1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics (Class XI, Semester - 1)
Module Name/Title	Trigonometry: Part 6
Module Id	kemh_10306
Pre-requisites	Knowledge about Trigonometric Functions.
Objectives	After going through this lesson, the learners will be able to understand the following: 1. Introduction 2. Transformation of sum and difference of trigonometric functions into product of trigonometric functions. 3. Transformation of the product of trigonometric functions into sum or difference of trigonometric functions. 4. Summary
Keywords	Trigonometric Functions

2. Development Team

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1. Introduction

In this module we will discuss of transformation of sum and difference of trigonometric functions into product of trigonometric functions. Also we will discuss how to Transformation of the product of trigonometric functions into sum or difference of trigonometric functions.

2. Transformation of sum and difference of trigonometric functions into product of trigonometric functions.

(i) $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

(ii) $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

(iii) $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

(iv) $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

We know that

$$\cos(x+y) = \cos x \cos y - \sin x \sin y \quad \text{... (1)}$$

and $$\cos(x-y) = \cos x \cos y + \sin x \sin y \quad \text{... (2)}$$

Adding and subtracting (1) and (2), we get

$$\cos(x+y) + \cos(x-y) = 2 \cos x \cos y \quad \text{... (3)}$$

and $$\cos(x+y) - \cos(x-y) = -2 \sin x \sin y \quad \text{... (4)}$$

Further $$\sin(x+y) = \sin x \cos y + \cos x \sin y \quad \text{... (5)}$$

and $$\sin(x-y) = \sin x \cos y - \cos x \sin y \quad \text{... (6)}$$

Adding and subtracting (5) and (6), we get

$$\sin(x+y) + \sin(x-y) = 2 \sin x \cos y \quad \text{... (7)}$$

$$\sin(x+y) - \sin(x-y) = 2\cos x \sin y \quad \text{... (8)}$$

Substituting the values of x and y in (3), (4), (7) and (8), we get

$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$

$$\sin\theta + \sin\phi = 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\sin\theta - \sin\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$

Since θ and ϕ can take any real values, we can replace θ by x and ϕ by y. Thus, we get

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}\ ;\ \cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2},$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}\ ;\ \sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}.$$

Example 1: Prove that

$$1 + \cos 2x + \cos 4x + \cos 6x = 4\cos x + \cos 2x + \cos 3x$$

L.H.S.:

$$= 1 + \cos 2x + \cos 4x + \cos 6x$$

$$= 2cos^2 x + 2\cos 5x \cos x$$

$$= 2\cos x\ (\cos x + \cos 5x)$$

$$= 2\cos x\ (2\cos 3x \cos 2x)$$

$$= 4\cos x \cos 2x \cos 3x = \text{R.H.S.}$$

Example 2: Prove that $\frac{\cos 9x - \cos 5x}{sin\ 17x - sin 3x} = -\frac{\sin 2x}{cos\ 10x}$

Sol. L.H.S $= \frac{\cos 9x - \cos 5x}{\sin 17x - sin 3x} = \frac{-2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x)}{2}\right)\sin\left(\frac{17x-3x}{2}\right)}$

$$= \frac{-2\sin 7x \sin 2x}{2\cos 10x \sin 7x} = -\frac{\sin 2x}{\cos 10x} = R.H.S$$

Hence Proved.

Example 3 : Prove that $\frac{\sin x - \sin 3x}{sin^2\ x - cos^2 x} = 2\sin x.$

Sol. L.H.S $= \frac{\sin x - \sin 3x}{sin^2 x - cos^2 x} = \frac{-(\sin 3x - sinx)}{-(cos^2 x - sin^2 x)}$

$$=\frac{2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)}{\cos 2x} = \frac{2\cos 2x \, sinx}{\cos 2x} = 2 \sin x = \text{R.H.S}$$

Example 4: Prove that $\frac{\cos 4x+cos3x+cos2x}{\sin 4x+sin3x+sin2x} = cot3x$.

Sol. L.H.S $\frac{\cos 4x+cos3x+\cos 2x}{\sin 4x+sin3x+\sin 2x}$

$$= \frac{2cos\frac{4x+2x}{2}.\ cos\frac{4x-2}{2}+\cos 3x}{2\ sin\frac{4x+2x}{2}.\ cos\frac{4x-2x}{2}+\ sin3x} = \frac{\cos 3x\ cosx+cos3x}{\sin 3x\ cosx+sin3x}$$

$$= \frac{cos3x\ (cosx+1)}{sin3x\ (cosx+1)} = \frac{\cos 3x}{\sin 3x} = \cot 3x = R.H.S$$

3. Transformation of the product of trigonometric functions into sum or difference of trigonometric functions.

As part of the above identities , the following results can be obtained.

(i) $2 \cos x \cos y = \cos (x + y) + \cos (x - y)$

(ii) $-2 \sin x \sin y = \cos (x + y) - \cos (x - y)$

(iii) $2 \sin x \cos y = \sin (x + y) + \sin (x - y)$

(iv) $2 \cos x \sin y = \sin (x + y) - \sin (x - y)$.

Example 5: Prove that: $\sin 10° \, sin50° \, sin60° \, sin70° = \frac{\sqrt{3}}{16}$

L.H.S = sin10° sin50° sin60° sin70°

$=\frac{\sqrt{3}}{2}$. sin10° . sin50° . sin70° [sin 60° = $\frac{\sqrt{3}}{2}$]

$$= \frac{\sqrt{3}}{2}\ (2\ sin10°\ sin50°).\ sin70° = \frac{\sqrt{3}}{4}\ (cos40° - cos60°).\ sin70°$$

$$= \frac{\sqrt{3}}{4}\left[\frac{1}{2}\ (2\ sin70°\ cos40°) - \frac{1}{2}\sin 70°\right] \quad \left[cos60° = \frac{1}{2}\right]$$

$$= \frac{\sqrt{3}}{8}\ [(\sin 110° + sin30°) - sin70°] = \frac{\sqrt{3}}{8}[sin70° + \frac{1}{2} - \ sin70°]$$

$= \frac{\sqrt{3}}{16}$ = R.H.S.

Example 6: Prove that: cos20° cos40° cos80° = $\frac{1}{8}$.

L.H.S. = $\cos 20° \cos 40° \cos 80°$

$= \frac{1}{2}(2\cos 20° . \cos 40°). \cos 80° = \frac{1}{2}(\cos 60° + \cos 20°) \cos 80°$

$= \frac{1}{2}\left(\frac{1}{2} + \cos 20°\right) \cos 80°$ [$\cos 60° = \frac{1}{2}$]

$= \frac{1}{4} \cos 80° + \frac{1}{2} \cos 20° \cos 80° = \frac{1}{4} \cos 80° + \frac{1}{4} (2\cos 20° \cos 80°)$

$= \frac{1}{4} \cos 80° + \frac{1}{4}(\cos 60° + \cos 100°)$

$= \frac{1}{4} \cos 80° + \frac{1}{4}\left[\frac{1}{2} + \cos(180° - 80°)\right]$ [$\cos 60° = \frac{1}{2}$]

$= \frac{1}{4} \cos 80° + \frac{1}{8} - \frac{1}{4} \cos 80°$ [$\cos(180° - \theta) = -\cos\theta$]

$= \frac{1}{8} = R.H.S.$

4. Summary:

In this module identities on sum and difference of trigonometric functions has been discussed. These have been further used to convert the product of trigonometric functions as their sum or difference. Questions based on these have been solved.