1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 01 (Class XI, Semester - 1)
Module Name/Title	Linear inequalities - Part 2
Module Id	kemh_10602
Pre-requisites	Basic knowledge of plotting equation on graph and solving simultaneous equations.
Objectives	After going through this lesson, the learners will be able to understand the following: ● Difference between equation and inequality. ● Solve system of linear inequalities in two variables. ● Plot a graph of systems of linear inequalities. ● Describe the shaded region when inequalities are plotted graphically.
Keywords	Linear inequalities, Graphical solutions, Shaded region

2. Development Team

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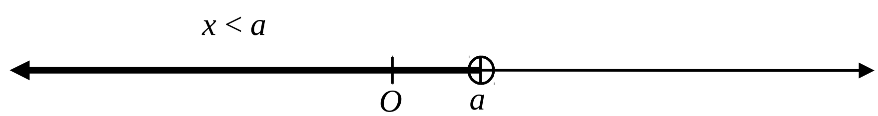
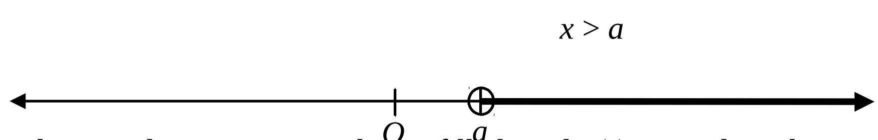
1. Graphical Representation of algebraic Solutions of Linear Inequalities in One Variable

Solution of a linear inequality in one variable can be represented on a number line. There are certain rules to draw number line and draw inequality.

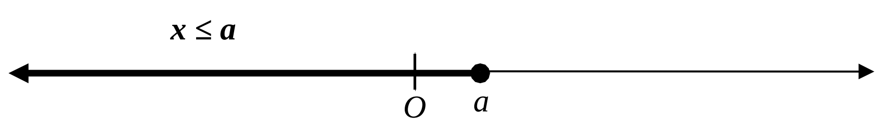
(i) If the inequality involves '>' or '<' then an open circle (o) is used on the number line to indicate that the number corresponding to the open circle is not included in the solution set. For example, to draw $x < a$



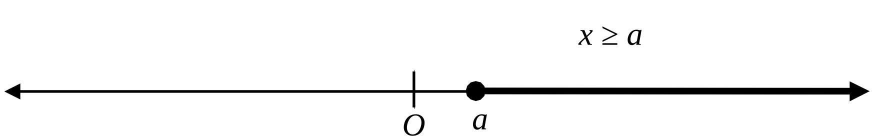
To draw $x > a$



(ii) If the inequality involves '≥' or '≤', then a filled circle (•) is used on the number line to indicate that the number corresponding to the filled circle is included in the solution set. For example, to draw $x \leq a$



To draw $x \geq a$



Example 1: Solve the inequality for real x: $3x + 7 > 5x - 9$ and represent the solution on number line.

Solution: Given $3x + 7 > 5x - 9$

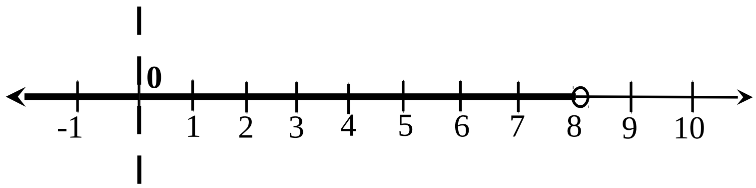
$\Rightarrow$ $7 + 9 > 5x - 3x$

$\Rightarrow$ $2x < 16$

$\Rightarrow$ $x < 8$

Thus, all real numbers x, which are less than 8, are the solutions of the given inequality.

Graphical representation of solution of given inequality on number line is



Example 2: Solve the inequality for real x: $-(5+2x) - 3 + 7x \leq 3(x - 2)$ and represent the solution on number line.

Solution: Given $\quad -(5+2x) - 3 + 7x \leq 3(x-2)$

$\Rightarrow \quad -5 - 2x - 3 + 7x \leq 3x - 6$

$\Rightarrow \quad -8 + 5x \leq 3x - 6$

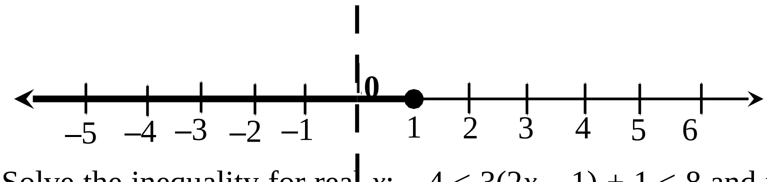
$\Rightarrow \quad 5x - 3x \leq 8 - 6$

$\Rightarrow \quad 2x \leq 2$

$\Rightarrow \quad x \leq 1$

Thus, all real numbers x, which are less than or equal to 1, are the solutions of the given inequality.

