1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 01 (Class XI, Semester - 1)
Module Name/Title	Relation and Functions - Part 1
Module Id	kemh_10201
Pre-requisite	Sets and its properties
Objectives	After going through this module, the learners will be able to understand the following: • Ordered pairs • Cartesian Product
Keywords	Cartesian Product

2. Development Team

Role	Name	Affiliation
National MOOC Coordinator (NMC)	Prof. Amarendra P. Behera	CIET, NCERT, New Delhi
Program Coodinator	Dr Indu Kumar	CIET, NCERT, New Delhi
Course Coordinator (CC) / PI	Dr. Til Prasad Sarma	DESM, NCERT, New Delhi
Subject Coordinator	Anjali Khurana	CIET, NCERT, New Delhi
Subject Matter Expert (SME)	Dr. Monika Sharma	Shiv Nadar University, Noida
Revised By	Dr. Sadhna Srivastava (Retd.)	PGT, KVS, Faridabad
Review Team	Prof. Bhim Prakash Sarrah Prof. Ram Avtar (Retd.) Prof. Mahendra Sankar	Assam University, Tezpur. DESM, NCERT, New Delhi DESM, NCERT, New Delhi

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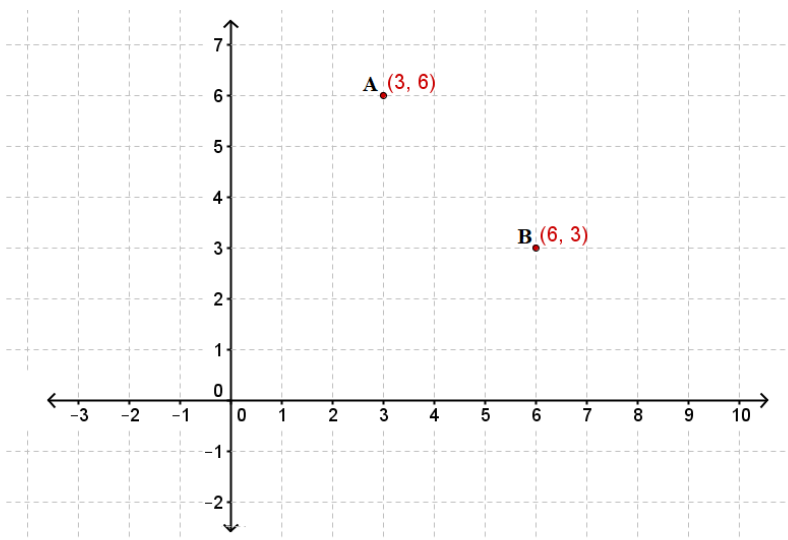
1. Introduction

We have learnt in earlier classes to plot points on Cartesian plane. In the following graph two points have been plotted with coordinates (3, 6) and (6, 3). You are quite familiar that these are the co-ordinates of two different points A and B as depicted below in the graph.



This shows that the order of the numbers in these two pairs is very important. Though the numbers are same but by changing the order we get two different points on Cartesian plane. So the order in which the numbers are written in pair is important. Such pairs are called ordered pairs.

2. Ordered Pairs

An **ordered pair** is a **pair** of numbers in a specific **order**.

Two ordered pairs of elements $(a, b)$and(u, v)will be same if and only if $a = u$ and $b = v$.

So the ordered pair (p, q) is always different from ordered pair (q, p) unless p = q. Thus an ordered pair is a pair of elements written in a specified order.

Graphically, ordered pair (4, 5) means abscissa x= 4 and ordinate y= 5and ordered pair (5,4) means abscissa x= 5 and ordinate y= 4 and therefore they represent two different points.

If A and B are two sets, then the ordered pair of elements where $a \in A$ and $b \in B$ is represented by pair (a, b) in that order, where a being the first element and b being the second element.

