1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 01 (Class XI, Semester - 1)
Module Name/Title	Algebra of Sets - Part 4
Module Id	kemh_10104
Pre-requisites	Understands the concept of a set, Represents a set into Roster and Set builder form, Comprehends the operations on sets which includes Union, Intersection and Complement
Objectives	After going through this lesson, the learners will be able to understand the following: • Applies properties of the Operation of Union of Sets • Applies properties of the Operation of Intersection of Sets • Applies properties of the Operation of Complement of Sets
Keywords	Operation of Union, Operation of Intersection, Complement Sets

2. Development Team

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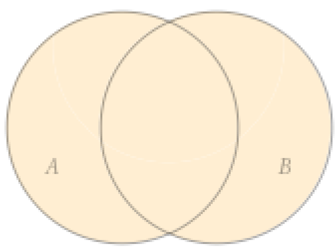
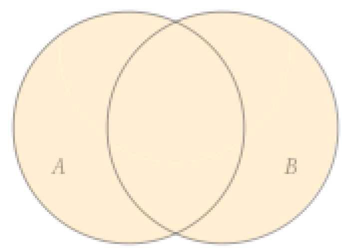
1. **Some Properties of the Operation of Union**

2) A ∪ B = B ∪ A (Commutative law)

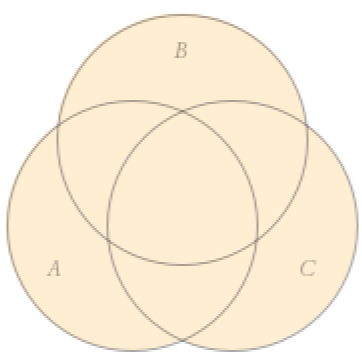


A ∪ B

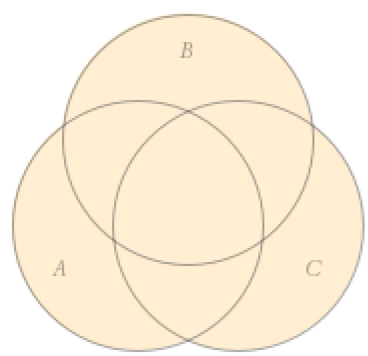


B ∪ A

3) (A ∪ B) ∪ C = A ∪ (B ∪ C) (Associative law)



$(A \cup B) \cup C$



$A \cup (B \cup C)$

4) A ∪ φ = A (Law of identity element, φ is the identity of ∪)
5) A ∪ A = A (Idempotent law)

6) $U \cup A = U$ (Law of U)

For Example

Let A = {0, 1, 2, 3, 4, 5}, B = {2, 4, 6, 8} and C = {1, 3, 5, 7}

Verify $(A \cup B) \cup C = A \cup (B \cup C)$

Solution:

$(A \cup B) \cup C = A \cup (B \cup C)$

L.H.S. = $(A \cup B) \cup C$

$A \cup B$ = {0, 1, 2, 3, 4, 5, 6, 8}

$(A \cup B) \cup C$ = {0, 1, 2, 3, 4, 5, 6, 7, 8} ……………….. (1)

R.H.S. = $A \cup (B \cup C)$

$B \cup C$ = {1, 2, 3, 4, 5, 6, 7, 8}

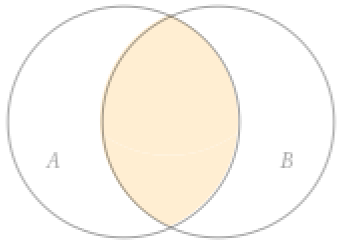
$A \cup (B \cup C)$ = {0, 1, 2, 3, 4, 5, 6, 7, 8} ……………….. (2)

Therefore, from (1) and (2), we find that;

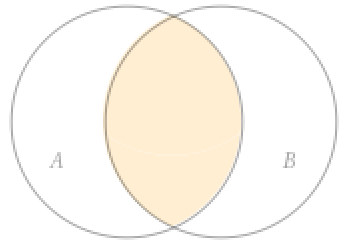
$(A \cup B) \cup C = A \cup (B \cup C)$ [*verified*]

2. Some Properties of Operation of Intersection

(1) $A \cap B = B \cap A$ (Commutative law).

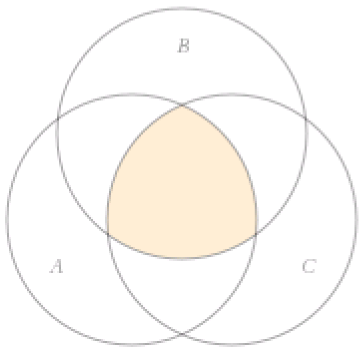


$A \cap B$

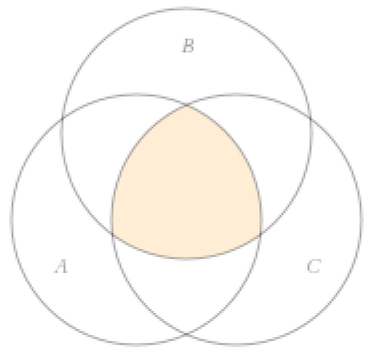


B ∩ A

(2) (A ∩ B) ∩ C = A ∩ (B ∩ C) (Associative law).



