1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 01 (Class XI, Semester - 1)
Module Name/Title	Linear inequalities - Part 3
Module Id	kemh_10603
Pre-requisites	Basic knowledge of drawing graphs of the linear equations and solving simultaneous equations.
Objectives	After going through this lesson, the learners will be able to understand the following: ● Difference between equation and inequality. ● Solve system of linear inequalities in two variables. ● Plot a graph of systems of linear inequalities. ● Describe the feasible regions when inequalities are plotted graphically.
Keywords	Linear inequality, Graphical solution, Shaded region

2. Development Team

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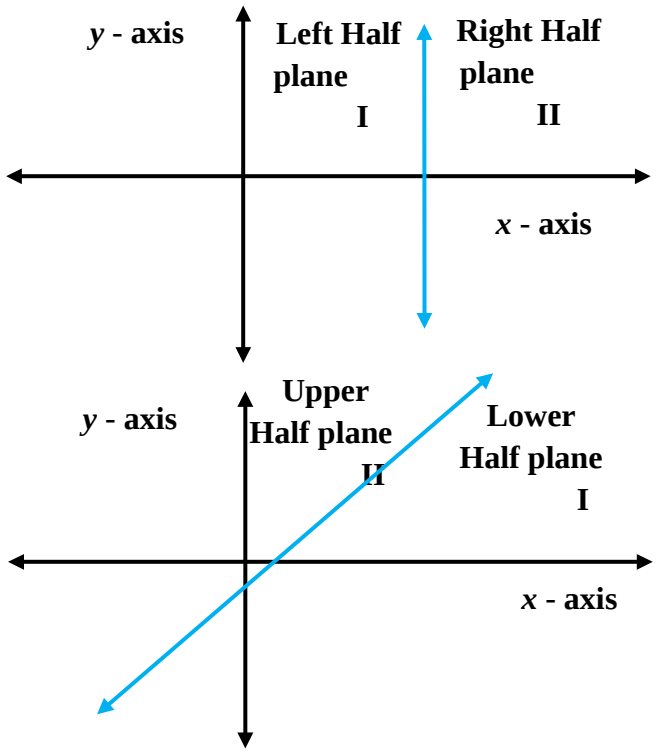
1. Graphical Solution of Linear Inequalities in Two Variables

The line divides the Cartesian plane into two parts. Each part is known as a half plane. A vertical

line divides the plane in left and right half planes and a non-vertical line divides the plane into

lower and upper half planes.



Graph of a linear inequality in one variable can be represented on number line as well as in the plane but the graph of a linear inequality in two variables can be represented in the plane only. There are four types of linear inequality in two variables.

(i) $ax + by > c$
(ii) $ax + by \geq c$
(iii) $ax + by < c$
(iv) $ax + by \leq c$ ($a \neq 0$, $b \neq 0$)

Working rule to graph inequalities of the form $ax + by > c$ or $ax + by < c$

Step 1: Draw the graph of the line $ax + by = c$

Using two points $(c/a, 0)$ and $(0, c/b)$ which are on x – axis and y – axis respectively.

Step 2: Substitute any point (h, k) in the inequality $ax + by > c$. If inequality is satisfied by the

point (h, k) then graph of inequality is shaded region containing the point (h, k), otherwise graph is the shaded region not containing (h, k). If line does not pass through origin, take the point (0, 0) to check for the region.