Example 1: Find the values of a and b, if

$(a+1, b-1) = (2,3)$

Solution: We know that ordered pairs are equal only if their corresponding elements are equal.

Hence, $(a+1, b-1) = (2,3)$

$\Rightarrow a+1=2$ and $b-1 =3$

thus, $a= 2-1 \;\Rightarrow a=1$ and $b=3+1 \;\Rightarrow b =4$

3. Cartesian Product of Sets:

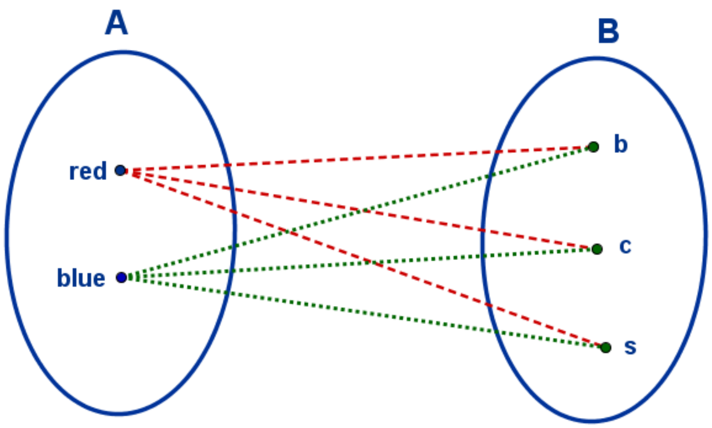
Suppose set A is a set of 2 colours and set B is a set of three objects, i.e.

A = {red, blue} and B = {b,c,s},

Where b, c and s represent a particular bag, coat and shirt respectively.

How many pairs of coloured objects can be made from these two sets?

Proceeding in a very orderly manner, we can see that there will be 6 distinct pairs as given below:



(red, b), (red, c),(red, s), (blue, b), (blue, c),(blue, s)

Thus, we get 6 distinct objects.

The set of all possible ordered pairs is,

{(red, b), (red, c),(red, s), (blue, b), (blue, c),(blue, s)}

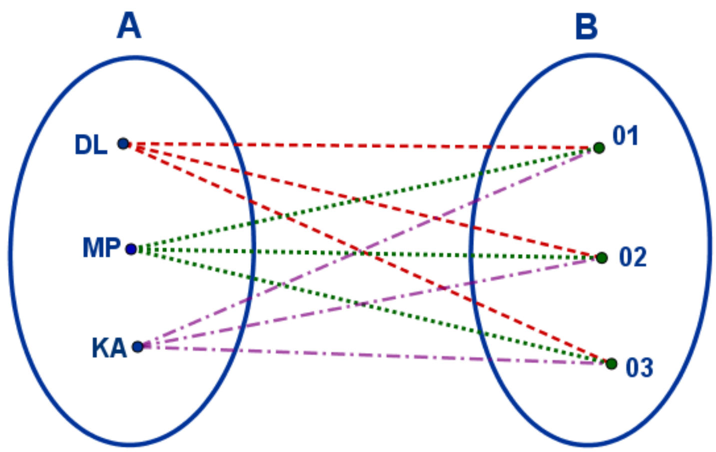
Let us consider another example of the two sets as below:

A={DL,MP,KA}, where DL,MP,KA represent Delhi, Madhya Pradesh and Karnataka respectively and B = {01,02,03} representing codes for the license plates of vehicles issued by DL, MP and KA.

Again, if the three states, Delhi, Madhya Pradesh and Karnataka were making codes for the license plates of vehicles, with the restriction that the code begins with an element from set A, then the available pairs will be;

(DL,01), (DL,02), (DL,03), (MP,01),(MP,02), (MP,03), (KA,01) (KA,02), (KA,03).

As is depicted in the following diagram;



The set containing all these possible pairs is;

{(DL,01), (DL,02), (DL,03), (MP,01),(MP,02), (MP,03), (KA,01), (KA,02), (KA,03)}

This set of all these possible ordered pairs from the two sets is known as Cartesian product of these two sets A and B.

