1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 01 (Class XI, Semester - 1)
Module Name/Title	Permutation and Combination- Part 1
Module Id	kemh_10701
Pre-requisites	Visualization skills, understanding application based problems
Objectives	After going through this lesson, the learners will be able to understand the following : ● Understand the need for formulas in solving problems based on Permutation and Combination. ● Understand the Factorial notation. ● Apply the concept of factorial in solving questions based on permutations. ● Picture Method- Tree diagram
Keywords	Permutations, Restrictions, Grouping

2. Development Team

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1. Introduction

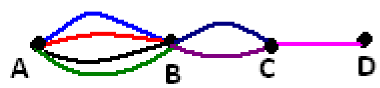
In our day to day life we come across many situations where we make use of permutations and combinations knowingly or unknowingly. Suppose you go for a conference where there are 250 student delegates across India, each student shakes hand with every other student present there, how many handshakes happened there in total? What comes to your mind? How would you calculate this?

What if you come to know that the combination lock which you have known over the years was not actually a combination lock but a permutation lock? Since the order in which the numbers are perceived is important.

Doesn't that sound interesting?

When you go to a restaurant you sieve through the menu list you choose items over the course of food, this involves permutations and combinations.

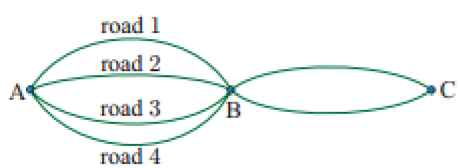
The road which you choose to travel from point A to point D, through path B and C also involves permutations and combinations.



But before getting into the mathematical depth of these concepts let us understand the basic fundamental principle of counting on which all these arrangements are based.

2. Fundamental principle of counting

Let us consider three cities A, B, C. There are four paths to go from A to B represented by the four different roads; there are 2 paths to go from B to C.



What are the total ways of going from A to C through B?

Suppose a person has to go from A to B, he can take road 1 then either of the alternative paths to C.

He can take Road 2 and either of the alternative paths to reach C

He can take road 3 and either of the alternative paths to reach C

He can take road 4 and either of the alternative paths to reach C

So, in all he has 2+2+2+2 = 8 alternatives

Or, in all he has 4×2 i.e. 8 alternatives.

Here 4 corresponds to the number of roads from A to B and 2 corresponds to the numbers of roads from B to C.

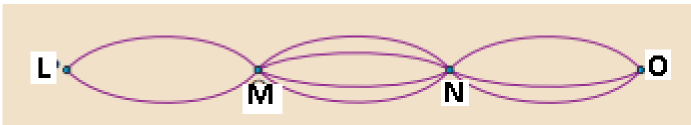
So putting it mathematically:

3. Product Principle: Two different events

If there are m different ways in which an operation can be performed and for each of these there are n different ways in which another independent operation can be performed, then in all there are mn different ways in which these two operations can be performed in succession.

Let us now consider one more example

Let L,M,N,O be the four places where the four friends Leena, Mohan, Naresh and Omi live. There are 2 ways to go from L to M, 4 ways to go from M to N and 3 ways to go from N to O. Then what are the different ways to go from L to O through M and N ?



By counting different ways to reach O from L we can can arrive to a conclusion that there will be

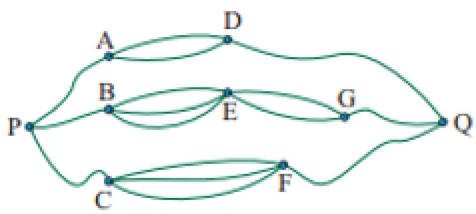
$2 \times 4 \times 3 = 24$ ways to reach from L to O through M and N.

4. Product Principle: Three different events

Thus for the **three events the principle of counting can be generalized as:**

If an event can occur in m different ways, following which another event can occur in n different ways following which a third event can happen in p different ways, then the total number of occurrence of all the three events simultaneously one followed by the other is expressed by the product $m \times n \times p$.

Consider the road system illustrated which shows the roads from P to Q.



From A to Q there are $2 \times 1 = 2$ paths.

From B to Q there are $3 \times 2 = 6$ paths.

From C to Q there are $3 \times 1 = 3$ paths.

