1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 01 (Class XI, Semester - 1)
Module Name/Title	Relation and Functions - Part 2
Module Id	kemh_10202
Pre-requisite	Sets and its properties, Cartesian Products
Objectives	After going through this lesson, the learners will be able to do the following: • Relation • Domain, Codomain and Range of Relation
Keywords	Cartesian Product, Relation, Domain, Codomain and Range

2. Development Team

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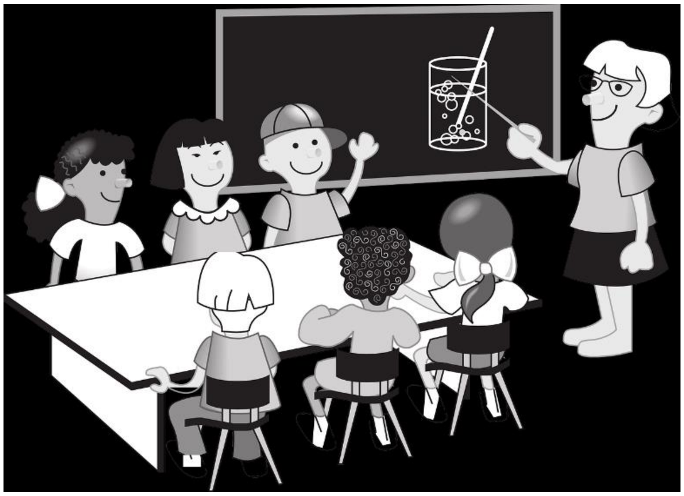
1. Introduction:

From our childhood we have learnt that Relation is a connection between or among two or more objects. For example the most common relation for a child which he learns is of father, mother and child.

The relation of brother and sister,

The next very common relation with which a child comes across is of teacher and student.



Have you ever thought, how do we define a relation?

Every relation has a pattern or property. And every relation will involve at least two identities.

2. What is Relation in Mathematics?

Today we are going to discuss what we mean by a relation in Mathematics.

In Mathematics, the term relation is used to relate the numbers, symbols, variables, sets, group of sets, etc.

In this module, we will learn how to link pairs of objects from two sets and introduce relations between elements of two sets. Finally, we will learn about special relations which will qualify to be functions in following module.

(i) Relations:

Let us consider two sets, set A being the set of boys and set B being the set of girls.

A= {Shyam, Ram, Monu, Sonu}

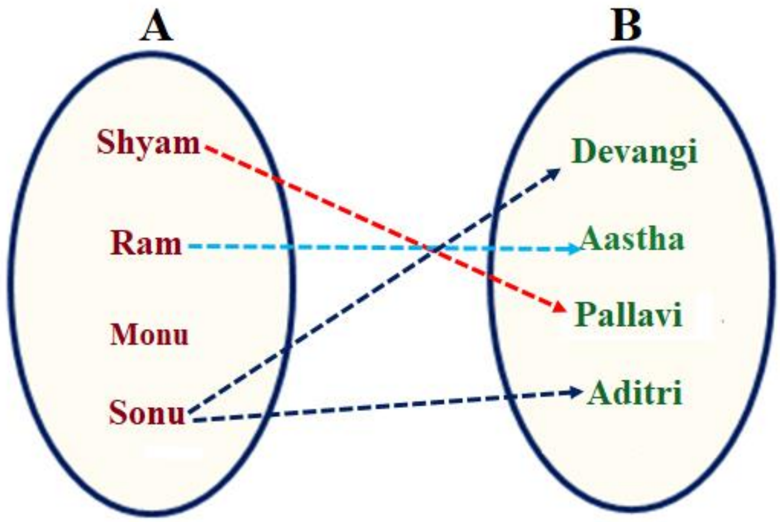
B = {Devangi, Aastha, Pallavi, Aditri}

As we have learnt in our previous module, if we write all the pairs from these two sets to get Cartesian product A×B, then there will ben(A) × n(B) = 4×4 =16 ordered pairs.