So, Cartesian product of sets A and B is,

{(DL, 01), (DL, 02), (DL, 03), (MP, 01), (MP, 02), (MP, 03), (KA, 01), (KA, 02), (KA, 03)}

(i) Definition:

Given two non-empty sets P and Q. The Cartesian product of the sets P and Q denoted by $P \times Q$ is the set of all ordered pairs (p, q) such that $p \in P$, $q \in Q$.

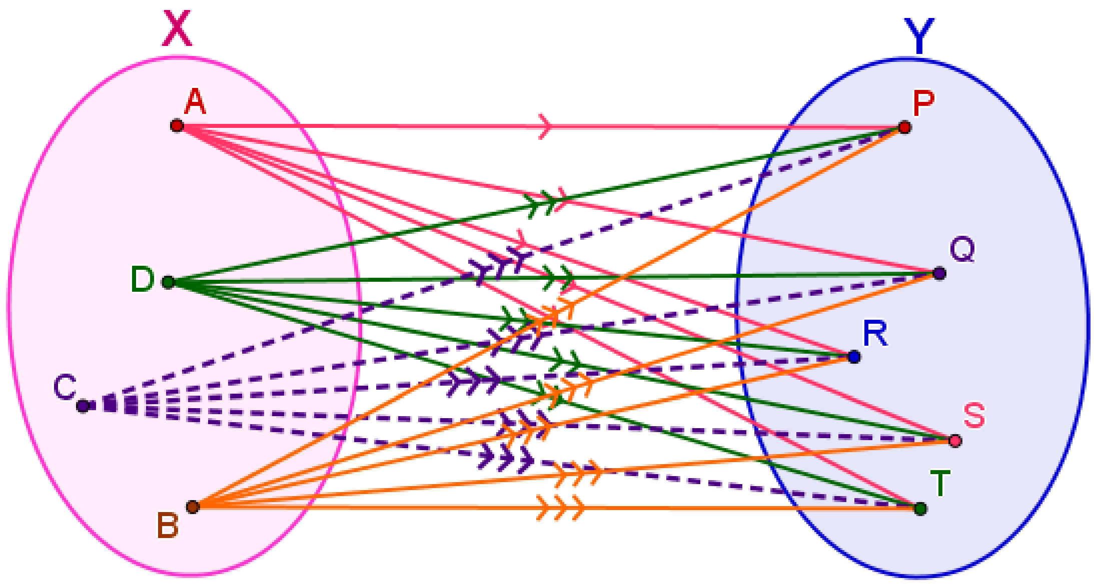
$$P \times Q = \{(p, q):\ p \in P, q \in Q\}.$$

Since Cartesian product is set of ordered pairs hence $P \times Q$ and $Q \times P$ will be different sets unless $P = Q$.

Following figure will illustrate the Cartesian product between two sets X and Y where,

X = {A, B, C, D} and Y = {P,Q, R, S, T}

Cartesian Product



X × Y = {(A,P), (A,Q), (A,R), (A,S), (A,T)
(B,P), (B,Q), (B,R), (B,S), (B,T)
(C,P), (C,Q), (C,R), (C,S), (C,T)
(D,P), (D,Q), (D,R), (D,S), (D,T) }

Observe in figure n(X×Y) = n(X) ×n(Y) = 20.

Remarks:

(i) A×B and B×A are different sets as A ≠ B.

(ii) Did you note the number of elements in set A×B? It is 9 because there are 3 elements in each of the sets A and B.

(iii) Also note the order in which these elements are paired, code (DL, 01) is different from code (01, DL).

(iv) If there are p elements in set A and q elements inset B, then there will be pq elements inset A×B.

i.e if n(A)=p and n(B) =q, then n(A×B)=pq.

