1. **Details of Module and its structure**

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics (Class XI, Semester - 1)
Module Name/Title	Trigonometry: Part 3
Module Id	kemh_10303
Pre-requisites	Knowledge about Trigonometric Functions.
Objectives	After going through this lesson, the learners will be able to understand the following: 1.Introduction 2. Domain and Range of Trigonometric functions 3.Graphs of Trigonometric functions 4.Summary
Keywords	Multiple of an angle, Submultiple of an angle

2. **Development Team**

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1. Introduction

In this module the domain and range of trigonometric functions are discussed. Using domain and range, the graphs will be plotted and periodicity will be checked.

2. Domain and Range of trigonometric functions

From the definition of sine and cosine functions, it can be seen that they are defined for all real numbers. Further, we observe that for each real number x, $-1 \leq \sin x \leq 1$ and $-1 \leq \cos x \leq 1$. Thus, domain of $y = \sin x$ and $y = \cos x$ is the set of all real numbers and range is the interval

$[-1, 1]$, i.e., $-1 \leq y \leq 1$. Since $\operatorname{cosec} x = \dfrac{1}{\sin x}$, the domain of $y = \operatorname{cosec} x$ is the set $\{x : x \in \mathrm{R}$ and $x \neq n\pi, n \in \mathrm{Z}\}$ and range is the set $\{y : y \in \mathrm{R}, y \geq 1$ or $y \leq -1\}$.

Similarly, the domain of $y = \sec x$ is the set $\{x : x \in \mathrm{R}$ and $x \neq (2n + 1)\dfrac{\pi}{2}, n \in \mathrm{Z}\}$ and range is the set $\{y : y \in \mathrm{R}, y \leq -1$ or $y \geq 1\}$.

The domain of $y = \tan x$ is the set $\{x : x \in \mathrm{R}$ and $x \neq (2n + 1)\pi/2, n \in \mathrm{Z}\}$ and range is the set of all real numbers.

The domain of $y = \cot x$ is the set $\{x : x \in \mathrm{R}$ and $x \neq n\pi, n \in \mathrm{Z}\}$ and the range is the set of all real numbers.

The values of $\sin x$ and $\cos x$ repeats after an interval of 2π. Hence, values of $\operatorname{cosec} x$ and $\sec x$ will also repeat after an interval of 2π.

Values of $\tan x$ will repeat after an interval of π. Since $\cot x$ is reciprocal of $\tan x$, its values will also repeat after an interval of π.

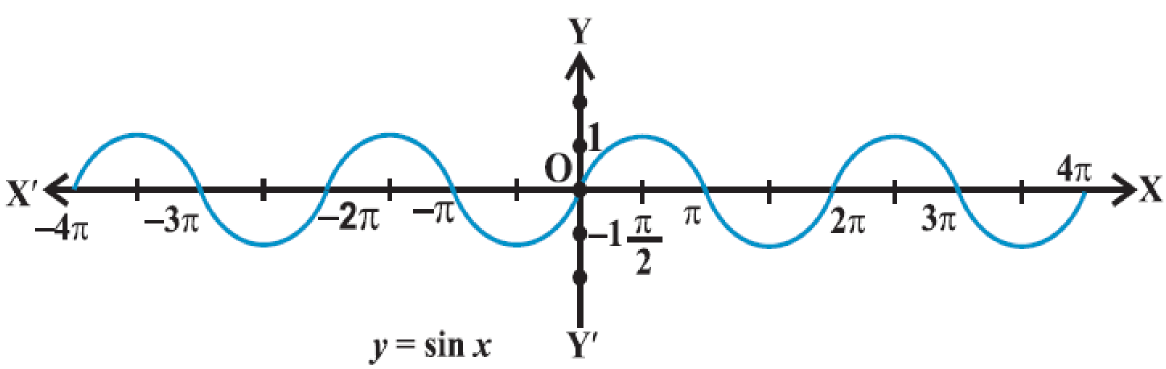
Table showing Domain and Range of Trigonometric Functions:

