1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 01 (Class XI, Semester - 1)
Module Name/Title	Linear inequalities - Part 1
Module Id	kemh_10601
Pre-requisites	Basic knowledge of plotting equation on graph and solving simultaneous equations.
Objectives	After going through this lesson, the learners will be able to understand the following: ● Difference between equation and inequality. ● Solve system of linear inequalities in two variables. ● Plot a graph of systems of linear inequalities. ● Describe the shaded region when inequalities are plotted graphically.
Keywords	Linear inequality, Graphical solution, Shaded region

2. Development Team

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Table of Contents :

1. Introduction

We already know that a statement can be written in the form of an equation. In an equation, two expressions are written with sign of equality ‘=’ between them. For example, cost of 3 mangoes and 4 apples is ` 100. If cost of one mango and one apple is represented by variables ‘x’ and ‘y’, then equation will be of the form, $3x + 4y = 100$.

On the other side, inequalities are the statements involving symbols such as ‘<’, ‘>’, ‘≤’, ‘≥’. For example, if cost of one chair is ` 45 then number of chairs which can be bought in ` 200 will be expressed as $45x \leq 200$ where x is the number of chairs bought. Thus inequalities are the statement in which both sides of the statement are not equal to each other.

2. Inequalities

Two real numbers or algebraic expressions involving symbols like ‘<’, ‘>’, ‘≤’, ‘≥’ form an inequality.

Reading of inequality symbols

Symbol	**Read as**
<	Less than
>	More than
≤	Less than or equal to
≥	More than or equal to

There are two types of inequalities.

(i) Numerical inequality
(ii) Literal inequality

Inequalities having numerals on both the sides of the symbol are known as numerical inequalities. For example, $3 < 6$; $8 > 2$. On the other side, inequalities having variable on one side of the symbol are known as literal inequalities. For example, $x < 5$; $y \geq 7$.

Inequalities can be further classified according to symbols.

a) **Strict inequality:** Inequalities which contain symbols '<' or '>' are called strict inequalities such as $ax < b$ or $ax > b$.

b) **Slack inequality:** Inequalities which contain symbols '≤' or '≥' are called slack inequalities such as $ax \leq b$ or $ax \geq b$.

Classification of inequalities according to degree

a) **Linear inequality:** When exponent of each variable is one then inequality is termed as linear inequality such as $ax + by \geq c$ or $ax < b$.

b) **Non-linear inequality:** When exponent of any of the variable involved is more than one then inequality is termed as non-linear inequality such as $ax^2 + bx + c \geq d$.

3. Algebraic Solutions of Linear Inequalities in One Variable

As it is known that every equation in one variable has unique solution which satisfies the equation also. But linear inequality can have one or more than solutions. These are the values of variable x, which satisfies the linear inequality. Let us consider the linear inequality $5x \leq 15$, where x is a natural number.

When $x = 1$ then $5x = 5 \times 1 = 5 \leq 15$ is true thus $x = 1$ is solution of inequality.

When $x = 2$ then $5x = 5 \times 2 = 10 \leq 15$ is true thus $x = 2$ is solution of inequality.

When $x = 3$ then $5x = 5 \times 3 = 15 = 15$ is true thus $x = 3$ is solution of inequality.

When $x = 4$ then $5x = 5 \times 4 = 20 > 15$ is false thus $x = 4$ is not the solution of inequality.

Thus all the solutions {1, 2, 3} of inequality form a set and called as the solution set.

Example 1: Find the solution set of linear inequality $5x \leq 15$, where x is an integer.

Solution: Integers are {…, –3, –2, –1, 0, 1, 2, 3 …}

When $x = 0$ then $5x = 5 \times 0 = 0 \leq 15$ is true thus $x = 0$ is solution of inequality.

When $x = -1$(negative number) then $5x = 5 \times (-1) = -5 \leq 15$ is true thus $x = -1$ is solution of inequality. Even inequality is satisfied for all negative integers.

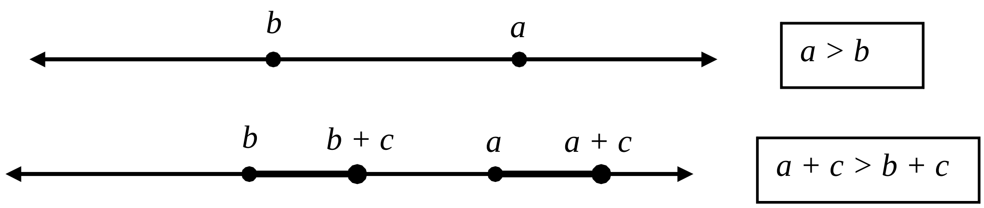
We have seen earlier that inequality is satisfied for integers 1, 2 and 3 only.