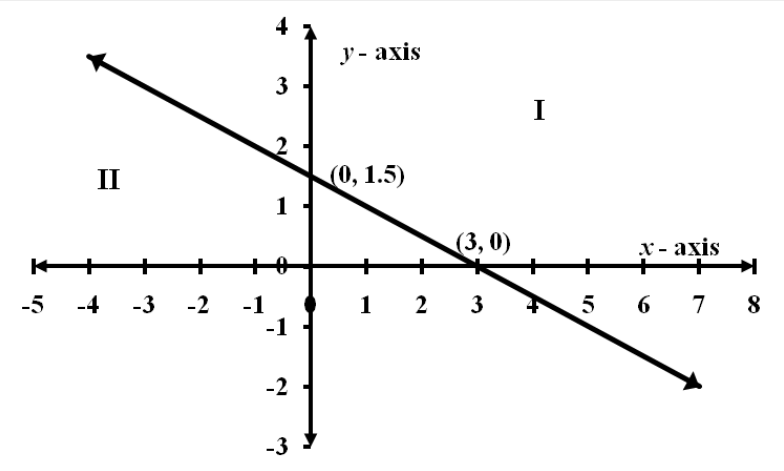
Note: Points on the line $ax + by = c$ are not to be included in the shaded region to draw the inequality $ax + by > c$ or $ax + by < c$. So draw a broken line or dotted line in the shaded region.

Working rule to graph inequalities of the form $ax + by \geq c$ or $ax + by \leq c$ ($a > 0$)

If an inequality is of the type $ax + by \geq c$ or $ax + by \leq c$, then draw the shaded region corresponding to inequality $ax + by > c$ or $ax + by < c$ as well as the points on the line $ax + by = c$. So a dark line is drawn in the shaded region in this case.

Example 1: Graph the inequality $x + 2y > 3$, where x and y are real numbers.

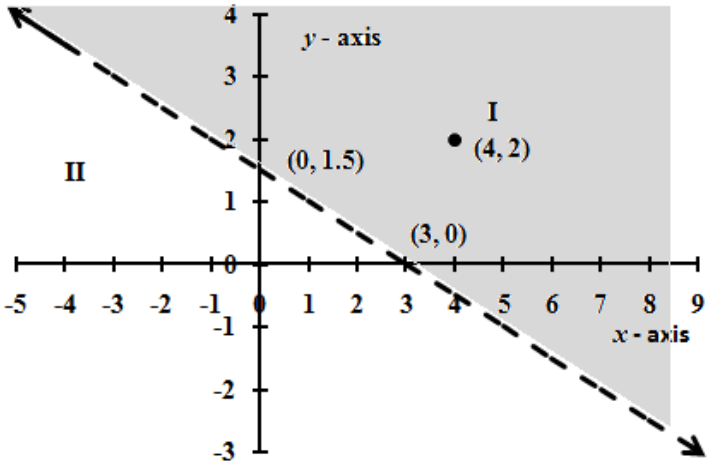
Solution: To draw the line represented by $x + 2y = 3$, consider the points (3, 0) and (0, 3/2) which satisfy the given equation and are on x – axis and y – axis respectively.



The graph of $x + 2y = 3$ is a line that divides the Cartesian plane into two parts; 'I' above the line and 'II' below the line. On taking any point in the part which is above the line say (4, 2) and substituting in the inequality $x + 2y > 3$,

$4 + 2(2) = 8 > 3$ which is true

Thus, shaded region will be the part containing the point (4, 2). Since inequality is strict inequality thus line $x + 2y = 3$ will be dotted line. Thus graph of inequality will be as



Example 2: Graph the inequality $y + 8 \geq 2x$, where x and y are real numbers.