Suppose the boys of set A are related with the girls of set B with the relation, “is a brother of”.

Suppose Shyam is the brother of Pallavi, Ram is the brother of Aastha, Monu doesn't have his sister in this set and Sonu has two sisters Devangi and Aditri.

A visual representation of this relation is shown below,



Observe now we will be having only four pairs with this relation

"is a brother of" applied from set A to set B.

This can be written in the form of a set in roaster form as:

R= {(Shyam, Pallavi), (Ram, Aastha),(Sonu, Devangi), (Sonu, Aditri)}

and set builder form as:

R ={(x,y) : x∈A, y∈B, where x is the brother of y}.

Thus, the relation "is the brother of", from the set A to the set B gives rise to a subset 'R' of Cartesian Product A×Bsuch that (x, y) ∈R if and only if x is related to y with the relation "is the brother of "and x∈A, y∈B.

You are quite familiar that in mathematics we have already learnt many such relations over the set of numbers, for example;

"is greater than", "is less than", "is equal to", "is square of" etc.

Let us consider one more example:

If we define a relation by "has its cube as" from the set N to N.

Then it can be written in roaster form as:

R= {(1,1),(2,8), (3,27), (4,64),…….} and in set builder form as :

R= $\{(x,y):x, y\in N$ and $y=x^3\}$.

Observe, the above relation "has its cube as" from N to N gives rise to subset "R" of N×N, such that $(x,y)\in R$, if and only if, $y=x^3$.

The related elements of the sets are written as,

1R1, 2R8, 3R27, 4R64 ,……..where R stands for the relation "has its cube as".

Let us consider two more sets,

P = {a, b, c} and Q = {Ali, Bhanu, Binoy, Chandra, Divya}.

The Cartesian product of P and Q has 3×5 (= 15) ordered pairs,

Because, $n(P\times Q) = n(P)\times n(Q) = 15$ which can be listed as,

P × Q = {(a, Ali), (a, Bhanu), (a, Binoy), ………., (c, Divya)}.

We will obtain a subset of P×Q if we introduce a relation R between the first element $x\in P$ and the second element $y\in Q$ for each ordered pair $(x, y)\in P\times Q$.

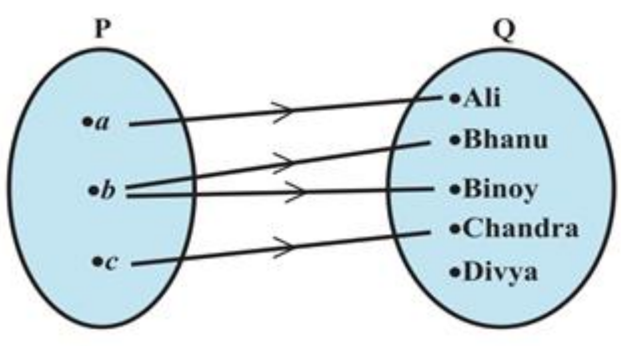
For example, consider relation, "x is the first letter of the name y", then we get the subset,

R = {(a, Ali), (b, Bhanu), (b, Binoy), (c, Chandra)}

And in the set builder form,

R= {(x,y): x is the first letter of the name y, $x\in P$, $y\in Q$}.

The visual representation of this relation is shown below,



Observe Divya is not associated with any of the element of set P, because first letter 'd'of the name 'Divya' does not belong to the set P.

From all these examples, we can now define the Relation as:

(ii) Definition:

A relation R from a non-empty set A to a non-empty set B is a subset of the Cartesian product $A \times B$. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in $A \times B$.

The second element is called the image of the first element and the first element of the ordered pair is called pre-image of the second element.

In the above example, "Ali" is image of element "a" for relation R: P →Q and "a" is pre-image of element "Ali" for same relation R defined from set P to set Q.

"Bhanu and Binoy" are two images of element "b" and "Chandra" is image of element "c" for relation R: P → Q and "b" and "c" are their respective pre-images.