Graphical representation of solution of given inequality on number line is



Example 3: Solve the inequality for real x: $-4 < 3(2x - 1) + 1 < 8$ and represent the solution on number line.

Solution: Given $\quad -4 < 3(2x-1) + 1 < 8$

$\Rightarrow \quad -4 < 6x - 3 + 1 < 8$

$\Rightarrow \quad -4 < 6x - 2 < 8$

$\Rightarrow \quad -4 + 2 < 6x - 2 + 2 < 8 + 2$

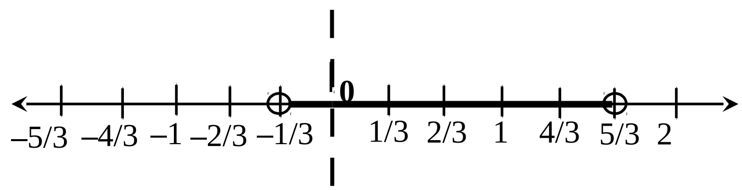
$\Rightarrow \quad -2 < 6x < 10$

$$\frac{-2}{6} < x < \frac{10}{6}$$

$$\frac{-1}{3} < x < \frac{5}{3}$$

Thus, all real numbers *x*, which are less than $\frac{5}{3}$ and more than $\frac{-1}{3}$, are the solutions of the given inequality.

Graphical representation of solution of given inequality on number line is



Example 4: Solve the inequality for real $x: \frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-8)}{5}$ and represent the solution on number line.

Solution: Given $\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-8)}{5}$

$$\Longrightarrow \frac{x}{2} \geq \frac{25x - 10 - 21x + 24}{15}$$

$$\Longrightarrow \frac{x}{2} \geq \frac{4x + 14}{15}$$

$$\Longrightarrow 15x \geq 2(4x + 14)$$

$$\Longrightarrow 15x \geq 8x + 28$$

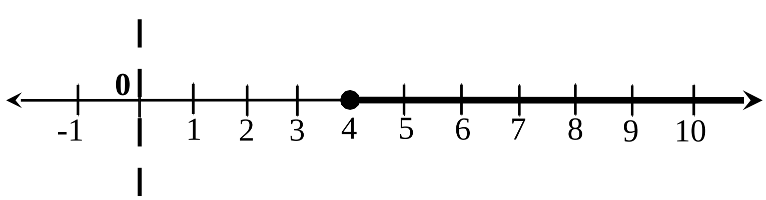
$$\Longrightarrow 15x - 8x \geq 28$$

$$\Longrightarrow 7x \geq 28$$

$$\Longrightarrow x \geq 4$$

Thus, all real numbers *x*, which are more than or equal to 4, are the solutions of the given inequality.

Graphical representation of solution of given inequality on number line is



Example 5: Solve the inequality for real $x: -2 < \frac{2-3x}{5} \leq 1 \left(x > 0\right)$ and represent the solution on number line.

Solution: Given $-2 < \frac{2-3x}{5} \leq 1$

$$\Longrightarrow \frac{2-3x}{5} \leq 1 \wedge -2 < \frac{2-3x}{5}$$

$$\Longrightarrow 2-3x \leq 5 \wedge -10 < 2-3x$$

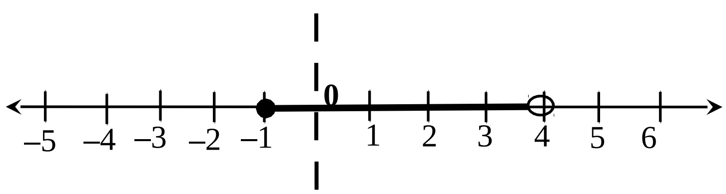
$$\Longrightarrow 2-5 \leq 3x \wedge 3x < 2+10$$

$$\Longrightarrow -3 \leq 3x \wedge 3x < 12$$

$$\Longrightarrow -1 \leq x \wedge x < 4$$

Thus, all real numbers *x*, which are more than or equal to –1 but less than 4, are the solutions of the given inequality.

Graphical representation of solution of given inequality on number line is



Example 6: Solve the inequality for real $x: \frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}, \left(x > 0\right)$ and represent the solution on number line.

