## 1. Details of Module and its structure

| Module Detail | |
|---|---|
| Subject Name | Mathematics |
| Course Name | Mathematics (Class XI, Semester - 1) |
| Module Name/Title | Trigonometric Functions: Part 1 |
| Module Id | kemh_10301 |
| Pre-requisites | Knowledge about Trigonometric Functions. |
| Objectives | After going through this lesson, the learners will be able to understand the following:<br>1. Angles<br>2. Measurement of Angles<br>3. Degree Measure<br>4. Radian Measure<br>5. Conversion from Degree to Radian<br>6. Conversion from Radian To Degree<br>7. Relationship between arc length, angle at the centre and radius<br>8. Summary |
| Keywords | Angle, Anticlockwise, Clockwise, Degrees, Radians. |

## 2. Development Team

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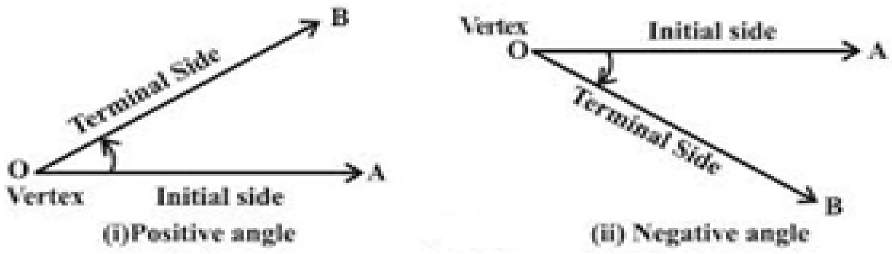
## 1. Introduction

The word ‘trigonometry’ is derived from the Greek words ‘trigon’ and ‘metron’ and it means ‘measuring the sides of a triangle’. The subject was originally developed to solve geometric problems involving triangles. It was studied by sea captains for navigation, surveyor to map out the new lands, by engineers and others. Currently, trigonometry is used in many areas such as the science of seismology, designing electric circuits, describing the state of an atom, predicting the heights of tides in the ocean, analysing a musical tone and in many other areas. In earlier class, we have already studied the trigonometric ratios of acute angles as the ratio of the sides of a right angled triangle. Now we will generalize the concept of trigonometric ratios to trigonometric functions and study their properties. Also we will study the relationship between $l, \theta \, and \, r$ where $l$ is the length of an arc, $r$ is radius of the circle and $\theta$ is the angle subtended by the arc at the centre of the circle.

### Concept Of An Angle

Angle is a measure of rotation of a given ray about its initial point.

The original ray is called the initial side and the final position of the ray after rotation is called the terminal side of the angle. The point of rotation is called the vertex.The measure of an angle is the amount of rotation performed to get the terminal side from the initial side.

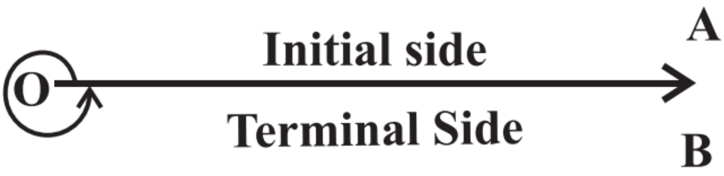



The angle measured in anticlockwise direction is taken to be positive and the angle measured in clockwise direction is taken to be negative.

**2. Measurement of Angles**

The measure of an angle is the amount of rotation performed to get the terminal side from the initial side.

The definition of an angle suggests a unit, viz. one complete revolution from the position of the initial side as indicated in Fig, given below:




There are two system for measuring angles:

**(i)** **Degree Measure**
**(ii)** **Radian System**

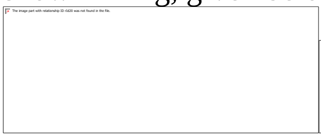
**3. Degree Measure (Sexagesimal System)**:

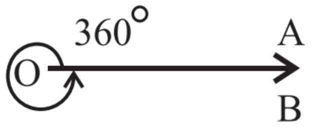
If a rotation from the initial side to terminal side is $\left(\frac{1}{360}\right)^{th}$ of a revolution, the angle is said to have a measure of one degree, written as 1°. A degree is divided into 60 minutes, and a minute is divided into 60 seconds . One sixtieth of a degree is called a minute, written as 1′, and one sixtieth of a minute is called a second, written as 1″.

