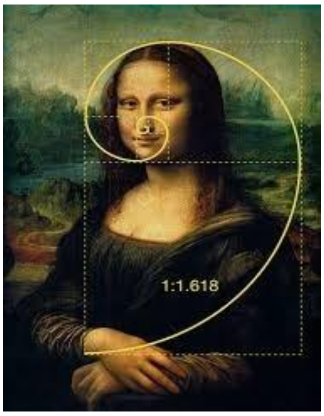


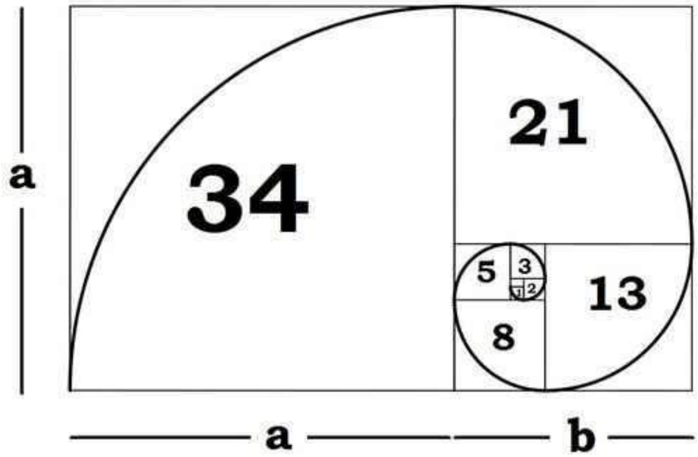
Fibonacci Sequence & Golden Ratio

The Fibonacci sequence is a recursive sequence given as 0, 1, 1, 2, 3, 5, 8, 13,. Every term is sum of preceding two terms and this is applicable second term onwards. This Fibonacci sequence is applicable in Golden ratio.

Fibonacci was a famous mathematician (c. 1170 – c. 1250) was an Italian mathematician. Fibonacci popularized the Hindu–Arabic numeral system to the Western World primarily through his composition in 1202 of *Liber Abaci*(*Book of Calculation*). He also introduced Europe to the sequence of Fibonacci numbers, which he used as an example in *Liber Abaci*.

We start with two squares of size 1. Along one edge, we add a new square of size 2, and then another square of size 3. We keep adding squares to the longest edge of the rectangle as shown below. Since the edge of every new square is the sum of the edges of the two previous squares, we get the Fibonacci numbers. If we trace a curve along the corners of these squares, we can make a spiral. This spiral approximates the **Golden Spiral**. Many similar *logarithmic spirals* appear in nature, for example Nautilus shells.



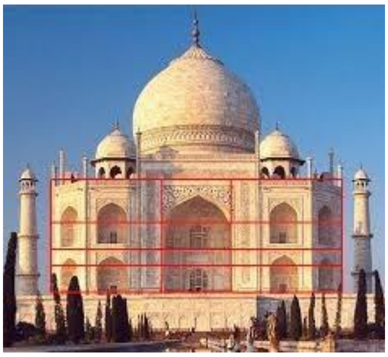


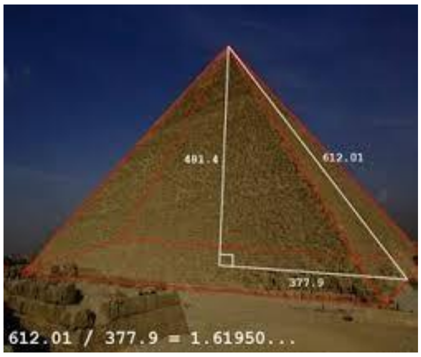
While adding more squares, the proportions of the rectangle become closer to a very special shape: the**Golden rectangle**. The ratio of the sides of the golden rectangle is called the**Golden ratio**. It is the limit of the ratio of consecutive Fibonacci Numbers.