Solution: Given $\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}$

$$\Longrightarrow \frac{4}{x+1} \leq 3 \wedge 3 \leq \frac{6}{x+1}$$

$$\Longrightarrow 4 \leq 3\left(x+1\right) \wedge 3\left(x+1\right) \leq 6$$

$$\Longrightarrow 4 \leq 3x+3 \wedge 3x+3 \leq 6$$

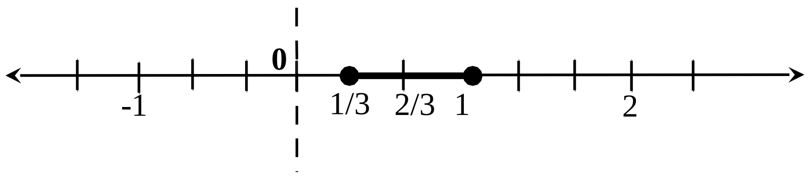
$$\Longrightarrow 4-3 \leq 3x \wedge 3x \leq 6-3$$

$$\Longrightarrow 1 \leq 3x \wedge 3x \leq 3$$

$$\Longrightarrow \frac{1}{3} \leq x \wedge x \leq 1$$

Thus, all real numbers x, which are less than or equal to 1 and more than or equal to $\frac{1}{3}$, are the solutions of the given inequality.

Graphical representation of solution of given inequality on number line is



Example 7: Solve the inequality for real x: $|x - 2| \leq 5$ and represent the solution on number line.

Solution: Given $|x - 2| \leq 5$

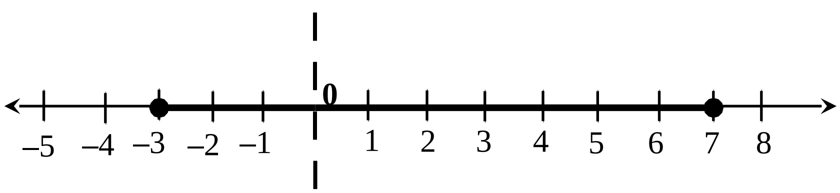
$\Rightarrow$ $-5 \leq x - 2 \leq 5$

$\Rightarrow$ $-5 + 2 \leq x - 2 + 2 \leq 5 + 2$

$\Rightarrow$ $-3 \leq x \leq 7$

Thus, all real numbers x, which are less than or equal to 7 and more than or equal to –3, are the solutions of the given inequality.

Graphical representation of solution of given inequality on number line is



Example 8: Solve the inequality for real x: $|2x + 7| > 13$ and represent the solution on number line.

Solution: Given $|2x + 7| > 13$

⇒ $2x + 7 > 13$ or $2x + 7 < -13$

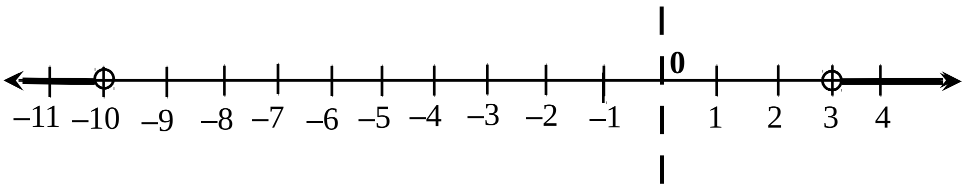
⇒ $2x > 13 - 7$ or $2x < -13 - 7$

⇒ $2x > 6$ or $2x < -20$

⇒ $x > 3$ or $x < -10$

Thus, all real numbers x, which are more than 3 or less than –10, are the solutions of the given inequality.

Graphical representation of solution of given inequality on number line is



Example 9: Solve the inequality for real $x : \vee \frac{x-3}{2} \vee \geq 4$ and represent the solution on number line.

Solution: Given ¿ $\frac{x-3}{2} \vee \geq 4$

$\Longrightarrow \vee x - 3 \vee \geq 8$

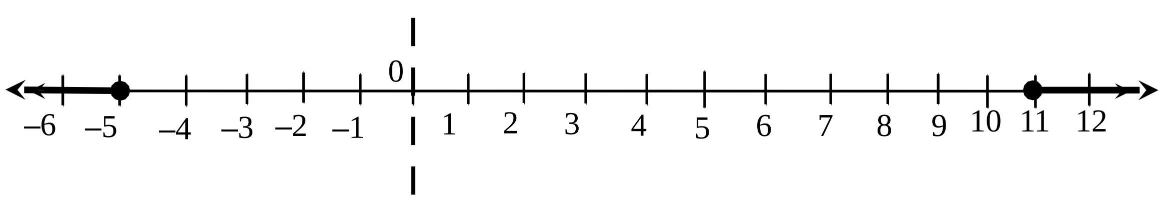
⇒ $x - 3 \geq 8$ or $x - 3 \leq -8$

⇒ $x \geq 8 + 3$ or $x \leq -8 + 3$

⇒ $x \geq 11$ or $x \leq -5$

Thus, all real numbers x, which are more than or equal to 11 or less than or equal to –5, are the solutions of the given inequality.

