1. **Details of Module and its structure**

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics (Class XI, Semester - 1)
Module Name/Title	Trigonometry: Part 7
Module Id	kemh_10307
Pre-requisites	Knowledge about Trigonometric Functions.
Objectives	After going through this lesson, the learners will be able to understand the following: 1.Introduction 2.Trigonometric equations 3. Summary
Keywords	Trigonometric Equations, Principal Solution, General Solution

2. **Development Team**

Role	**Name**	**Affiliation**
National MOOC Coordinator (NMC)	Prof. Amarendra P. Behera	CIET, NCERT, New Delhi
Program Coordinator	Dr. Indu Kumar	CIET, NCERT, New Delhi
Course Coordinator (CC) / PI	Prof. Til Prasad Sarma	DESM, NCERT, New Delhi
Course Co-Coordinator	Ms. Anjali Khurana	CIET, NCERT, New Delhi
Subject Matter Expert (SME)	Ms. Kiran Seth	HOD (Math), Amity International School, Saket, New Delhi.
Review Team	Dr. S.K.S. Gautum	Retd. Professor DESM, NCERT, New Delhi

TABLE OF CONTENTS

1. Introduction

In this module we will discuss trigonometric equations, their principal and general solution.

Equations involving trigonometric functions of a variable are called trigonometric equations. In this section, we shall find the solutions of such equations. It is already discussed in the previous modules that the values of sinx and cosx repeat after an interval of 2π and the values of tanx repeat after an interval of π. The solutions of trigonometric equations for which $0 \le x < 2\pi$ are called principal solutions. The expression involving integer 'n' which gives all solutions of a trigonometric equation is called the general solution. We shall use 'Z' to denote the set of integers.

2. Trigonometric Equations

Theorem 1:

For any real numbers x and y,

$\sin x = \sin y \Rightarrow x = n\pi + (-1)^n y,\ n \in Z$

Proof:

$If \ \ \sin x = \sin y$

$\Rightarrow \sin x - \sin y = 0$

$\Rightarrow 2\cos(\frac{x+y}{2})\sin(\frac{x-y}{2}) = 0$

which gives $\cos(\frac{x+y}{2}) = 0 \ or \sin(\frac{x-y}{2}) = 0$

Therefore $\frac{x+y}{2}=(2n+1)\frac{\pi}{2}$ *or* $\frac{x-y}{2}=n\pi$, where n$\in$ Z.

i.e. $x=(2n+1)\pi-y$ *or* $x=2n\pi+y$, where n$\in$ Z.

Hence $x=(2n+1)\pi+(-1)^{2n+1}y$ *or* $x=2n\pi+(-1)^{2n}y$ where n$\in$ Z.

Combining the two results, we get

$x=n\pi+(-1)^{n}y$, where n$\in$ Z.

Theorem 2:

For any real numbers x and y,

$\cos x=\cos y \Rightarrow x=2n\pi\pm y$, where n$\in$ Z.

Proof:

If $\cos x=\cos y$, then

$\cos x-\cos y=0$

i.e. $-2\sin(\frac{x+y}{2})\sin(\frac{x-y}{2})=0$

Thus $\sin(\frac{x+y}{2})=0$ *or* $\sin(\frac{x-y}{2})=0$

Therefore $\frac{x+y}{2}=n\pi$ *or* $\frac{x-y}{2}=n\pi$, where n$\in$ Z.

i.e. $x=2n\pi-y$ *or* $x=2n\pi+y$, where n$\in$ Z.

Hence $x=2n\pi\pm y$, where n$\in$ Z.

Theorem 3:

Prove that if x and y are not odd multiple of $\frac{\pi}{2}$, then $\tan x = \tan y \Rightarrow x = n\pi + y,$ where n$\in$ Z.

Proof: If $\tan x = \tan y \Rightarrow \tan x - \tan y = 0$

Or $\frac{\sin x \cos y - \cos x \sin y}{\cos x \cos y} = 0$

which gives $\sin(x-y) = 0$.

Therefore $x - y = n\pi, i.e.,\ x = n\pi + y$, where n$\in$ Z.

Example 1: Find the principal solutions of the equation $\tan x = -\frac{1}{\sqrt{3}}$.

Solution:

We know that, $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$.

Thus, $\tan(\pi - \frac{\pi}{6}) = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$

and $\tan(2\pi - \frac{\pi}{6}) = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$

Thus, $\tan\frac{5\pi}{6} = \tan\frac{11\pi}{6} = -\frac{1}{\sqrt{3}}$

Therefore, principal solutions are $\frac{5\pi}{6}$ *and* $\frac{11\pi}{6}$.

Example 2: Find the solution of $\sin x = -\frac{\sqrt{3}}{2}$.

Solution: We have

$$\sin x = -\frac{\sqrt{3}}{2} = -\sin\frac{\pi}{3}$$

$$= \sin(\pi + \frac{\pi}{3})$$

$$= \sin\frac{4\pi}{3}$$

which gives

$x = n\pi + (-1)^n \frac{4\pi}{3}$, where $n \in$ Z.

Example 3: Find the principal solutions of $\sec x = 2$.

Solution: We have

$$\sec x = 2$$

$\Rightarrow \cos x = \frac{1}{2}$ which is positive so x lies in first or fourth quadrant.

Therefore, $\cos x = \frac{1}{2} = \cos\frac{\pi}{3} \, or \cos\frac{5\pi}{3}$

Hence principal solutions are $x = \frac{\pi}{3} \, or \, \frac{5\pi}{3}$.

Example 4: Solve for x, if $\cot x = -\sqrt{3}$.

Solution: We have

$$\cot x = -\sqrt{3}$$

$$\Rightarrow \tan x = -\frac{1}{\sqrt{3}}$$

$\Rightarrow \tan x = \frac{5\pi}{6} \, or \, \frac{11\pi}{6}$

$\Rightarrow x = \frac{5\pi}{6} \, or \, \frac{11\pi}{6}$.

So, principal solutions are $x = \frac{5\pi}{6} \, or \, \frac{11\pi}{6}$

And the general solution is $x = n\pi + \frac{5\pi}{6}$, where n$\in$ Z.

Example 5: Solve for x, if $\cos 4x = \cos 2x$.

Solution: We have

$\cos 4x = \cos 2x$

$\Rightarrow 4x = 2n\pi \pm 2x$

$\Rightarrow 4x - 2x = 2n\pi \; or \; 4x + 2x = 2n\pi$

$\Rightarrow 2x = 2n\pi \; or \; 6x = 2n\pi$

$\Rightarrow x = n\pi \; or \; x = \frac{1}{3} n\pi$

Example 6: Solve for x, if $\cos 3x + \cos x - \cos 2x = 0$.

Solution: Here $\cos 3x + \cos x - \cos 2x = 0$.

$$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0$$

$$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$$

$$\Rightarrow \cos 2x(2\cos x - 1) = 0$$

$$\Rightarrow either \cos 2x = 0 \; or \; (2\cos x - 1) = 0$$

$$\Rightarrow \textit{either } 2x = (2n+1)\frac{\pi}{2} \textit{ or } \cos x = \frac{1}{2} = \cos\frac{\pi}{3}, \text{ where } n \in \text{ Z.}$$

$$\Rightarrow \textit{either } x = (2n+1)\frac{\pi}{4} \textit{ or } x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in \text{ Z.}$$

3. Summary

In this module, trigonometric equations were taken and their solution derived. Principal solution and general solution of trigonometric equations were discussed.