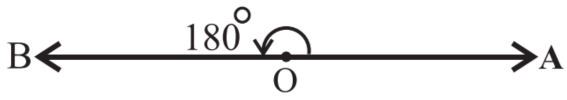
Thus, 1° = 60′, 1′ = 60″

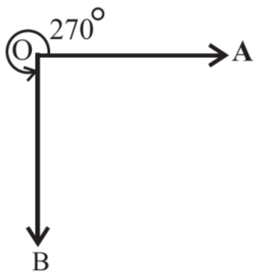
In this system, a right angle is divided into 90 equal parts, called Degrees i.e., 1 right angle = 90°

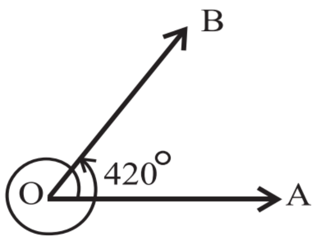
Some of the angles whose measures are 360°,180°, 270°, 420°, – 30°, – 420° are shown in Fig, given below

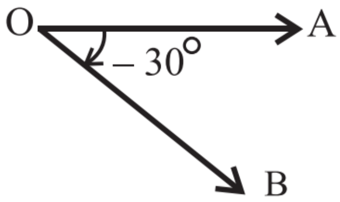


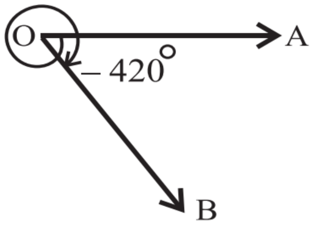



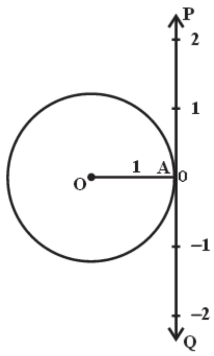




**4. Radian System (Circular System):**

In this system, angle is measured in radian.
A radian is the angle subtended at the centre of a circle by an arc, whose length is equal to the radius of the circle. Let us try to understand it.

**One to one correspondence between Radians and Real Numbers:**
Consider the unit circle with centre O. Let A be any point on the circle. Consider OA as initial side of an angle. Then the length of an arc of the circle will give the radian measure of the angle which the arc will subtend at the centre of the circle. Consider the line PAQ which is tangent to the circle at A. Let the point A represent the real number zero, AP represents positive real number and AQ represents negative real numbers as given in the figure given below. If we rope the line AP in the anticlockwise direction along the circle, and AQ in the clockwise direction, then every real number will correspond to a radian measure and conversely. Thus, radian measures and real numbers can be considered as one and the same.




Since a circle subtends at the centre an angle whose radian measure is 2π and its degree measure is 360°, it follows that 2π radian = 360° or π radian = 180°.
The above relation enables us to express a radian measure in terms of degree
measure and a degree measure in terms of radian measure. Using approximate value π as 22/7, we do the conversions.

### 5. Conversion from Degree to Radian

**Example 1**: Covert $30^o$ into radian.
Solution: We know that $360^o$ is equal to $2\pi$ radian, so

$$1^0 = \frac{2\pi}{360^0} \text{ radian}$$

$$\therefore\ 30^0 = \frac{2\pi}{360^0} \times 30^0 = \frac{\pi}{6} \text{ radian.}$$

Similarly, we can also show that

$$45^0 = \frac{\pi}{4} radian$$

$$60^0 = \frac{\pi}{3} radian$$

$$90^0 = \frac{\pi}{2} radian$$

**Example 2:** Convert $40^0 20'$ into radian.
Solution: We know that $180^0 = \pi\ radian$.

Now $40^0 20' = (40\frac{1}{3})^0$ $\qquad \{\because 1^0 = 60'\}$

$$= (40\frac{1}{3})^0 = (\frac{121}{3})^0 = \frac{121}{3} \times \frac{\pi}{180}\ radian$$

$$= \frac{121\pi}{540}\ radian$$

**Example 3: Convert $-37^0 30'$ into radian.**

**Solution:** $-37^0 30' = -(37\frac{1}{2})^0 = -(\frac{75}{2})^0 = -(\frac{75}{2} \times \frac{\pi}{180})\,radian$

$-(\frac{5\pi}{24})\,radian$

**Example 4:** Convert $5^0 37' 30''$ into radian.

Solution: We know that $180^0 = \pi\ radian$.

Now $30'' = \left(\frac{30}{60}\right)' = \left(\frac{1}{2}\right)'$ $\{\because 1' = 60''\}$

$\Rightarrow 37'30'' = (37\frac{1}{2})' = (\frac{75}{2})' = (\frac{75}{2} \times \frac{1}{60})^0$ $\{\because 1^0 = 60'\}$

$\Rightarrow 5^0 37' 30'' = \left(5\frac{5}{8}\right)^0 = \left(\frac{45}{8}\right)^0$

$= \left(\frac{45}{2} \times \frac{\pi}{180}\right) radian$

$= \left(\frac{\pi}{32}\right) radian$

**6. Conversion from Radian To Degree**

We know that

$\pi\, radian = 180^0$

$\Rightarrow 1 radian = (\frac{180}{\pi})^0$

Using this we will solve the following questions.