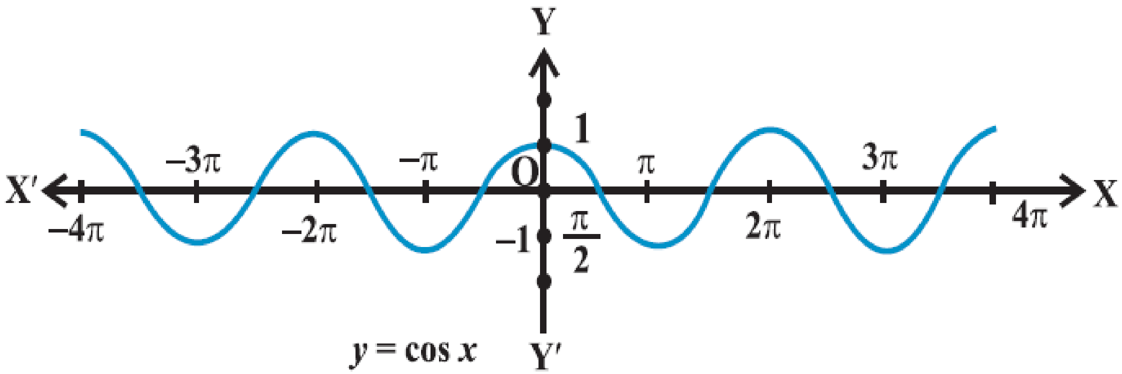
Functions	Domain	Range
sine	$\mathbf{R}$	$[-1, 1]$
cosine	$\mathbf{R}$	$[-1, 1]$
tan	$\mathbf{R} - \{(2n+1)\dfrac{\pi}{2} : n \in \mathbf{Z}\}$	$\mathbf{R}$
cot	$\mathbf{R} - \{n\pi : n \in \mathbf{Z}\}$	$\mathbf{R}$
sec	$\mathbf{R} - \{(2n+1)\dfrac{\pi}{2} : n \in \mathbf{Z}\}$	$\mathbf{R} - (-1, 1)$
cosec	$\mathbf{R} - \{n\pi : n \in \mathbf{Z}\}$	$\mathbf{R} - (-1, 1)$

3. Graphs of Trigonometric Functions:

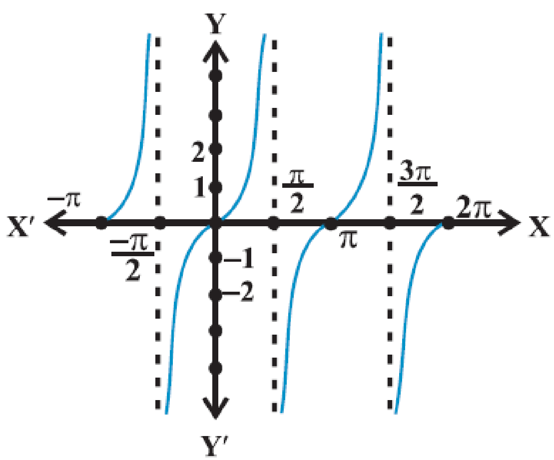
Using this knowledge and behaviour of trigonometric functions, we can sketch the graph of these functions.

The graph of these functions are given below.

