1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 01 (Class XI, Semester - 1)
Module Name/Title	Application of algebra on Sets - Part 5
Module Id	kemh_10105
Pre-requisite	Understands the concept of a set, Comprehends the operations on sets which includes Union, Intersection and Complement, Represents the relationships between sets using Venn Diagrams
Objectives	After going through this lesson, the learners will be able to do the following: • Perform Algebra of Sets Operations involving union and intersection. • Apply this knowledge to solve basic problems on Sets. • Apply this knowledge to solve practical problems.
Keywords	Algebra of Sets Operation, Problems using Venn Diagrams

2. Development Team

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1. Algebra of sets operations

Let A and B be finite sets. If $A \cap B = \varphi$, then

(i) $\mathbf{n(A \cup B) = n(A) + n(B)}$ **... (1)**

The elements in $A \cup B$ are either in A or in B but not in both as $A \cap B = \varphi$.

So, (1) follows immediately.

In general, if A and B are finite sets, then

(ii) $\mathbf{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$ **... (2)**

Therefore, $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

$= n(A - B) + n(A \cap B) + n(B - A) + n(A \cap B) - n(A \cap B)$

$= n(A) + n(B) - n(A \cap B)$, which verifies (2)

If A, B and C are finite sets, then

(iii) $\mathbf{n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)}$ **... (3)**

In fact, we have $n(A \cup B \cup C) = n(A) + n(B \cup C) - n[A \cap (B \cup C)]$ [by (2)]

$= n(A) + n(B) + n(C) - n(B \cap C) - n[A \cap (B \cup C)]$

Since $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, we get

$n[A \cap (B \cup C)] = n(A \cap B) + n(A \cap C) - n[(A \cap B) \cap (A \cap C)]$

$= n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)$

Therefore,

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

This proves (3).

Some problems on sets:

1. Let A and B be two finite sets such that $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 36$, find $n(A \cap B)$.

Solution:

Using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

then $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 20 + 28 - 36$

$= 48 - 36$

$= 12$

2. If $n(A - B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, then find $n(B)$.

Solution:

Using the formula $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

$70 = 18 + 25 + n(B - A)$

$70 = 43 + n(B - A)$

$n(B - A) = 70 - 43$

$n(B - A) = 27$

Now $n(B) = n(A \cap B) + n(B - A)$

$= 25 + 27$

$= 52$

Some practical problems

3. In a group of 60 people, 27 like tea and 42 like coffee and each person likes at least one of the two drinks. How many like both coffee and tea?

Solution:

Let A = Set of people who like tea.

and B = Set of people who like coffee.

Given

$(A \cup B) = 60$, $n(A) = 27$ and $n(B) = 42$ then;

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 27 + 42 - 60$

$= 69 - 60 = 9$

$= 9$

Therefore, 9 people like both tea and coffee.

4. There are 35 students in art class and 57 students in dance class. Find the number of students who are either in art class or in dance class.

a) **When two classes meet at different hours and 12 students are enrolled in both activities.**

b) **When two classes meet at the same hour.**

Solution:

$n(A) = 35$, $n(B) = 57$, $n(A \cap B) = 12$

(Let A be the set of students in art class.

B be the set of students in dance class.)

(i) When 2 classes meet at different hours $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 35 + 57 - 12$

$= 92 - 12$

$= 80$

(ii) When two classes meet at the same hour, $A \cap B = \emptyset$; $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= n(A) + n(B)$

$= 35 + 57$

$= 92$

5. In a group of 100 persons, 72 people can speak English and 43 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French?

Solution:

Let A be the set of people who speak English.

B be the set of people who speak French.

A - B be the set of people who speak English and not French.

B - A be the set of people who speak French and not English.

$A \cap B$ be the set of people who speak both French and English.

Given,

$n(A) = 72 \quad n(B) = 43 \quad n(A \cup B) = 100$

Now, $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 72 + 43 - 100$

$= 115 - 100$

$= 15$

Therefore, Number of persons who speak both French and English = 15

$n(A) = n(A - B) + n(A \cap B)$

$\Rightarrow n(A - B) = n(A) - n(A \cap B)$

$= 72 - 15$

$= 57$

and $n(B - A) = n(B) - n(A \cap B)$

$= 43 - 15$

$= 28$

Therefore, the number of people speaking English only = 57, and the

number of people speaking French only = 28

6. In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories?

Solution:

Let A = set of persons who got medals in dance.

B = set of persons who got medals in dramatics.

C = set of persons who got medals in music.