Thus solution set = {…, –3, –2, –1, 0, 1, 2, 3}

Properties

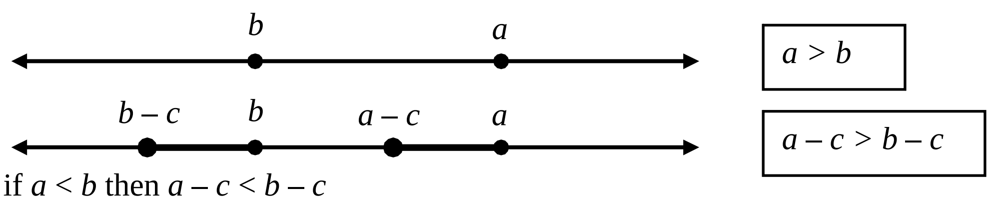
There are certain properties which are satisfied by linear inequality. Solution of any linear inequality can be obtained using these properties.

- **Transitive Property:** For any real numbers a, b and c, if $a < b$ and $b < c$ then $a < c$.
- **Addition of two inequalities:** For any real numbers a, b, c and d, if $a < b$ and $c < d$ then $a + c < b + d$.
- **Addition of a constant to an inequality:** On adding same number to both sides of inequality, sign of inequality does not change. For any real numbers a, b and c, if $a > b$ then $a + c > b + c$



  Similarly, if $a < b$ then $a + c < b + c$

- **Subtraction property of inequality:** On subtracting same number from both sides of inequality, sign of inequality does not change. For any real numbers a, b and c, if $a > b$ then $a - c > b - c$



  Similarly, if $a < b$ then $a - c < b - c$

- **Multiplication property of inequality:** On multiplying same positive number to both sides of inequality, sign of inequality remains same.

  However, sign of inequality changes when a negative number is multiplied to both sides of inequality.

  For any real numbers a, b and c,

$$\text{When } c \text{ is positive number} \begin{cases} \text{if } a > b \text{ then } ac > bc \\ \text{if } a < b \text{ then } ac < bc \end{cases}$$

$$\text{When } c \text{ is negative number} \begin{cases} \text{if } a > b \text{ then } ac < bc \\ \text{if } a < b \text{ then } ac > bc \end{cases}$$

- **Division property of inequality:** On dividing both sides of inequality by same positive number, sign of inequality remains same.

However, sign of inequality changes when both sides of inequality are divided by same negative number.

For any real numbers a, b and c,

$$When\ c\ is\ positive\ number\ \{if\ a>b\ then\ \frac{a}{c}>\frac{b}{c}\ if\ a<b\ then\ \frac{a}{c}<\frac{b}{c}$$

$$When\ c\ is\ negative\ number\ \{if\ a>b\ then\ \frac{a}{c}<\frac{b}{c}\ if\ a<b\ then\ \frac{a}{c}>\frac{b}{c}$$

Note: All the properties of linear inequalities mentioned above are true if symbol < (strict inequality) is replaced by symbol ≤ (slack inequality) and symbol > is replaced by symbol ≥.

Important result

- If a and b are two real number and $b \neq 0$, then

$$(i)\ ab>0 \vee \frac{a}{b}>0 \Rightarrow a \wedge b\ are\ of\ same\ sign.$$

$$(ii)\ ab<0 \vee \frac{a}{b}<0 \Rightarrow a \wedge b\ are\ of\ opposite\ sign.$$

Example 2: Find the solution set of linear inequality $25x < 101$, when (i) x is an integer (ii) x is a natural number.

Solution: Given inequality is $25x < 101$

$$\Longrightarrow \frac{25x}{25}<\frac{101}{25}\ \{Using\ multiplication\ property\}$$

$$\Longrightarrow x<\frac{101}{25}$$

(i) When x is an integer, then solutions of given inequality are …, –3, –2, –1, 0, 1, 2, 3, and 4.

Thus, solution set of the given inequality is {…, –3, –2, –1, 0, 1, 2, 3, 4}.

(ii) When x is a natural number, then solutions of given inequality are 1, 2, 3, and 4.

Thus, solution set of the given inequality is {1, 2, 3, 4}.

Example 3: Find the solution set of linear inequality $-12x \geq 35$, when (i) x is an integer (ii) x is a natural number.

Solution: Given inequality is $-12x \geq 35$

$$\Longrightarrow \frac{-12x}{12} \geq \frac{35}{12}$$

$$\Longrightarrow -x \geq \frac{35}{12}$$

$$\Longrightarrow (-1)(-x) \leq (-1)\frac{35}{12} \{Multiplication\, by\, a\, negative\, number\}$$

$$\Longrightarrow x \leq \frac{-35}{12}$$

(i) When x is an integer, then integers satisfying given inequality are …, –5, –4 and –3.

Thus, solution set of the given inequality is {…, –5, –4, –3}.