Graphical representation of solution of given inequality on number line is



Example 10: Solve the following inequalities for real x and represent the solution graphically on number line: $3x - 7 > 2(x - 6)$, $6 - x > 11 - 2x$.

Solution: Given first inequality as: $3x - 7 > 2(x - 6)$

$\Longrightarrow \quad 3x - 7 > 2x - 12$

$\Longrightarrow \quad 3x - 2x > -12 + 7$

$\Longrightarrow \quad x > -5 \qquad$ ---(1)

Given second inequality as: $6 - x > 11 - 2x$

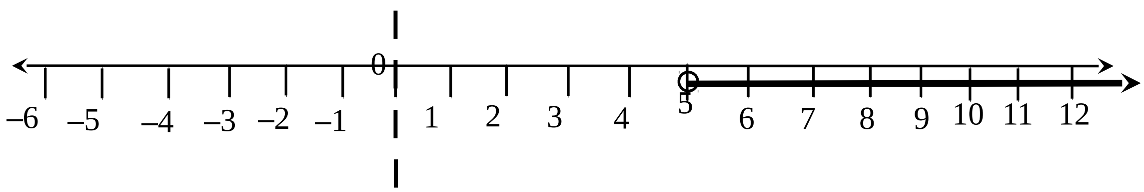
$\Rightarrow \quad 2x - x > 11 - 6$

$\Rightarrow \quad x > 5 \qquad$ ---(2)

Thus, combining (1) and (2), we get, $x > 5$

Thus, all real numbers x, which are more than 5, are the solutions of the given inequalities.

Graphical representation of solution of given inequalities on number line is



Example 11: Solve the following inequalities for real x and represent the solution graphically on number line:

$5(2x - 7) - 3(2x + 3) \leq 0,\ 2x + 15 \leq 5x + 30$

Solution: Given first inequality as: $5(2x - 7) - 3(2x + 3) \leq 0$

$\Longrightarrow \quad 10x - 35 - 6x - 9 \leq 0$

$\Longrightarrow \quad 4x - 44 \leq 0$

$\Longrightarrow \quad x - 11 \leq 0$

$\Longrightarrow \quad x \leq 11 \qquad$ ---(1)

Given second inequality as: $2x + 15 \leq 5x + 30$

$\Rightarrow \quad 15 - 30 \leq 5x - 2x$

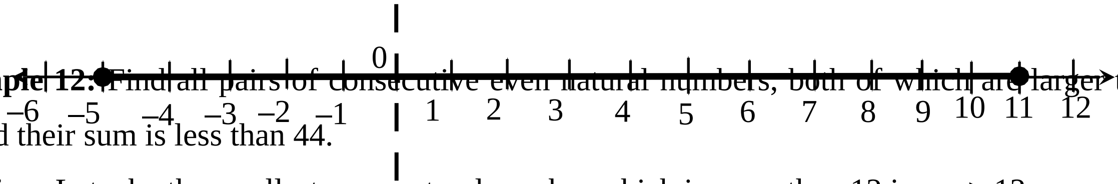
$\Rightarrow \quad -15 \leq 3x$

$\Rightarrow \quad -5 \leq x \qquad$ ---(2)

Thus, combining (1) and (2), we get, $-5 \leq x \leq 11$

Thus, all real numbers x, which are more than or equal to –5 and less than or equal to 11, are the solutions of the given inequalities.

Graphical representation of solution of given inequalities on number line is



Example 12: Find all pairs of consecutive even natural numbers, both of which are larger than 12 and their sum is less than 44.

Solution: Let x be the smallest even natural number which is more than 12 i.e., $x > 12$.

So, consecutive even natural number will be $x + 2$.

Given that their sum is less than 44.

$\Rightarrow \quad x + x + 2 < 44$

$\Rightarrow \quad 2x + 2 < 44$

$\Rightarrow \quad 2x < 42$

$\Rightarrow \quad x < 21$

Thus x lies between 12 and 21 or $12 < x < 21$

So, pairs of consecutive even natural numbers whose sum is less than 44 are

(14, 16), (16, 18), (18, 20) and (20, 22)

Example 13: IQ of a person is given by the formula

$$IQ = \frac{MA}{CA} \times 100$$

Where MA is mental age and CA is chronological age. If $60 \le IQ \le 120$ for a group of 10 years old children, find the range of their mental age.