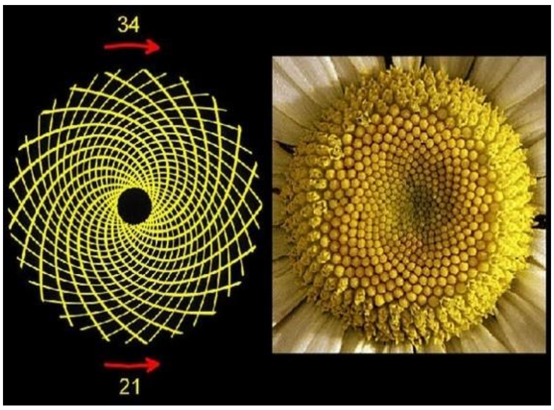
11= 1,21= 2,32= 1.5,53= 1.67,85= 1.6,138= 1.63, …

You can see that these ratios get closer and closer to a particular number around 1.6. This is the Golden ratio and its actual value is 1.61803…

Golden ratio is present all around us in nature, humans, paintings, music, architecture, galaxy and everywhere.







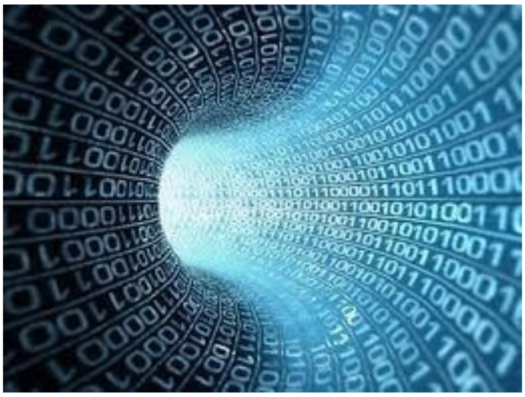
Action sequence Photography

This is something latest in the field of photography. As the technology is advancing, cameras are also evolving with better effects. Composite photography is also leading among photographers who love to shoot moving images of specially athlete and composite all such still images into a single one to give the mesmerising effect to their art and talent.

Multi-disciplinary Applications

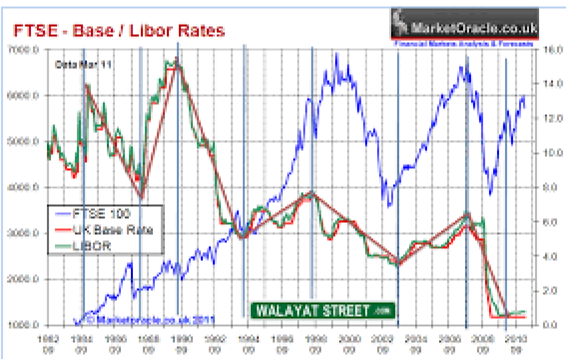
Computers

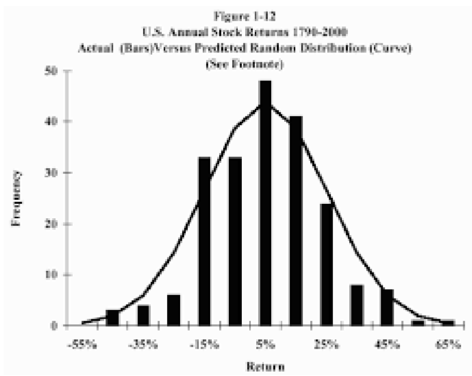
Sequences are significant in other disciplines and studies such as computers where software is built up by using sequence of programs. Encrypting is also based on specific sequences to be followed.