Solution: Given $y + 8 \geq 2x$

$\Rightarrow$ $8 \geq 2x - y$

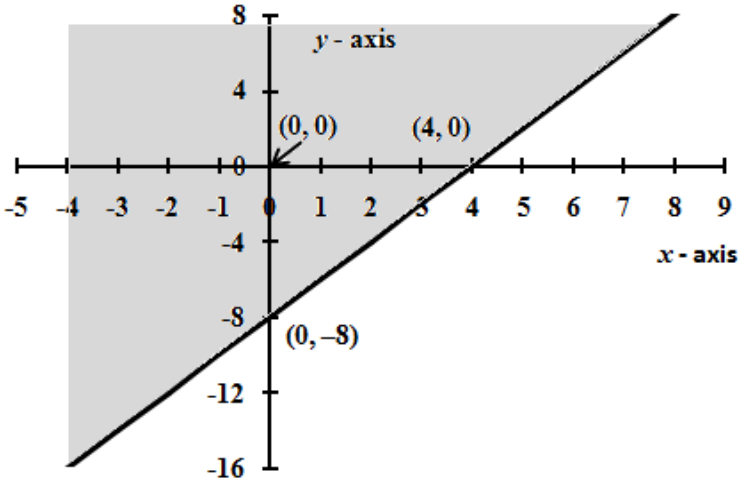
$\Rightarrow$ $2x - y \leq 8$

To draw the graph of $2x - y = 8$, consider the points (4, 0) and (0, –8) which are on x – axis and y – axis respectively.

Graph of $2x - y = 8$ is a line that divides the Cartesian plane into two parts. On taking any point say (0, 0) and substituting in the inequality $2x - y \leq 8$, we get

$2(0) - (0) = 0 \leq 8$ which is true.

Thus, shaded region will be the part containing the point (0, 0). Since inequality is slack inequality thus line $2x - y = 8$ will be dark line. Thus graph of inequality will be as



Example 3: Graph the inequality $3y - 5x < 30$, where x and y are real numbers.

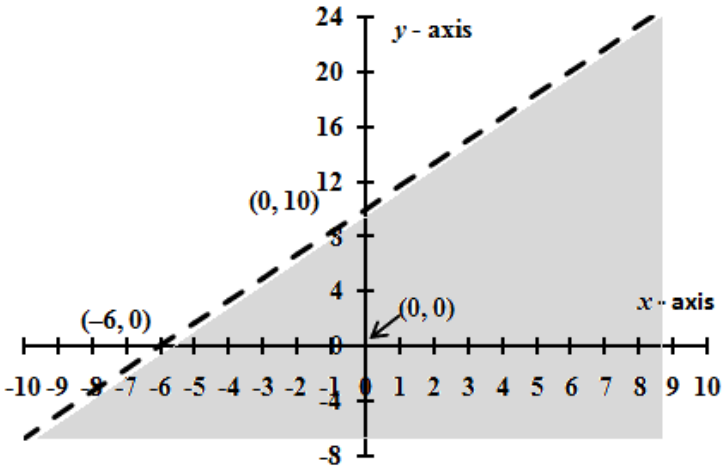
Solution: Given $3y - 5x < 30$

To draw the line $3y - 5x = 30$, consider the points $(-6, 0)$ and $(0, 10)$ which are on x – axis and y – axis respectively.

Line $3y - 5x = 30$ divides the Cartesian plane into two parts. On taking any point say $(0, 0)$ and substituting in the inequality $3y - 5x < 30$, we get

$3(0) - 5(0) = 0 < 30$ which is true.

Thus, shaded region will be the part containing the point $(0, 0)$. Since inequality is strict inequality thus line $3y - 5x = 30$ will be dotted line. Thus graph of inequality will be as



Example 4: Graph the inequality $2x - 3y > 6$, where x and y are real numbers.

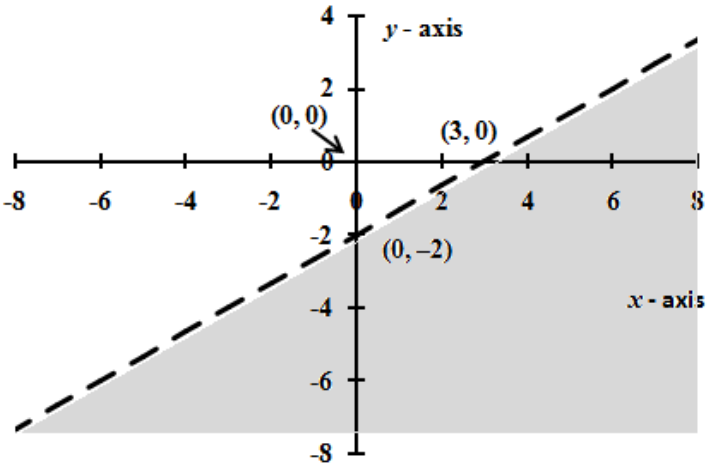
Solution: Given $2x - 3y > 6$

To draw the line $2x - 3y = 6$, consider the points $(3, 0)$ and $(0, -2)$ which are on x – axis and y – axis respectively.