**Example 5:** Convert $\left(\frac{\pi}{8}\right)$ radian into degrees.

Solution: $(\frac{2\pi}{15})$ radian = $\left(\frac{2\pi}{15} \times \frac{180}{\pi}\right)^0 = 24^0$

**Example 6:** Convert 1 radian into degrees

Solution: 1 radian = $\left(\frac{180}{\pi}\right)^0 = \left(\frac{180}{22} \times 7\right)^0$

= 57° 16′ approximately.

**Example 7**: Convert $(-2)$ radian into degrees.

Solution: $(-2)$ radian $= \left(\frac{180}{\pi}\times(-2)\right)^0 = \left(\frac{180}{22}\times 7\times(-2)\right)^0$

$$= \left(-114\frac{6}{11}\right)^0 = \{-114^0(\frac{6}{11}\times 60)'\}$$

$$= \{-114^0(32\frac{8}{11})'\} = \{-114^0(32)'(\frac{8}{11}\times 60)''\}$$

$$= -\{114^0 32' 44''\}$$

**Example 8:** Convert $\left(\frac{11}{16}\right)$ radian into degrees.

Solution: $(\frac{11}{16})$ radian $= \left(\frac{180}{\pi}\times\frac{11}{16}\right)^0 = \left(\frac{180}{22}\times 7\times\frac{11}{16}\right)^0$

$$= \left(\frac{315}{8}\right)^0 = \{39\frac{3}{8}\}^0$$

$$= 39^0(\frac{3}{8}\times 60)' = 39^0 22'(\frac{1}{2}\times 60)''$$

$$= 39^0 22' 30''$$

### 7. Relationship between arc length, angle at the centre and Radius

More generally, in a circle of radius r, an arc of length r will subtend an angle of 1 radian. It is well-known that equal arcs of a circle subtend equal angle at the centre. Since in a circle of radius r, an arc of length r subtends an angle whose measure is 1radian, an arc of length $l$ will subtend an angle whose measure is $\frac{l}{r}$ radian. Thus, if in a circle of radius r, an arc of length $l$ subtends an angle θ radian at the centre we have

$$\theta = \frac{l}{r} \text{ or } l = r\theta .$$

**Example 9**: Find in degrees the angle through which a pendulum swings if its length is 50cm and the tip describes an arc of length 10cm.

Solution: Here, r=50cm and s=10cm, therefore

$$\theta = (\frac{l}{r}) radian$$

$$= \left(\frac{1}{5}\right) radian = \left(\frac{1}{5}\times\frac{180}{\pi}\right)^0$$

$$= \left(\frac{36}{22}\times 7\right)^0 = \left(11\frac{5}{11}\right)^0 \quad = 11^0 27' 16''.$$

**Example 10:** A circular wire of radius 7.5 cm is bent so as to lie along the circumference of a hoop whose radius is 120cm. Find in degrees the angle which is subtended at the centre of the hoop.

Solution: Radius of the circular wire=7.5cm

Length of the circular wire= $2\pi \times 7.5$ cm= 15 $\pi$ cm.

Radius of the hoop=120cm.

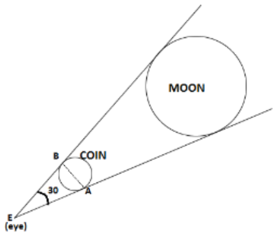
Let $\theta$ be the angle subtended by the wire at the centre of the hoop. Then,

$$\theta = \frac{arc}{radius} \Rightarrow \theta = (\frac{15\pi}{120}) radian$$

$$\theta = (\frac{\pi}{8} \times \frac{180}{\pi})^0 = 22^0 30'.$$

**Example 11:** If the angular diameter of the moon be 30', how far from the eye a coin of diameter 2.2cm be kept to hide the moon?

Solution: Suppose the coin be kept at a distance r from the eye to hide the moon completely. Let E be of the eye of the observer and let AB be the diameter of the coin. Then , arcAB =approximately equal to diameter AB = 2.2cm.




Now, $\theta = 30' = \left(\frac{30}{60}\right)^0 = \left(\frac{1}{2} \times \frac{\pi}{180}\right) radian = \left(\frac{\pi}{360}\right) radian$

But, $\theta = \frac{arc}{radius} \Rightarrow \frac{\pi}{360} = \frac{2.2}{r}$

$$\Rightarrow r = \frac{2.2 \times 360 \times 7}{22} = 252 cm.$$

**8. Summary**

- The angle measured in anticlockwise direction is taken to be positive and the angle measured in clockwise direction is taken to be negative.
- In the degree system, 1° = 60′, 1′ = 60″.
- For the conversion of degree to radian and radian to degree, 2π radian = 360° or π radian = 180° is used.
- Thus, if in a circle of radius r, an arc of length $l$ subtends an angle θ radian at the centre we have $\theta = \frac{l}{r}$ or $l = r\theta$.