Economics

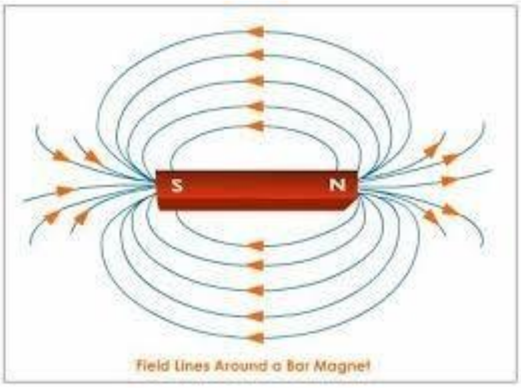
It is useful in the case of economics where patterns in stock market decide future of a company. The normal probability curve is also using same pattern on both the sides of the peak. If any stock has sequence of getting up and down in a specific period of time, then definitely this study is very helpful to the investors.

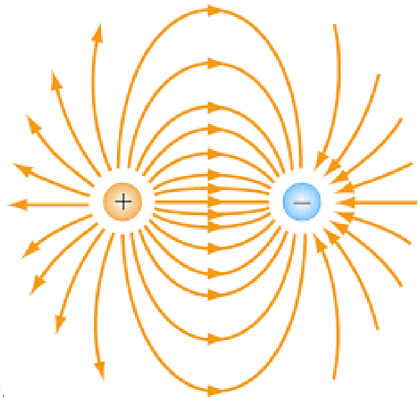


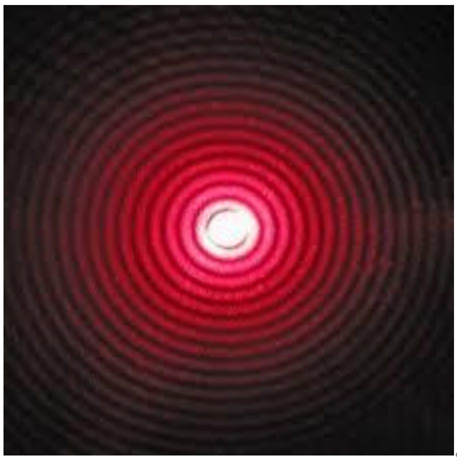


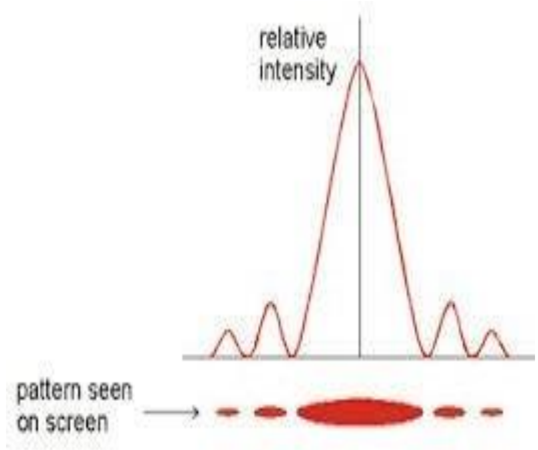
Physics

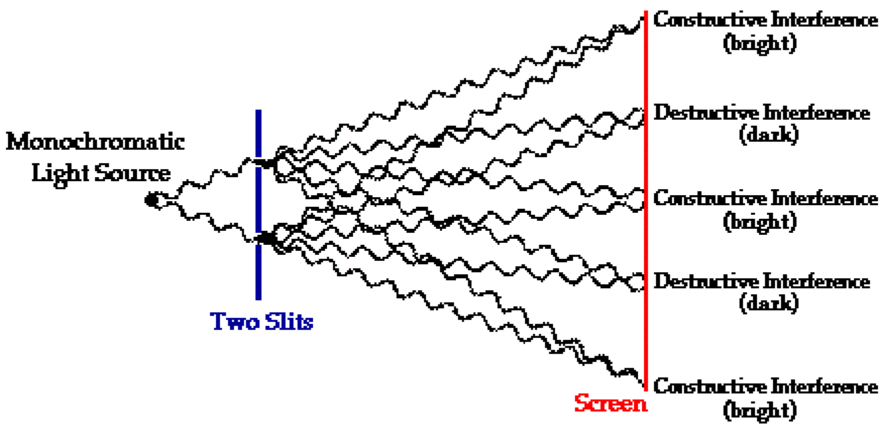
In the field of physics, reflection of light, diffraction pattern, sound waves, light waves, electric field, magnetic field all represent patterns. Oscillating waves, pendulum motion, optics are all based on sequences. Have not you ever tried rainbow pattern, which is caused by reflection of light.