Line $2x - 3y = 6$ divides the Cartesian plane into two parts. On taking any point say $(0, 0)$ and substituting in the inequality $2x - 3y > 6$,

$2(0) - 3(0) = 0 < 6$ thus inequality is not true.

Thus, shaded region will be the part not containing the point $(0, 0)$. Since inequality is strict inequality thus line $2x - 3y = 6$ will be dotted line. Thus, graph of inequality will be as



Example 5: Graph the inequality $x - y \leq 3$, where x and y are real numbers.

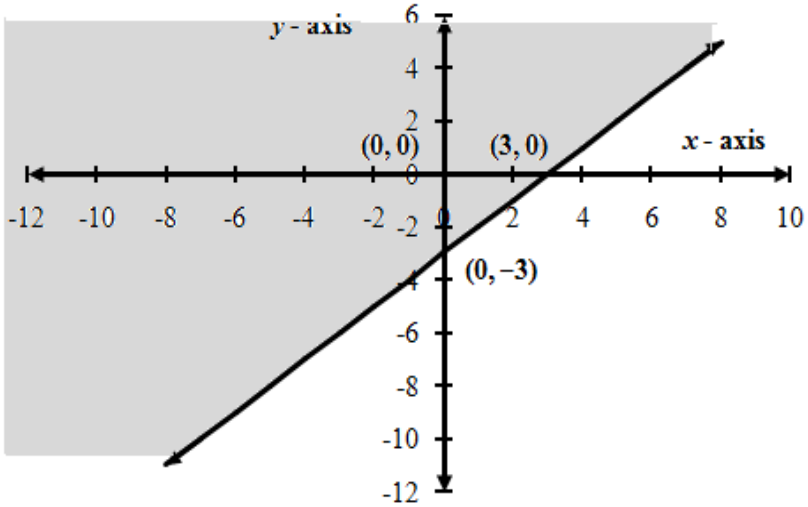
Solution: Given $x - y \leq 3$

To draw the line $x - y = 3$, consider the points (3, 0) and (0, –3) which are on x – axis and y – axis respectively.

Line $x - y = 3$ divides the Cartesian plane into two parts. On taking any point say (0, 0) and substituting in the inequality $x - y \leq 3$,

$(0) - (0) = 0 \leq 3$ thus inequality is true.

Thus, shaded region will be the part containing the point (0, 0). Since inequality is slack inequality thus line $x - y = 3$ will be dark line. Thus, graph of inequality will be as



Example 6: Graph the inequality $3x - 2y \leq 6$, where x and y are real numbers.