Therefore from P to Q there are 2 + 6 + 3 = 11 paths.

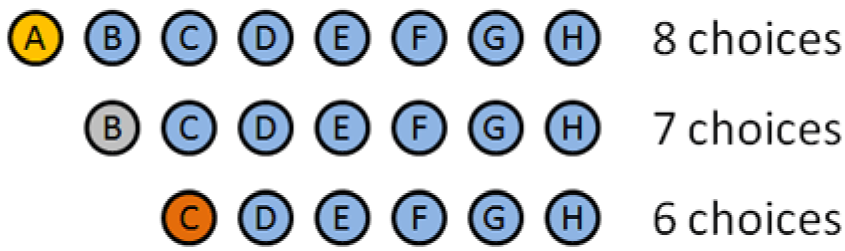
- When going from B to G, we go from B to E **and** then from E to G. We **multiply** the possibilities.
- When going from P to Q, we must first go from P to A **or** P to B **or** P to C. We **add** the possibilities from each of these first steps.

The word **and** suggests *multiplying* the possibilities.
The word **or** suggests *adding* the possibilities.

Note: <u>You can generalize the principle for any finite number of events</u>.

Example

Suppose we have eight contestants battling for a First, Second and Third position in a 100m race. How many ways can we award a 1st, 2nd and 3rd place prize among eight contestants? (Gold / Silver / Bronze)



Gold medal: 8 choices: A B C D E F G H (Anyone can get the gold, lets mark A as the person who wins the gold)

Silver medal: 7 choices: B C D E F G H. Let's say B wins the silver.

Bronze medal: 6 choices: C D E F G H. Let's say… C wins the bronze.

So we had 8 choices at first, then 7 choices for the second, then 6 choices for the third. The total number of options are $8 \times 7 \times 6 = 336$. _____

Problems – With And Without Restrictions

You may come across problems where there are restrictions or probably no restrictions involved-

- When there are restrictions involved, the restrictions should be considered first.
- When there is no restriction involved, the problems can be solved in any order.

Some Problems based on Counting Principle:

Problem 1:

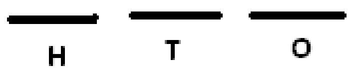
How many 3 – digit numbers can be formed from the digits 1, 2, 3, 4 and 5

Assuming that

(1) Repetitions of digits are allowed

Solution :

Lets visualize the problem by drawing 3 dashes _ _ _



The ones places can be filled by any of the 5 digits. (i.e. 1, 2, 3, 4, 5)

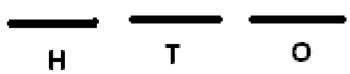
Since repetition is allowed, for the tens place also we have all the 5 digits available (i.e. 1, 2, 3, 4, 5)

Similarly for the Hundreds place also we have all the 5 digits available (i.e. 1, 2, 3, 4, 5)

Thus we can have 5 × 5 × 5 = 125, 3 digit numbers possible with repetition.

(2) Repetitions of digits are not allowed

Solution :



The ones places can be filled by any of the 5 digits. (i.e. 1, 2, 3, 4, 5)

Since repetition is not allowed for the tens place we now have 4 digits available.

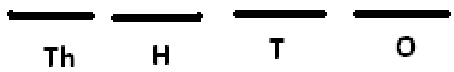
Now for the Hundreds place we have 3 digits available.

Thus we can have 5 × 4 × 3 = 60, 3 digit numbers possible, without repetition.

Problem 2:

How many 4 –digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6, where the digits cannot be repeated.

Solution : Let us again visualize the problem by considering the diagram



First thing to be kept in mind now is that the 4 – digit number has to be an even number

So we have to choose an even number from the set of digits provided. So we can choose any one out of 2, 4, 6 (only one of them)

In the tens place we have now the availability of 5 digits.

Hundreds place we have the availability of 4 digits.

Thousands place we have the availability of 3 digits.

Thus total number of 4 digit even numbers that can be formed with the digits 1, 2, 3, 4 , 5, 6 are

$3 \times 4 \times 5 \times 3 = 180$

6. Understanding With The Picture Method – Tree Diagram

Visualising problems always have been the best way of understanding things.