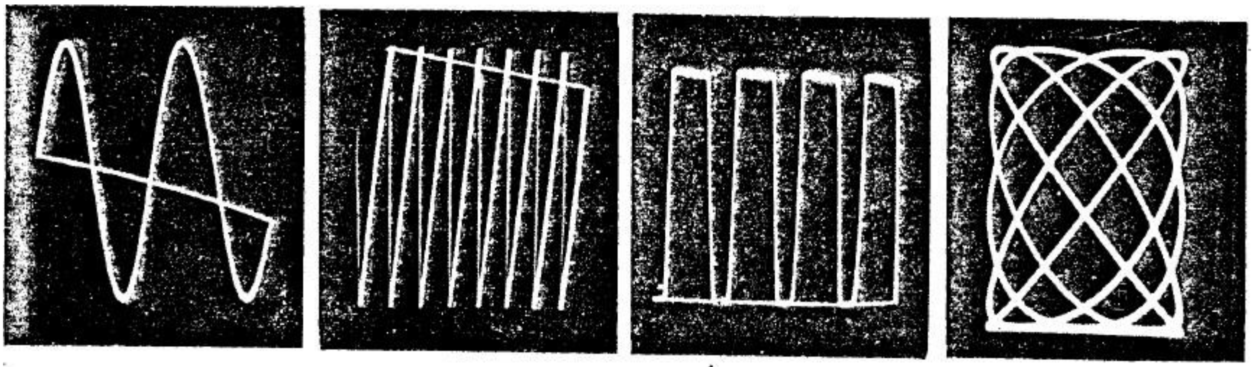








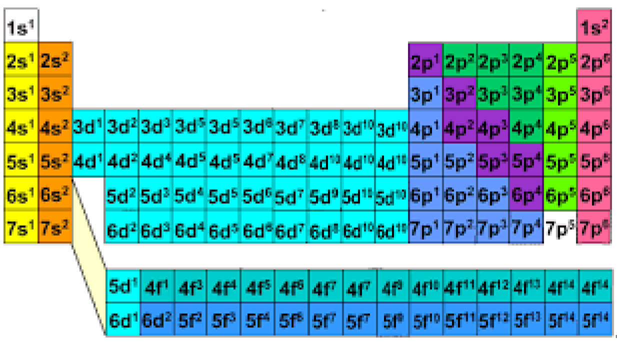
A two-point source interference pattern creates an alternating pattern of bright and dark lines when it is projected onto a screen.

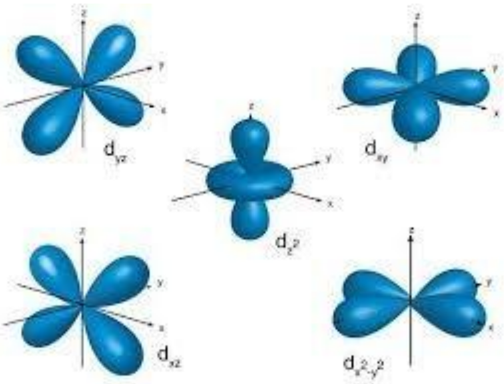


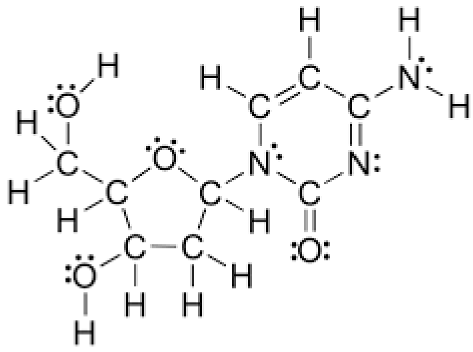
Sine-wave   Sawtooth   Square Wave   Lissajous Figure

Chemistry

In the field of chemistry, periodic table shows patterns or sequence. Elements of the same block has same sequence of characteristics. Orbitals, electron movements, bonds, organic compounds are all based on sequences.