(ii) When x is a natural number, then, no natural number satisfies the given inequality.

Thus, solution set of the given inequality is empty.

Example 4: Solve the linear inequality for real x: $4x + 3 \geq 2x + 17$

Solution: Given $4x + 3 \geq 2x + 17$

$\Rightarrow$ $4x + 3 - 2x \geq 2x + 17 - 2x$ {Subtracting $2x$ from both sides}

$\Rightarrow$ $2x + 3 \geq 17$

$\Rightarrow$ $2x + 3 - 3 \geq 17 - 3$ {Subtracting 3 from both sides}

$\Rightarrow$ $2x \geq 14$

$\Rightarrow$ $x \geq 7$ {Dividing both sides by 2}

Thus, all real numbers x, which are more than or equal to 7, are the solutions of the given inequality.

Hence, solution set of given inequality is $[7, \infty)$.

Example 5: Solve the linear inequality for real x: $2(2x + 3) - 10 < 6(x - 2)$

Solution: Given $2(2x + 3) - 10 < 6(x - 2)$

$\Rightarrow \quad 4x + 6 - 10 < 6x - 12$

$\Rightarrow \quad 4x - 4 < 6x - 12$

$\Rightarrow \quad 12 - 4 < 6x - 4x$

$\Rightarrow \quad 8 < 2x$

$\Rightarrow \quad x > 4$

Thus, all real numbers x, which are more than 4, are the solutions of the given inequality.

Hence, solution set of given inequality is $(4, \infty)$.

Example 6: Solve the linear inequality for real x: $3(2 - x) \geq 2(1 - x)$

Solution: Given $3(2 - x) \geq 2(1 - x)$

$\Rightarrow \quad 6 - 3x \geq 2 - 2x$

$\Rightarrow \quad 6 - 2 \geq 3x - 2x$

$\Rightarrow \quad 4 \geq x$

$\Rightarrow \quad x \leq 4$

Thus, all real numbers x, which are less than or equal to 4, are the solutions of the given inequality.

Hence, solution set of given inequality is $(-\infty, 4]$.

Example 7: Solve the linear inequality for real x: $x+\frac{x}{4}+\frac{x}{5}<58$

Solution: Given $x+\frac{x}{4}+\frac{x}{5}<58$

$\Longrightarrow x\left(1+\frac{1}{4}+\frac{1}{5}\right)<58$

$\Longrightarrow x\left(\frac{20+5+4}{20}\right)<58$

$\Longrightarrow x\left(\frac{29}{20}\right)<58$

$$\Longrightarrow x < \frac{58 \times 20}{29}$$

$$\Longrightarrow x < 40$$

Thus, all real numbers *x*, which are less than 40, are the solutions of the given inequality.

Hence, solution set of given inequality is (–∞, 40).

Example 8: Solve the linear inequality for real *x*: $1 + \frac{5x}{2} \geq x - 2$

Solution: Given $1 + \frac{5x}{2} \geq x - 2$

$$\Longrightarrow \frac{5x}{2} - x \geq -2 - 1$$

$$\Longrightarrow \frac{5x - 2x}{2} \geq -3$$

$$\Longrightarrow \frac{3x}{2} \geq -3$$

$$\Longrightarrow 3x \geq -6$$

$$\Longrightarrow x \geq -2$$

Thus, all real numbers *x*, which are more than or equal to –2, are the solutions of the given inequality.

Hence, solution set of given inequality is [–2, ∞).

Example 9: Solve the linear inequality for real *x*: $\frac{-1}{5}(2x-4) < \frac{2}{3}(x-2)$

Solution: Given $-\frac{1}{5}(2x-4) < \frac{2}{3}(x-2)$

$$\Longrightarrow \frac{-2x}{5} + \frac{4}{5} < \frac{2x}{3} - \frac{4}{3}$$

$$\Longrightarrow \frac{4}{3} + \frac{4}{5} < \frac{2x}{3} + \frac{2x}{5}$$

$$\Longrightarrow \frac{20+12}{15} < \frac{10x+6x}{15}$$

$$\Longrightarrow \frac{32}{15} < \frac{16x}{15}$$

$$\Longrightarrow \frac{32}{16} < x$$

$$\Longrightarrow 2 < x \vee x > 2$$

Thus, all real numbers x, which are more than 2, are the solutions of the given inequality.

Hence, solution set of given inequality is $(2, \infty)$.

Double Inequality or Compound Inequality

A compound inequality or double inequality is combination of two simple inequalities joined by the word "and" or "or".

For example,

$-3 \leq x < 2$ is compound inequality using "and" i.e. $-3 \leq x$ and $x < 2$.