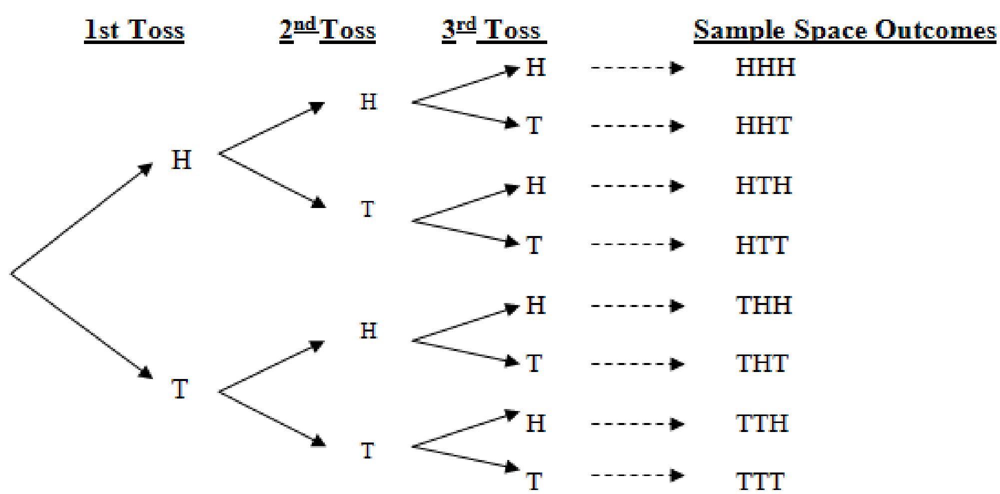
Problem

A coin is tossed three times and the outcomes are recorded. How many possible outcomes are there?

Solutions:

Sometimes drawing tree diagram helps in understanding the problems in a better way.

Consider the first toss and record the outcomes, the second toss and its outcomes, third toss and the outcomes.



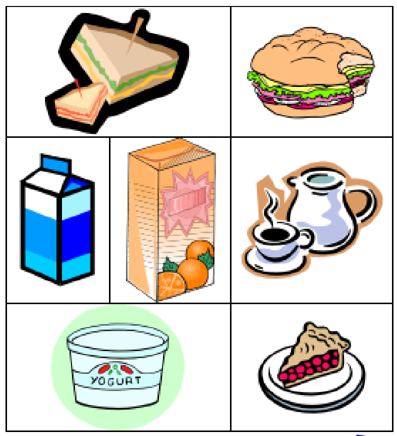
Thus we can say that when we toss a coin thrice, in all we have eight outcomes.

In other words you can also say that it has 2^3 outcomes.

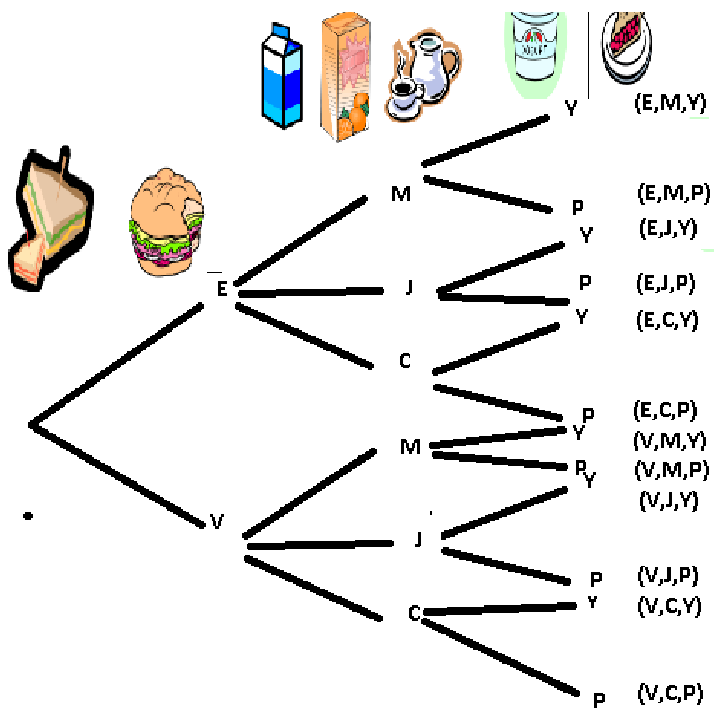
What will be the outcomes if the coin is tossed 4 times , 8 times, …? In that case perhaps the tree diagram may be quite lengthy but we can find that the total number of outcomes are 2^4, 2^8 …. respectively.