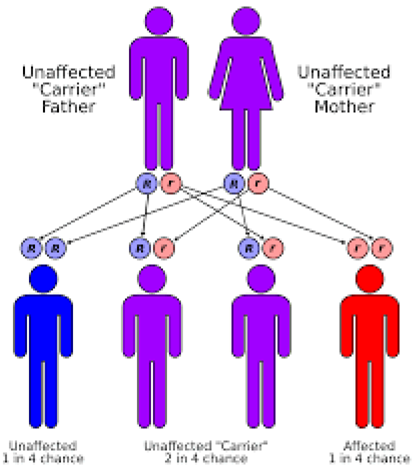


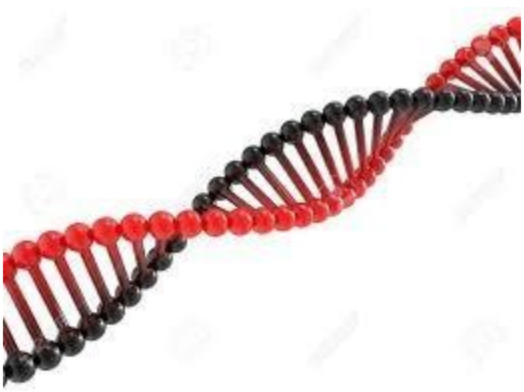


2'-deoxycytidine

Genetics

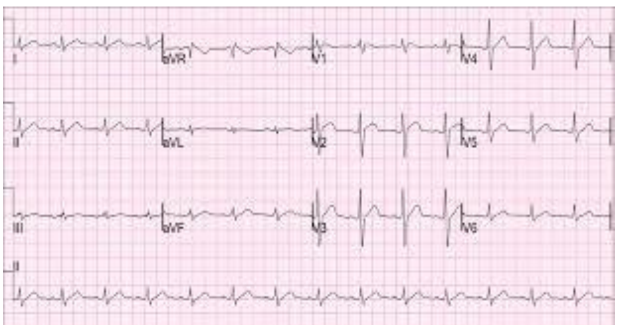
The study of genetics is based on studying patterns only. You can recall spiral pattern of chromosomes in DNA.





Medical

Entire medical line is based on study of sequence. Do not you think that your heart and circulatory system follows a specific sequence which is visible in the case of ECG also. If this sequence is abrupted due to any reason then it causes to heart diseases. This follow-up of set sequence of internal organs and physical systems is a must for sound health.



7. Summary

- A sequence can be defined as a function whose domain is the set of natural numbers or some subset of natural numbers. Sometimes, we use the functional notation a(n) for a_n.
- Let $a_1, a_2, a_3, a_4, a_5, \ldots\ldots a_n$ be any sequence then the expression $a_1 + a_2 + a_3 + a_4 + a_5 + \ldots + a_n$ is the series corresponding to the sequence.
- Series is denoted by Greek letter *Sigma* as $\sum_{k=1}^{n} a_k$
- A progression is a type of a sequence in which all the successive terms are increasing or decreasing according to a fixed rule.
- Every progression is a sequence but vice-versa is not true.
- Fibonacci sequence is a type of recursive sequence in which first two terms are 0 and 1 and every subsequent term can be obtained by adding previous two terms.
- There are many types of sequences like finite, infinite, recursive, square, cube, arithmetic, geometric, harmonic, increasing and decreasing.
- Sequences are useful in our real life, action sequence photography, nature, physics, chemistry, economics genetics, biology, medical etc.