(v) Two ordered pairs are equal, if and only if the corresponding first elements are equal and the corresponding second elements are also equal.

(vi) If A and B are non-empty sets and either A or B is an infinite set, then $A \times B$ will be an infinite set.

(vii) If either P or Q is a null set *i.e.*, $P = \phi$ or $Q = \phi$, then

$P \times Q = \phi$

(viii) And if both P and Q are not the null sets *i.e.*, $P \neq \phi$ and $Q \neq \phi$, then

$P \times Q \neq \phi$.

(ii) Graphical Representation of Cartesian Products of Sets:

Let's represent the order pair (p,q) graphically, where $p \in P$, $q \in Q$,

P and Q are two non-empty sets of real numbers.

We draw lines x=p and y=q and find the intersection point of the perpendicular lines at x=p and y=q. The intersecting point of the lines x=p and y = q is represented by the order pair (p,q).

Example 2:

Let P = {3,4,5} and Q ={1,2,3} then find

(i) $P \times Q$ and $Q \times P$

(ii) Is $P \times Q = Q \times P$?

(iii) Is $n(P \times Q) = n(Q \times P)$?

Solution :

We have,

P = {3,4,5} and Q = {1,2,3},then

(i) $P \times Q$ = {(3,1),(3,2),(3,3),(4,1),(4,2),(4,3),(5,1),(5,2),(5,3)}

and $Q \times P$ = {(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,3),(3,4),(3,5)}

(ii) We see that $P \times Q$ and $Q \times P$ do not have exactly the same ordered pairs, hence,

$P \times Q \neq Q \times P$

(iii) We observe that sets $P \times Q$ and $Q \times P$ have same number of ordered pairs, hence

$n(P \times Q) = n(Q \times P)$ because, $n(P \times Q) = n(P) \times n(Q) = n(Q) \times n(P) = n(Q \times P) = 3 \times$x3 = 9.

Example 3:

If A ×X B = {(3,p),(4,q), (5,r),(6,s)}. Then determine the set A and the set B?

Solution:

A ×X B = {(3,p),(4,q), (5,r),(6,s)}.

Set of first elements is {3,4,5,6} and

set of second elements is{p,q,r,s}

Which implies that,

A = {3,4,5,6} and B ={p,q,r,s}

Example 4:

If A × B= {(p,q), (p,r), (m,q) (m,r)}, find A and B.

Solution :

According to the definition,

The Cartesian product A × Bof two sets A and B is the set of all ordered pairs of elements from sets A and B such that,

A × B = {(*x*,*y*):*x*∈A, y∈B}, thus

A = set of all first elements of ordered pairs of A × B = {p,m}

B= set of all second elements of ordered pairs of A × B = {q,r}

(iii) Ordered Triplets and Cartesian Product of three sets:

Consider three sets A, B and C, then the set of all ordered triplets (a,b,c); where a ∈ A, b ∈ B and c ∈ C in that particular order is called **Cartesian Product** of three sets A,B,C and is represented by A×B×C.

Symbolically we write it as,

A×B×C = {(a, b, c) :a ∈ A, b ∈ B and c ∈ C }

Example5:

If, A= {−-2,1}, then find A×A×A.

Solution:

A×A = {−2,1}× x{−2,1}

$= \{(-2,-2),(-2,1),(1,-2),(1,1)\}$

$A \times A \times A = \{(-2,-2),(-2,1),(1,-2),(1,1)\} \times x\{-2,1\}$

$= \{(-2,-2,-2), (-2,1,-2),(1,-2,-2),(1,1,-2), (-2,-2,1),(-2,1,1),(1,-2,1),(1,1,1)\}$

Ordered n-tuples for Cartesian products of n sets:

In general, if $A_1, A_2, \ldots\ldots\ldots A_n$ are n sets , then by **ordered n-tuples** of elements we mean,$(a_1, a_2 \ldots\ldots\ldots a_n)$; $a_1 \in A_1$, $a_2 \in A_2$, $a_n \in A_n$ in that order.