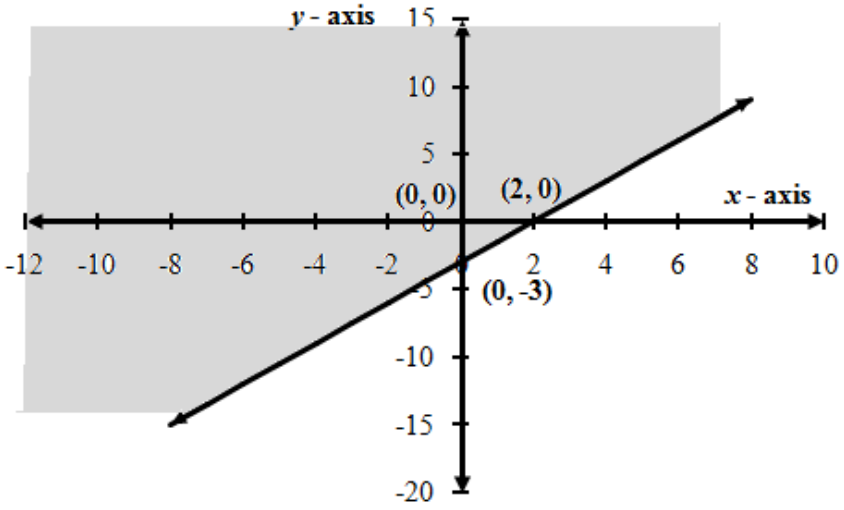
Solution: Given $3x - 2y \leq 6$

To draw the line $3x - 2y = 6$, consider the points (2, 0) and (0, –3) which are on x – axis and y – axis respectively.

Line $3x - 2y = 6$ divides the Cartesian plane into two parts. On taking any point say (0, 0) and substituting in the inequality $3x - 2y \leq 6$,

$(0) - (0) = 0 \leq 6$ thus inequality is true.

Thus, shaded region will be the part containing the point (0, 0). Since inequality is slack inequality thus line $3x - 2y = 6$ will be dark line. Thus, graph of inequality will be as



Example 7: Graph the inequality $5x - 3y \leq 15$, where x and y are real numbers.

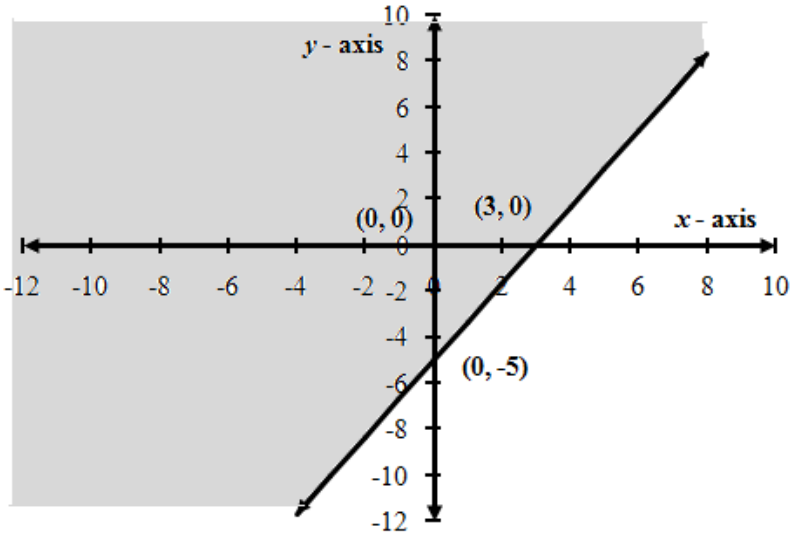
Solution: Given $5x - 3y \leq 15$

To draw the line $5x - 3y = 15$, consider the points (3, 0) and (0, –5) which are on x – axis and y – axis respectively.

Line $5x - 3y = 15$ divides the Cartesian plane into two parts. On taking any point say (0, 0) and substituting in the inequality $5x - 3y \leq 15$,

$(0) - (0) = 0 < 15$ thus inequality is true.

Thus, shaded region will be the part containing the point (0, 0). Since inequality is slack inequality thus line $5x - 3y = 15$ will be dark line. Thus, graph of inequality will be as



Example 8: Graph the inequality $3x - 2y \leq 0$, where x and y are real numbers.

Solution: Given $3x - 2y \leq 0$

Line $3x - 2y = 0$ or $3x = 2y$ passes through origin (0, 0).

To draw the line, consider the points.