Investigate: Counting Without Counting

1. 1. A café has a lunch special consisting of an egg or a vegetable sandwich (E or V); milk, juice, or coffee (M, J, or C); and yogurt or pie for dessert (Y or P). a) One item is choosen from each category. List all possible meals. Use a tree diagram to organize your work.



Solution :



b) How many possible meals are there? Count the ends of the branches of your tree diagram.

Solution : On seeing the end of the branches we can see that there are 12 possible meals in all.

c) How can you determine the number of possible meals without listing all of them?

Solution : To determine the number of possible meals without listing them we can see that at first we had an option of 2 meals (Egg and Vegetable sandwich), then we had three options of 3 drinks(Milk, Juice, coffee), then we had an two options for desserts (Yoghurt and pie).

Using Product Principle, we have **$2 \times 3 \times 2 = 12$ possible meals.**

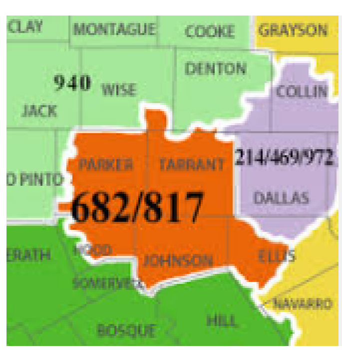
Example: You are taking a multiple-choice test that has eight questions. Each of the questions has four answer choices, with one correct answer per question. If you select one of these four choices for each question and leave nothing blank, in how many ways can you answer the questions? The number of different ways you can answer the questions is: $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^8 = 65,536$

7. Examples where restrictions are involved

Telephone numbers in a country begin with three- digit area codes, followed by seven-digit local telephone numbers. Area codes and local telephone numbers cannot begin with 0 or 1. How many different telephone numbers are possible?

Solution : This situation involves making choices with ten groups of items.

Here are the choices for each of the ten groups of items:



Area Code Local Telephone Number: The area code will have 8 × 10 × 10 = 800 choices,

Telephone numbers will have 8 × 10 × 10 × 10 × 10 × 10 × 10 choices

Thus the total number of different telephone numbers is: (8 ×10×10) × 8 × 10×10×10 ×10 × 10×10 = 6,400,000,000.

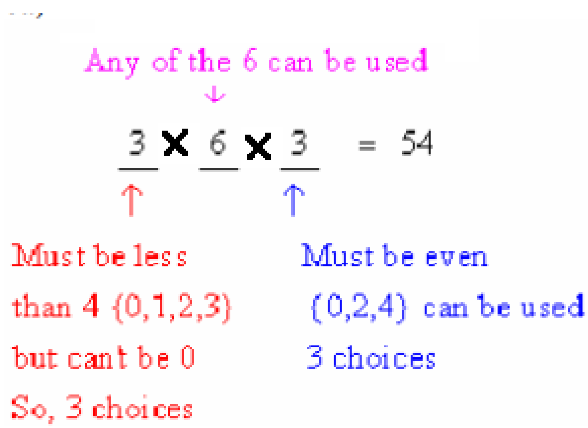
Question with restrictions :

Example: How many positive even three-digit integers less than 400 can be formed from the digits {0 , 1 , 2 , 3 , 4 , 5 } if:

a) Repetition is allowed?

b) No digit is repeated ?