Given,

$n(A) = 36$ $n(B) = 12$ $n(C) = 18$

$n(A \cup B \cup C) = 45$ $n(A \cap B \cap C) = 4$

We know that number of elements belonging to exactly two of the three sets A, B, C

$= n(A \cap B) + n(B \cap C) + n(A \cap C) - 3n(A \cap B \cap C)$

$= n(A \cap B) + n(B \cap C) + n(A \cap C) - 3 \times 4$ ……..(i)

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Therefore, $n(A \cap B) + n(B \cap C) + n(A \cap C) = n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cup B \cup C)$

From (i) required number

$= n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cup B \cup C) - 12$

$= 36 + 12 + 18 + 4 - 45 - 12$

$= 70 - 67$

$= 3$

7. Each student in a class of 40 plays at least one indoor game chess, carrom and scrabble. 18 play chess, 20 play scrabble and 27 play carrom. 7 play chess and scrabble, 12 play scrabble and carrom and 4 play chess, carrom and scrabble. Find the number of students who play (i) chess and carrom. (ii) chess, carrom but not scrabble.

Solution:

Let A be the set of students who play chess

B be the set of students who play scrabble

C be the set of students who play carrom

Therefore, We are given $n(A \cup B \cup C) = 40$,

$n(A) = 18$, $n(B) = 20$ $n(C) = 27$,

$n(A \cap B) = 7$, $n(C \cap B) = 12$ $n(A \cap B \cap C) = 4$

We have

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

Therefore, $40 = 18 + 20 + 27 - 7 - 12 - n(C \cap A) + 4$

$40 = 69 – 19 - n(C \cap A)$

$40 = 50 - n(C \cap A)$ $n(C \cap A) = 50 - 40$

$n(C \cap A) = 10$

Therefore, the number of students who play chess and carrom is 10.
Also, the number of students who play chess, carrom and not scrabble.

$= n(C \cap A) - n(A \cap B \cap C)$

$= 10 – 4$

$= 6$

Solving problems using Venn Diagrams

Example 8: In a group of 100 customers at Big Red's Pizza Emporium, 80 of them ordered mushrooms on their pizza and 72 of them ordered pepperoni. 60 customers ordered both mushrooms and pepperoni on their pizza.

a. How many customers ordered mushrooms but no pepperoni?

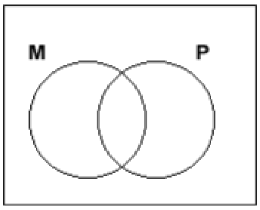
b. How many customers ordered pepperoni but no mushrooms?

c. How many customers ordered neither of these two toppings?

Solution

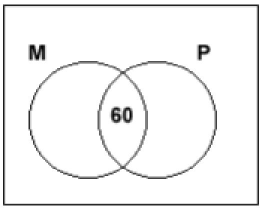
Create a Venn diagram with two sets.

To do this, first draw two intersecting circles inside a rectangle.

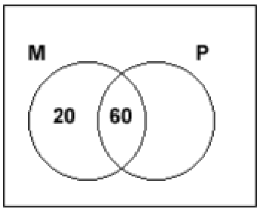


Then, make sure to work from the inside out.

That is, first place the 60 customers that fall into the intersection of the two sets. These 60 customers ordered BOTH mushrooms AND pepperoni on their pizza, so they go into the center region.

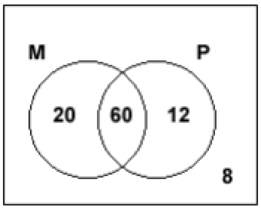


Next, we know we need to have a total of 80 customers inside the "M" circle. We already have 60 of them in there, so we have to put 20 more of them into circle for set M, being sure they are also NOT in the circle for set P.



Similarly, we know we need a total of 72 customers inside the "P" circle. We already have 60 of them in there, so we need to put 12 more into the circle for set P, being sure they are NOT in the circle for set M.

Finally, there are supposed to be 100 people on our diagram. Up to this point we have accounted for 92 of them (20 + 60 + 12), so the remaining 8 customers must go into region outside both of the circles, but still inside our rectangle.



Now, we can answer the questions.

a. 20 ordered mushrooms but not pepperoni.

b. 12 ordered pepperoni but not mushrooms.

c. 8 ordered neither of these two toppings.

Example 9:

At Dan's Automotive Shop, 50 cars were inspected. 23 of the cars needed new brakes, 34 needed new exhaust systems, and 6 cars needed neither repair.

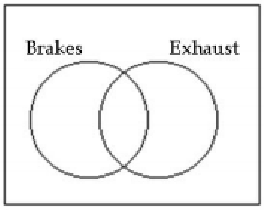
a. How many cars needed both repairs?

b. How many cars needed new brakes, but not a new exhaust system?