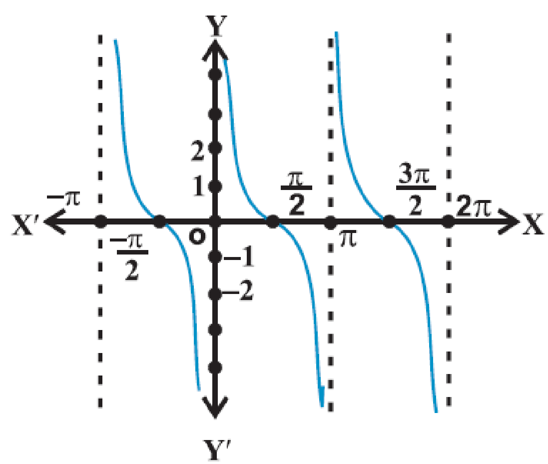




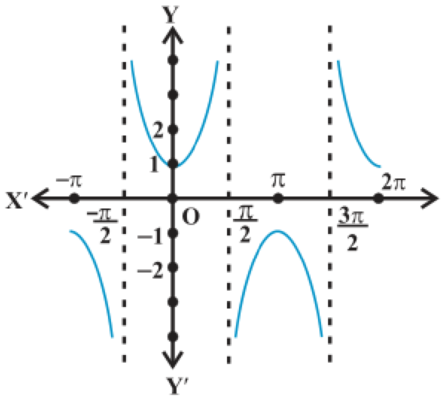
y = cos *x*



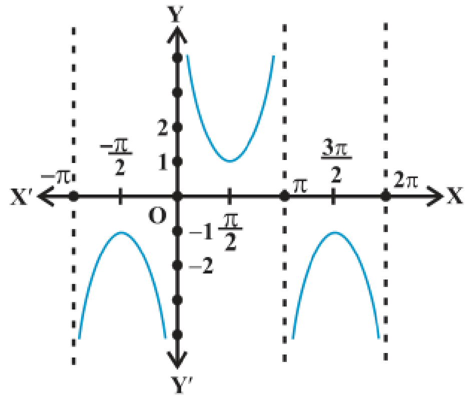
y = tan *x*



y = cot *x*



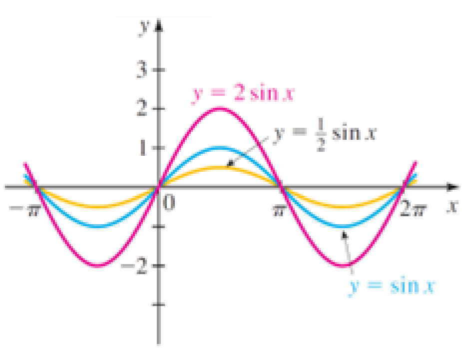
$y = \sec x$



$y = \operatorname{cosec} x$

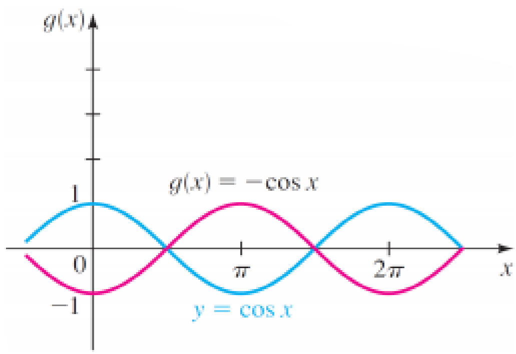
Example 1: Draw the graphs of $\sin x$, $2\sin x$ and $\frac{1}{2}\sin x$ on the same graph and compare.

Solution: To draw the graph of $y = 2\sin x$, start with the graph of $y = \sin x$ and multiply the y-coordinate of each point by 2. This has the effect of stretching the graph vertically by a factor of 2. For the graph of $y = \frac{1}{2}\sin x$, we start with the graph of $y = \sin x$ and multiply the y-coordinate of each point by $\frac{1}{2}$. This has the effect of stretching the graph vertically by a factor of $\frac{1}{2}$.



Example2: Show that the graph of $y = -\cos x$ is reflection of the graph of $y = \cos x$, along x-axis.

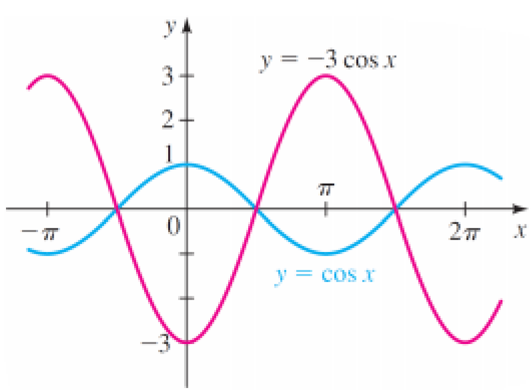
Solution: Draw the graphs of $y = -\cos x$ and $y = \cos x$, on the same axis.



It can be seen that the two graphs are reflection of each other along the x-axis.

Example 3: Draw the graph of $y = \cos x$ and $y = -3\cos x$, on the same graph.

Solution: The graphs are given below:



Example 4: Compare the graphs of the curve y = sink x , where k can take different values.

Solution: To see how the value of k effects the graphs of y = sink x . We know that

if $f(x)$ is periodic with period λ , then period of $f(x)$ is $\frac{\lambda}{k}$.

Let us observe the graph of the function y = sin2 x . Since

The period of sin2 x is $\frac{2\pi}{2} = \pi$, the graph completes one period in the interval $0 \le x \le \pi$.

For the graph of the function $y = \sin\frac{1}{2}x$, the period of is $\frac{2\pi}{\frac{1}{2}} = 4\pi$, and so the graph completes one period in the interval $0 \le x \le 4\pi$.

4. Summary:

In his module we have seen how to get the domain and range of trigonometric functions . Also with the help of domain and range, the graphs of all trigonometric functions were plotted. We also examined the changes in the graphs of sine and cosine functions when they were multiplied by a constant or an angle was multiplied by the same.