If (a,b) ∈ R, then we say that "**a is related to b**" and we denote it as aRb and read as "a is in relation R with b".

If (a,b) ∉ R, then we say "a is not in relation R with b".

3. Domain:

The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R.

In above discussion, Domain in the examples considered are;

(i) Domain = {Shyam, Ram, Monu, Sonu}

(ii)Domain ={1,2,3,4……..}

(iii) Domain= {a, b, c}

Thus, Domain R = {a: (a,b) ∈ R}

4. Range and Co-Domain:

The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the codomain of the relation R. It is to note that Range ⊆ Codomain.

In above discussion, range in different examples are;

(i) Range= {Devangi, Aastha, Pallavi, Aditri}

(ii) Range ={1,8,27,64, …………..}

(iii) Range={Ali, Bhanu, Binoy, Chandra}

Thus, Range R = {b: (a,b) ∈ R}.

Remarks:

(*i*) A relation may be represented algebraically either by the Roster method or by the Set-builder method.

(*ii*) An arrow diagram is a visual representation of a relation.

(*iii*) In particular, any subset of A×A defines a relation on A.

Let us now take some examples to understand these definitions.

IfR is a relation defined on N by“a is multiple of b”,where (a,b) ∈ R, then R ⊆N×N,

R= {(a,b):a∈ N,b∈ N,a is multiple of b}

Set R will contain ordered pairs(1,1),(2,1),(3,1),……,as 1, 2, 3, are multiples of 1.

- Now, set R will contain ordered pairs (4,2),(6,2), (8, 2), ……as 4, 6, 8, are all multiples of 2, and so on. There will be countless ordered pairs in N×N satisfying this property for the relation.

Note, ordered pairs (1,2),(1, 3),(2,3),(2, 4),(2, 5),(2, 6),(2, 8),…. etc., will not belong to R as first element in all these ordered pairs is not a multiple of second element of the ordered pairs.

Example 1:

Let A = {1, 2, 3, 4, 5, 6},a relation R is defined from A to A by

R = {(x, y): $y = x + 1$ }

(i) Depict this relation using an arrow diagram.

(ii) Write down the domain, codomain and range of R.

Solution:

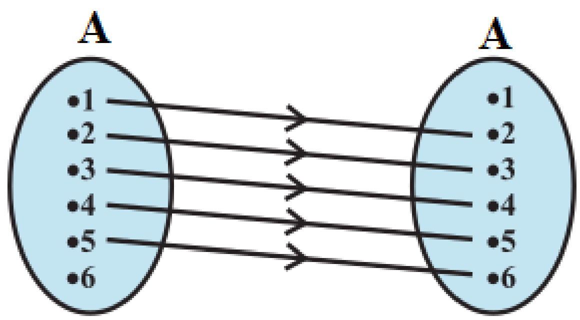
Given, A = {1, 2, 3, 4, 5, 6}

Relation R is defined from A to A by

R = {(x, y): $y = x + 1$}

Then using definition of the relation we get, R = {(1,2), (2,3), (3,4), (4,5), (5,6)}

The visual representation of this relation using arrow diagram is,



(ii) The set of all first elements of the ordered pairs in relation R form the set called domain of relation R, hence

Domain R ={1, 2, 3, 4, 5,}

Relation R is defined from A to A, entire second set A is defined as Codomain. Therefore,

Codomain R = {1, 2, 3, 4, 5, 6}

The set of all second elements of the ordered pairs in relation R form the set called Range of relation R, hence Range R = {2, 3, 4, 5, 6}

Comparing the Range and Codomain of the relation we can understand the difference between Range and Codomain.

Example 2:

Let R be a relation defined on N, defined by aRb, if "a–b is a multiple of a number n ∈ N". Which of the following ordered pairs belong to the set R.