When $x = 1$, $y = \frac{3}{2} = 1.5$

When $x = 2$, $y = 3$

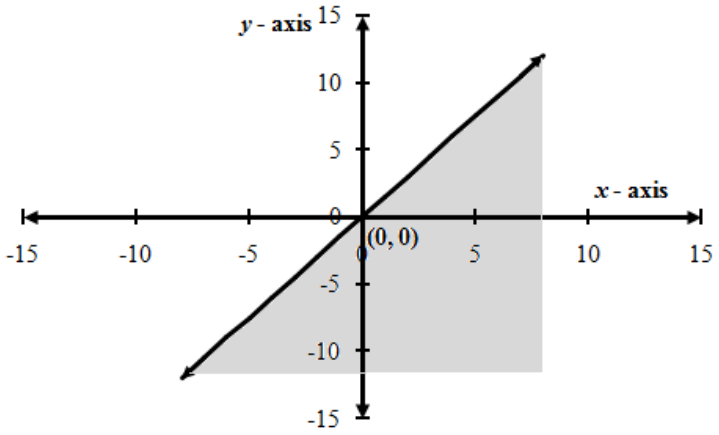
When $x = -1$, $y = \frac{-3}{2} = -1.5$

When $x = -2$, $y = -3$

Line $3x - 2y = 0$ divides the Cartesian plane into two parts. Since line passes through origin, thus, on taking any point say (1, 1) and substituting in the inequality $3x - 2y \leq 0$,

$3(1) - 2(1) = 3 - 2 = 1 > 0$ thus inequality is not true.

Thus, shaded region will be the part not containing the point (1, 1). Since inequality is slack inequality thus line $3x - 2y = 0$ will be dark line. Thus, graph of inequality will be as



Example 9: Graph the inequality $x - 3y < 12$, where x and y are real numbers.

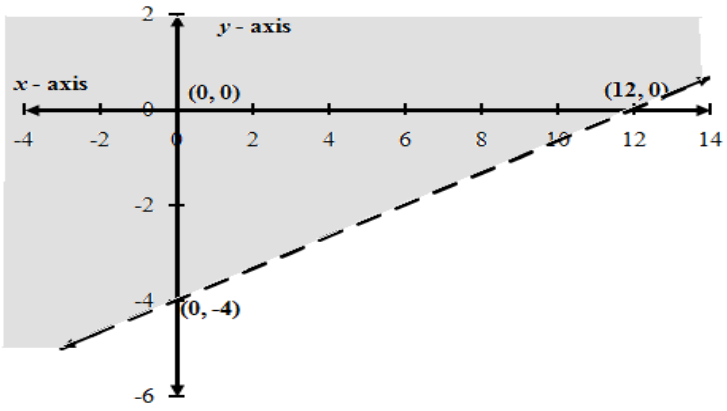
Solution: Given $x - 3y < 12$

To draw the line $x - 3y = 12$, consider the points (12, 0) and (0, –4) which are on x – axis and y – axis respectively.

Line $x - 3y = 12$ divides the Cartesian plane into two parts. On taking any point say (0, 0) and substituting in the inequality $x - 3y < 12$,

$(0) - (0) = 0 < 12$ thus inequality is true.

Thus, shaded region will be the part containing the point (0, 0). Since inequality is strict inequality thus line $x - 3y = 12$ will be dotted line. Thus, graph of inequality will be as



Example 10: Graph the inequality $2x + y > 4$, where x and y are real numbers.