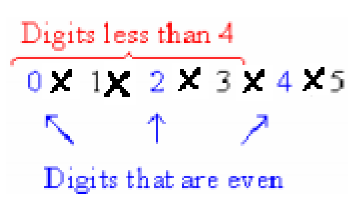
Solution a) Vizualise the problem as follows



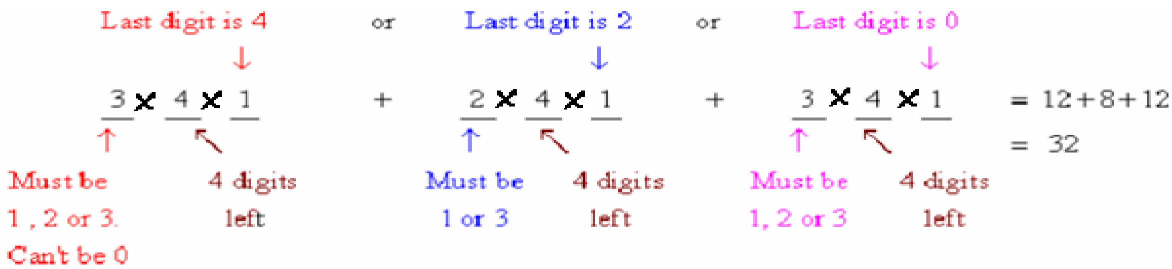
Hence, total 3 – digit even numbers less than 400 are 3 × 6 ×3 = 54

Solution b)

b) Remember, fill in restrictions first, but in this question you have overlapping restrictions. (restrictions that affect the other restrictions)

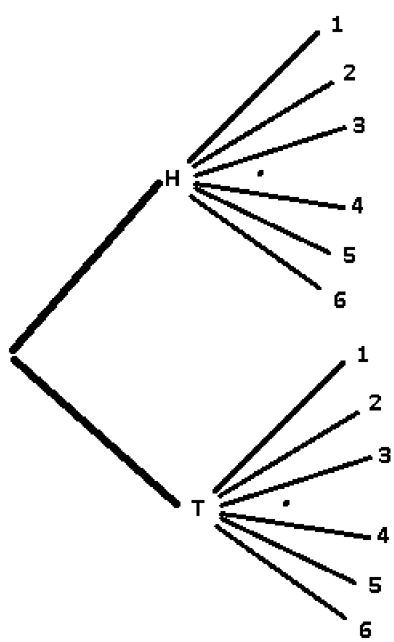


So, break up the question into parts – this will eliminate a restriction.



Hence, there are $3 \times 4 \times 1 + 2 \times 4 \times 1 + 3 \times 4 \times 1 = 12 + 8 + 12 = 32$ possible even numbers
less than 400 when no digit is repeated.

Example : If a coin is tossed followed by the throw of a die then the tree diagram can be as follows:



We see that the total number of possibilities are $2 \times 6 = 12$. The number of ways in which a series of successive things can occur is found by multiplying the number of ways in which each thing can occur.

From day to day life/ activities: Examples:

- The number of possible outfits from 2 pairs of jeans, 3 T-shirts and 2 pairs of sneakers are:

$2 \times 3 \times 2 = 12$ ways

- In a 6 course meal, you have 3 appetizer choices, 2 soup choices, and 4 salad choices, along with 5 main course choices, 10 beverage choices, and 3 dessert choices. How many unique 6-course meals you can make?

  Number of possible unique meals

  $= 3 \times 2 \times 4 \times 5 \times 10 \times 3 = 3{,}600$ possible unique meals

- License plates of cars have two letters from English alphabet followed by four digits. How many license plates can be made?

  The first two places must be letters from 26 alphabets and the remaining 4 places can be any of the 10 digits.

  Therefore the number of license plates

  $= 26 \times 26 \times 10 \times 10 \times 10 \times 10 = 6{,}760{,}000$

8. Summary

Many problems in probability and statistics require careful analysis of complex events. Combinatorics basic roots are to develop systematic ways of counting. These systematic counting methods will allow the solving of complex counting problems that are useful in all facets of life.

The Fundamental Counting Principle:

- If we can perform a first task in x different ways
- If we can perform a second task in y different ways
- If we can perform a third task in z different ways, and so on . . .

Then the first task followed by the second and so on can be performed in $x \cdot y \cdot z \dots$ different ways.

- Counting objects with restrictions We will continue to use the fundamental counting principle and use “blanks” instead of the tree method, but it is important that we count the restricted value first!
- If the problem is without a restriction then you can start with any blank or in any order.
- If data is small then the tree method helps visualize problems better.
- ‘AND’ refers to multiplication whereas ‘OR’ refers to addition.