Example 10: Solve the linear inequality for real x: $-2 < 5x + 8 \leq 18$

Solution: Given $-2 < 5x + 8 \leq 18$

On subtracting 8 from each part, we get

$-2 - 8 < 5x + 8 - 8 \leq 18 - 8$

$\Rightarrow \quad -10 < 5x < 10$

$\Rightarrow \quad -2 < x < 2$ {On dividing each part by 5}

Thus, all real numbers x, which are more than –2 and less than 2, are the solutions of the given inequality.

$\Rightarrow$ Hence, solution set of given inequality is $(-2, 2)$.

Example 11: Solve the linear inequality for real x: $3x + 2 < 8$ or $2x - 9 > 3$.

Solution: Given $3x + 2 < 8$ or $2x - 9 > 3$

$\Rightarrow$ $3x < 8 - 2$ or $2x > 3 + 9$

$\Rightarrow$ $3x < 6$ or $2x > 12$

$\Rightarrow$ $x < 2$ or $x > 6$

Thus, all real numbers x, which are less than 2 or more than 6, are the solutions of the given inequality.

$\Rightarrow$ Hence, solution set of given inequality is $(-\infty, 2) \cup (6, \infty)$.

Example 12: Solve the linear inequality for real $x: -5 < \frac{2-3x}{5} \leq 4$.

Solution: Given $-5 < \frac{2-3x}{5} \leq 4$

$\Longrightarrow -25 < 2-3x \leq 20$ {Multiplying all parts by 5}

$\Longrightarrow -25-2 < 2-3x-2 \leq 20-2$ {Subtracting 2 from all parts}

$\Longrightarrow -27 \leftarrow 3x \leq 18$

$\Longrightarrow \frac{-27}{-3} > \frac{-3x}{-3} \geq \frac{18}{-3} \{Division\, by\, negative\, number\}$

$\Longrightarrow 9 > x \geq -6$

Thus, all real numbers x, which are less than 9 and more than or equal to –6, are the solutions of the given inequality.

$\Rightarrow$ Hence, solution set of given inequality is $[-6, 9)$.

Inequalities involving absolute values

If a is any positive real number, i.e., $a > 0$, then

(i) $|x| < a \Leftrightarrow -a < x < a$

(ii) $|x| \leq a \Leftrightarrow -a \leq x \leq a$

(iii) $|x| > a \Leftrightarrow x < -a$ or $x > a$

(iv) $|x| \geq a \Leftrightarrow x \leq -a$ or $x \geq a$

Example 13: Solve the linear inequality for real x: $|x - 2| \leq 5$

Solution: Given $|x - 2| \leq 5$

$\Rightarrow$ $-5 \leq x - 2 \leq 5$

$\Rightarrow$ $-5 + 2 \leq x - 2 + 2 \leq 5 + 2$ {Adding 2 to all parts}

$\Rightarrow$ $-3 \leq x \leq 7$

Thus, all real numbers x, which are less than or equal to 7 and more than or equal to –3, are the solutions of the given inequality.

⇒ Hence, solution set of given inequality is $[-3, 7]$

Example 14: Solve the linear inequality for real x: $2 < |x - 2|$

Solution: Given $2 < |x - 2|$ or $|x - 2| > 2$

⇒ $x - 2 > 2$ or $x - 2 < -2$

⇒ $x - 2 + 2 > 2 + 2$ or $x - 2 + 2 < -2 + 2$ {Adding 2 to all parts}

⇒ $x > 4$ or $x < 0$

Thus, all real numbers x, which are less than 0 or more than 4, are the solutions of the given inequality.

⇒ Hence, solution set of given inequality is $(-\infty, 0) \cup (4, \infty)$.

Example 15: Solve the linear inequality for real x: $|2x - 3| < 15$

Solution: Given $|2x - 3| < 15$

⇒ $-15 < 2x - 3 < 15$

⇒ $-15 + 3 < 2x - 3 + 3 < 15 + 3$ {Adding 3 to all parts}

⇒ $-12 < 2x < 18$

⇒ $-6 < x < 9$ {Dividing all parts by 2}

Thus, all real numbers x, which are less than 9 and more than –6, are the solutions of the given inequality.

⇒ Hence, solution set of given inequality is $(-6, 9)$.

Example 16: Solve the linear inequality for real x: $|x - 10| \geq 3$

Solution: Given $|x - 10| \geq 3$

⇒ $x - 10 \geq 3$ or $x - 10 \leq -3$

⇒ $x - 10 + 10 \geq 3 + 10$ or $x - 10 + 10 \leq -3 + 10$ {Adding 10 to all parts}

⇒ $x \geq 13$ or $x \leq 7$

Thus, all real numbers x, which are less than or equal to 7 or more than or equal to 13, are the solutions of the given inequality.

⇒ Hence, solution set of given inequality is $(-\infty, 7] \cup [13, \infty)$.