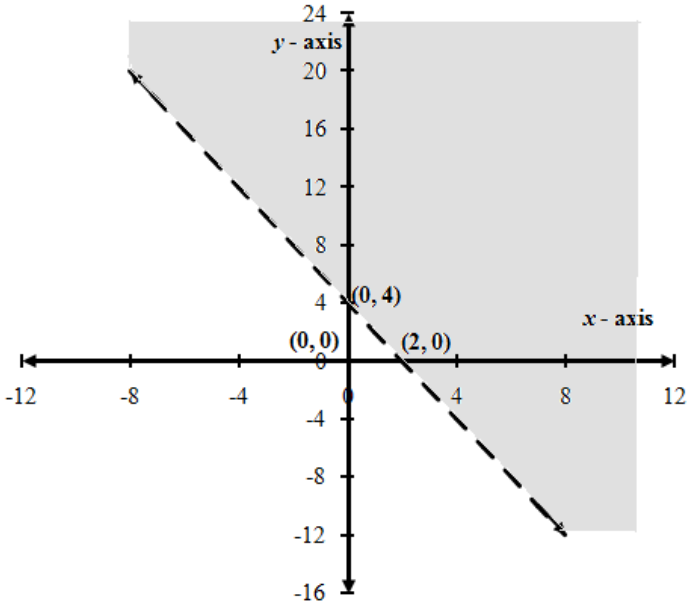
Solution: Given $2x + y > 4$

To draw the line $2x + y = 4$, consider the points (2, 0) and (0, 4) which are on x – axis and y – axis respectively.

Line $2x + y = 4$ divide the Cartesian plane into two parts. On taking any point say (0, 0) and substituting in the inequality $2x + y > 4$,

$2(0) + (0) = 0 < 4$ thus inequality is not true.

Thus, shaded region will be the part not containing the point (0, 0). Since inequality is strict inequality thus line $2x + y = 4$ will be dotted line. Thus, graph of inequality will be asshown below



Example 11: Graph the inequality $x – 5y < –5$, where x and y are real numbers.

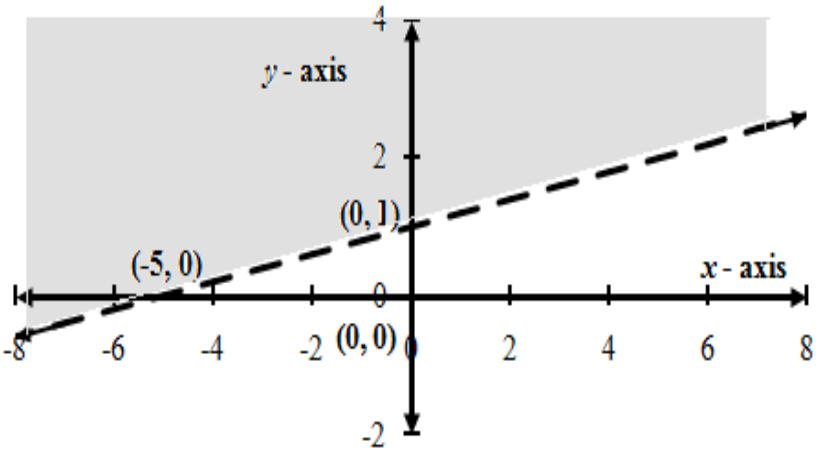
Solution: Given $x – 5y < –5$

To draw the line $x – 5y = –5$, consider the points (–5, 0) and (0, 1) which are on x – axis and y – axis respectively.

Line $x – 5y = –5$ divides the Cartesian plane into two parts. On taking any point say (0, 0) and substituting in the inequality $x – 5y < –5$,

$(0) – (0) = 0 > –5$ thus inequality is not true.

Thus, shaded region will be the part not containing the point (0, 0). Since inequality is strict inequality thus line $x – 5y = –5$ will be dotted line. Thus, graph of inequality will be as



Example 12: Graph the inequality $3y - 2x > 24$, where x and y are real numbers.

Solution: Given $3y - 2x > 24$

To draw the line $3y - 2x = 24$, consider the points (–12, 0) and (0, 8) which are on x – axis and y – axis respectively.

Line $3y - 2x = 24$ divides the Cartesian plane into two parts. On taking any point say (0, 0) and substituting in the inequality $3y - 2x > 24$,

$3(0) - 2(0) = 0 < 24$ thus inequality is not true.

Thus, shaded region will be the part not containing the point (0, 0). Since inequality is strict inequality thus line $3y - 2x = 24$ will be dotted line. Thus, graph of inequality will be as