The set of all ordered n-tuples $(a_1, a_2 \ldots\ldots\ldots a_n)$; $a_1 \in A_1$, $a_2 \in A_2$, ……… $a_n \in A_n$ is called the **Cartesian products of n sets** $A_1, A_2 \ldots\ldots\ldots A_n$ and is denoted by $A_1 \times A_2 \times \ldots\ldots\ldots \times A_n$,

It is represented by $\prod_{i=1}^{n} A_i$, where $\prod$ stands for the product.

In symbolic form,

$A_1 \times A_2 \times \ldots\ldots\ldots \times A_n = \{(a_1, a_2 \ldots\ldots\ldots a_n) ; a_i \in A_i, 1 \le i \le n\}$

Theorem:

If P and Q are any two non-empty sets, then prove that

$P \times Q = Q \times P$ implies and is implied by $P=Q$

Proof:

Let us consider, $P \times Q = Q \times P$

Then, we have to prove $P=Q$.

Let x be any element belonging to set P.

Then,$x \in P$ implies if $(x,a) \in P \times Q$ then $a \in Q$,

since we have assumed $P \times Q = Q \times P$,

$\therefore (x,a) \in P \times Q \Rightarrow (x,a) \in Q \times XP \Rightarrow x \in Q$

Hence, $x \in P \Rightarrow x \in Q$

$\Rightarrow$ P is subset of set Q.

Similarly taking y being any element belonging to setQ, we can show that,

Q is subset of set P.

Combining the two we get, $P=Q$.

(*ii*)

Let us now take, P=Q

Then,we have to prove that $P \times Q = Q \times P$

Now, P=Q $\Rightarrow P \times Q = P \times P$

and $Q \times P = P \times P$ (as P=Q)

Combining the two we get,

$P \times Q = Q \times P$

Hence we can say that if P and Q are any two non-empty sets, then

$P \times Q = Q \times P$ implies and is implied by P=Q

Example 6:

Let P be a non-empty set such that $P \times Q = P \times R$

prove that,Q= R.

Solutin:

Let q be any element of Q.

Then, $(p,q) \in P \times Q \Rightarrow (p,q) \in P \times R$ for all $p \in P$ (as $P \times Q = P \times R$)

Which implies $q \in R$

Therefore, $q \in Q \Rightarrow q \in R$

Hence, Q is a subset of R………………………………………..(*i*)

Now,

Let r be any element of R.

Then,

$(p,r) \in P \times R \Rightarrow (p, r) \in P \times Q$ for all $p \in P$ (as $P \times Q = P \times R$)

$\Rightarrow r \in Q$

Thus, $r \in R \Rightarrow r \in Q$

Therefore, R is the subset of Q …………………………(*ii*)

From (*i*) and (*ii*), we get

Q=R.

Example7:

If R is the set of all real numbers, what do the Cartesian Products

R×XR and R×XR×XR represent?

Solution:

The Cartesian product R×R represents the set,

R×R = $\{(x,y):x,y \in R\}$

which represents the coordinates of all the points in two dimensional space and the Cartesian product R×R×R represents the set,

R×R×R=$\{(x,y,z):x, y,z \in R\}$

which represents the coordinates of all the points in the three-dimensional space.

4. Summary:

1) A pair of elements grouped together in a particular order is defined as ordered pair.
2) Cartesian product A × B of two sets A and B is given by

   $A \times B = \{(a, b): a \in A, b \in B\}$
3) In particular $R \times R = \{(x, y): x, y \in R\}$ and

   $R \times R \times R = (x, y, z): x, y, z \in R\}$
4) If $(a, b) = (x, y)$, then $a = x$ and $b = y$.
5) If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.
6) For any set A,

   $A \times \varphi = \varphi$
7) In general, $A \times B \neq B \times A$.