Solution

Create a Venn diagram with two sets.

To do this, first draw two intersecting circles inside a rectangle. Be sure to label the circles accordingly.



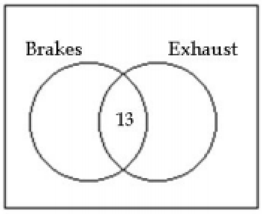
Now, work from the inside out.

That is, begin by determining the number of cars in the intersection of the two sets. Since 6 out of the 50 cars needed no repairs, leaving 44 cars that did need repairs. 23 needed brakes and 34 needed exhaust systems.

That makes 57 cars (23+34) that got worked on, which is too many; we know only 44 cars were worked on.

This means is 13 cars (57-44) got counted twice, which means that 13 cars get placed into the overlapping part of the Venn diagram (the intersection).

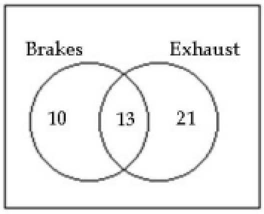
These 13 cars needed both brakes AND exhaust systems. Then look at the circle that corresponds to Brakes.



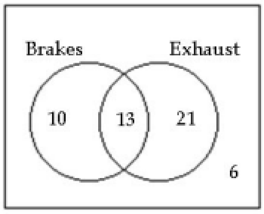
There should be 23 cars inside that circle.

13 are already accounted for, so the remaining 10 must be added into Brakes circle, but still are outside of the Exhaust circle.

Likewise, 34 vehicles must appear in the Exhaust circle, so 21 more must be placed inside that circle, but not be in the Brakes circle.



Finally, 6 cars need to be indicated outside the circles, but still inside the rectangle.



Now, by looking at the completed Venn diagram, answer the original questions.

a. 13 cars needed both repairs.

b. 10 cars needed brakes, but not an exhaust system.

Applications with three-circle Venn diagrams are a bit longer and, consequently, a bit more involved. However, the strategy remains the same – work from the inside out.

Example 10: A survey of 85 students asked them about the subjects they liked to study. Thirty five students liked math, 37 liked history, and 26 liked physics. Twenty liked math and history, 14 liked math and physics, and 3 liked history and physics. Two students liked all three subjects.

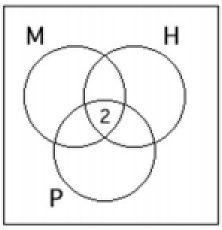
a. How many of these students like math or physics?

b. How many of these students didn't like any of the three subjects?

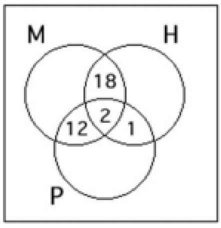
c. How many of these students liked math and history but not physics?

Solution :

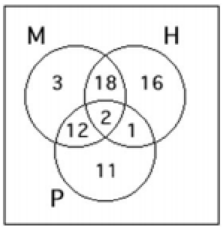
Create a Venn diagram with three sets, and label the circle M for math, H for history, and P for physics. Then make sure to work from the inside out. Start by placing the 2 students that like all three subjects into the center, which is the part of the diagram that represents the intersection of all three sets.



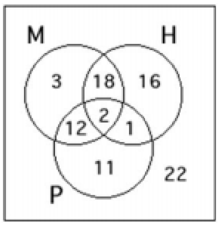
We know 20 students like math and history, so the intersection of those two sets must contain 20 students. We already have 2 of them in that intersection, so we put the remaining 18 in the intersection of the M and H circles, but not in the portion that also intersects the P circle. Using similar reasoning, we put 12 students and 1 student into the regions shown on the diagram.



Next, we know we need to have a total of 35 students inside the M circle. We already have 32 in there, so we put 3 students into Region I – the part of the Math circle that does not intersect with any other region. Similarly, we put 16 students into remaining section of the H circle and 11 students into the remaining section of the P circle.



Finally, there are supposed to be 85 students included in our diagram. Up to this point we have included 63 of them, so the remaining 22 students must go portion of the diagram that is outside all of the circles, but still in the rectangle.



Now, we can answer the original questions:

a. 47 of the students like math or physics.

b. 22 of the students didn't like any of these subjects.

c. 18 of the students liked math and history but not physics.

Summary

Let A and B be finite sets. If $A \cap B = \varphi$, then

(i) $\mathbf{n(A \cup B) = n(A) + n(B)}$

In general, if A, B and C are finite sets, then

(ii) $\mathbf{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$

(iii) $\mathbf{n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)}$