(6, 2), (5,3), (2,6), (3,5), (8,5), (8, 7), (7, 8), (7, 7)

Solution:

In set builder form,

R = {(a,b): a, b∈ N, a–b is a multiple of a number n ∈ N}

Let us consider given ordered pairs one by one, considering pair (6, 2) we get,

6 – 2 = 4 which is multiple of at least one number which belongs to N

Hence, (6, 2) ∈R,

Similarly, (5,3)∈R, (8, 5)∈R, (8, 7)∈R

Let us now consider the pair (2,6), 2– 6 = –4, although–4 is multiple of 1, 2, 4 but –4 ∉N, hence it does not satisfy the required condition, therefore (2, 6)∉R, (3, 5)∉R, (7, 8)∉R.

Now consider the pair (7,7), 7– 7 = 0, 0 ∉N, hence it does not satisfy the required condition, therefore (7, 7)∉R.

Thus out of the given ordered pairs (6, 2), (5,3), (8, 5), (8, 7) belong to the set R and (2,6), (3,5), (7, 8), (7, 7) do not belong to the set R.

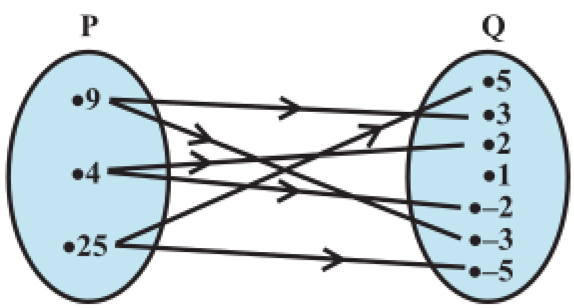
Example 3:

The figure below shows a relation between the sets P and Q. Write this relation (i) in set-builder form (ii) in roster form.

What is its domain, codomain and range?

Solution:

Studying the figure carefully, it is obvious that R is a relation from set P to set Q such that element belonging to set P is the square of the element belonging to set Q.



In set-builder form,

R = {(x,y):x∈ P, y∈ Q, x is the square of y"}.

In roster form,

R = {(9, 3), (9, –3), (4, 2), (4, –2), (25, 5), (25, –5)}

The domain of this relation is,

Domain = {4, 9, 25}.

The range of this relation is,

Range ={– 2, 2, –3, 3, –5, 5}

And the codomain is,

Codomain ={1, – 2, 2, –3, 3, –5, 5}= Q

We note that the element 1 is not related to any element in set P.

So the Range of the given relation is subset of co-domain Q.

5. Number of Relations:

Let P and Q be any two non-empty finite sets containing *m* and *n* elements respectively.

then, number of ordered pairs in $P \times Q$,

$n(P \times Q) = n(P) \times n(Q) = mn$

we have studied that if we define any relation $R: P \rightarrow Q$,

then $R \subseteq P \times Q$.

Hence, The total number of relations that can be defined from a set A to a set B is the number of all possible subsets of $A \times B$. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

Therefore total number of all possible subsets of $A \times B = 2^{pq}$, therefore total number of relations that can be defined from a set A to a set B is 2^{pq}. Among these 2^{pq} relations, the empty relation ϕ and the universal relation are also included.

Example 4:

Let A = {1, 2} and B = {3, 4}. Find the number of relations from A to B.

Solution:

We have, $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$.

Since $n(A \times B) = 4$,

$\therefore$ Number of subsets of $A \times B = 2^4$.

Hence, the number of relations from A into B will be 2^4.

Note:

I. If, $R = \phi$, then R is called empty relation or void relation .
II. If $R = A \times B$, then R is called the universal relation .
III. A relation R from A to A is also stated a relation on A.

6. Summary:

1) A relation R from a set A to a set B is a subset of the Cartesian product $A \times B$ obtained by describing a relationship between the first element x and the second element y of the ordered pairs in the set $A \times B$.
2) The image of an element x under a relation R is y, where $(x, y) \in R$ and x is called the pre-image of y.
3) The domain of R is the set of all first elements of the ordered pairs in a relation R.
4) The range of the relation R is the set of all second elements of the ordered pairs in a relation R.
5) If relation R is defined from a set A to a set B then set B is codomain of R.