Solution: Given that for a group of 10 years old children, $60 \le IQ \le 120$.

Here CA = 10

$$60 \le \frac{MA}{CA} \times 100 \le 120$$

$$60 \le \frac{MA}{10} \times 100 \le 120$$

$$6 \le MA \le 12$$

Thus range of mental age is $6 \le MA \le 12$.

Example 14: The heights of two-thirds of the members of a population satisfy the inequality

$$¿\frac{h-65.5}{2.4}\vee\leq 1$$

Where, height h is measured in inches. Determine the interval on the real number line in which these heights lie.

Solution: Given that heights of two-thirds of the members of a population satisfy

$$¿\frac{h-65.5}{2.4}\vee\leq 1$$

$$\Longrightarrow -1\leq\left(\frac{h-65.5}{2.4}\right)\leq 1$$

$\Longrightarrow$ $-2.4 \leq h - 65.5 \leq 2.4$

$\Longrightarrow$ $-2.4 + 65.5 \leq h - 65.5 + 65.5 \leq 2.4 + 65.5$

$\Longrightarrow$ $63.1 \leq h \leq 67.9$

Thus range of heights is [63.1, 67.9].

Example 15: The revenue from selling x units of a product is $R = 24.5x$. The cost of producing x units is $C = 15x + 15000$. To obtain a profit, the revenue must be greater than the cost. For what values of x will this product return a profit?

Solution: Given Revenue is $R = 24.5x$ and Cost is $C = 15x + 15000$

For profit, R > C

$\Rightarrow 24.5x > 15x + 15000$

$\Rightarrow 24.5x - 15x > 15000$

$\Rightarrow 9.5x > 15000$

$$x>\frac{15000}{9.5}$$

$\Rightarrow x > 1578.947$

Hence, the number of units should be such that $x \geq 1579$.

Example 16: Rabbit has a lifespan of 9 to 15 years. The inequality $|x - 12| \leq c$ gives the number of years a rabbit may live. What is the value of c?

Solution: Given $|x - 12| \le c$

$\Rightarrow \quad -c \le (x - 12) \le c$

$\Rightarrow \quad -c + 12 \le x - 12 + 12 \le c + 12$

$\Rightarrow \quad -c + 12 \le x \le c + 12 \quad$ --- (1)

Since lifespan of a rabbit is 9 to 15 years.

$\Rightarrow \quad 9 \le x \le 15 \quad$ --- (2)

On comparing (1) and (2), we get

$-c + 12 = 9$ and $c + 12 = 15$

$\Rightarrow \quad c = 12 - 9 = 3$ or $c = 15 - 12 = 3$

$\Rightarrow \quad c = 3$

Example 17: The length in meters of one side of a rectangle is 3 times another side of rectangle. If the perimeter of the rectangle is at least 64 m, find the minimum length of all sides.

Solution: Let length of smaller side of rectangle be x.

$\Rightarrow$ Length of bigger side of rectangle = $3x$

Thus, Perimeter of rectangle = $2(x + 3x) = 2(4x) = 8x$

Given that Perimeter ≥ 64

$\Rightarrow 8x \ge 64$

$\Rightarrow x \ge 8$

Thus minimum length of two sides is 8 m and 24 m respectively.

Example 18: Aman wants to construct a triangle of three different lengths from a single piece of rod of length 87 cm. The length of second side is to be 3 cm longer than the shortest side and the length of third side is to be twice as long as the shortest side. What are the possible lengths of the shortest side if length of third side is to be at least 5 cm longer than the second?

Solution: Let length of shortest side be x.

$\Rightarrow$ Length of second side = $x + 3$, Length of third side = $2x$

Length of rod is 87 cm.

$\Rightarrow$ Perimeter of triangle ≤ 87

$\Rightarrow x + x + 3 + 2x \leq 87$

$\Rightarrow 4x + 3 \leq 87$

$\Rightarrow 4x \leq 87 - 3$

$\Rightarrow 4x \leq 84$

$\Rightarrow x \leq 21$ ---(1)

Given: length of third side is to be at least 5 cm longer than the second.

i.e., $2x \geq x + 3 + 5$

$\Rightarrow x \geq 8$ ---(2)

Thus, combining (1) and (2), we get

$8 \leq x \leq 21$

Thus, length of shortest side should be greater than or equal to 8 and less than or equal to 21.