$(A \cap B) \cap C$

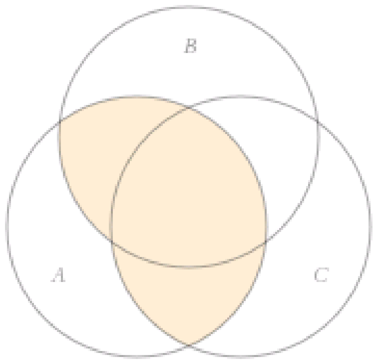


$A \cap (B \cap C)$

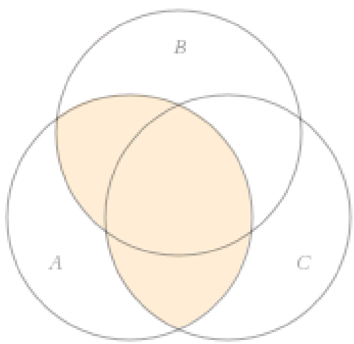
(3) φ ∩ A = φ, U ∩ A = A (Law of φ and U).

(4) A ∩ A = A (Idempotent law)

(5) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) (Distributive law) i. e., ∩ distributes over ∪



$A \cap (B \cup C)$ (A ∩ B) ∪ (A ∩ C)



For example

Let A = {0, 1, 2, 3, 4, 5}, B = {2, 4, 6, 8} and C = {1, 3, 5, 7}

Verify (A ∩ B) ∩ C = A ∩ (B ∩ C)

Solution:

<u>For (A ∩B)∩C = A∩(B∩C) :</u>

L.H.S. = (A∩B)∩C

A∩B = {2, 4}

$(A \cap B) \cap C = \emptyset$ ……………….. (1)

R.H.S. = $A \cap (B \cap C)$

$B \cap C = \emptyset$

$A \cap \{B \cap C\} = \emptyset$ ……………….. (2)

Therefore, from (1) and (2), we conclude that;

$(A \cap B) \cap C = A \cap (B \cap C)$[*verified*]

1. **Some Properties of Complement Sets**

Complement laws: (i) $A \cup A' = U$

(ii) $A \cap A' = \varphi$

De Morgan's law: (i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Law of double complementation : $(A')' = A$

Laws of empty set and universal set $\varphi' = U$ and $U' = \varphi$.

For example

Let A = {3, 5, 7}, B = {2, 3, 4, 6} and C = {2, 3, 4, 5, 6, 7, 8}

(i) Verify $(A \cap B)' = A' \cup B'$

(ii) Verify $(A \cup B)' = A' \cap B'$

Solution:

(i)(A ∩ B)' = A' ∪ B'

L.H.S. = (A ∩ B)'

A ∩ B = {3}

(A ∩ B)' = {2, 4, 5, 6, 7, 8} ……………….. (1)

R.H.S. = A' ∪ B'

A' = {5, 7, 8}

B' = {2, 4, 6}

A' ∪ B' = {2, 4, 5, 6, 7, 8} ……………….. (2)

From (1) and (2), we conclude that;

(A ∩ B)' = (A' ∪ B')

(ii)(A ∪ B)' = A' ∩ B'

L.H.S. = (A ∪ B)'

A∪B = {2, 3, 4, 5, 6, 7}

$(A \cup B)' = \{8\}$ ……………….. (1)

R.H.S. = $A' \cap B'$

$A' = \{2, 4, 6, 8\}$

$B' = \{5, 7, 8\}$

$A' \cap B' = \{8\}$ ……………….. (2)

From (1) and (2), we conclude that;

$(A \cup B)' = A' \cap B'$

2. **Some more examples**

If $A=\{1, 3, 5\}$, $B = \{3, 5, 6\}$ and $C = \{1, 3, 7\}$

Verify that:

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Solution:

<u>(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$</u> :

L.H.S. = $A \cup (B \cap C)$

$B \cap C = \{3\}$

$A \cup (B \cap C) = \{1, 3, 5\} \cup \{3\} = \{1, 3, 5\}$ ……………….. (1)

R.H.S. = $(A \cup B) \cap (A \cup C)$

$A \cup B = \{1, 3, 5, 6\}$

$A \cup C = \{1, 3, 5, 7\}$

$(A \cup B) \cap (A \cup C) = \{1, 3, 5, 6\} \cap \{1, 3, 5, 7\} = \{1, 3, 5\}$ ……………….. (2)

From (1) and (2), we conclude that;

$A \cup (B \cap C) = A \cup B \cap (A \cup C)$ [*verified*]

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S. = $A \cap (B \cup C)$

$B \cup C = \{1, 3, 5, 6, 7\}$

$A \cap (B \cup C) = \{1, 3, 5\} \cap \{1, 3, 5, 6, 7\} = \{1, 3, 5\}$ ……………….. (1)

R.H.S. = $(A \cap B) \cup (A \cap C)$

$A \cap B = \{3, 5\}$

$A \cap C = \{1, 3\}$

$(A \cap B) \cup (A \cap C) = \{3, 5\} \cup \{1, 3\} = \{1, 3, 5\}$ ……………….. (2)

From (1) and (2), we conclude that;

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [*verified*]

3.Summary

- **Some Properties of the Operation of Union**

i. $A \cup B = B \cup A$ (Commutative law)
ii. $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)
iii. $A \cup \varphi = A$ (Law of identity element, φ is the identity of $\cup$)
iv. $A \cup A = A$ (Idempotent law)
v. $U \cup A = U$ (Law of U)

(i) **Some Properties of Operation of Intersection**

i. $A \cap B = B \cap A$ (Commutative law).
ii. $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law).
iii. $\varphi \cap A = \varphi$, $U \cap A = A$ (Law of φ and U).
iv. $A \cap A = A$ (Idempotent law)
v. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law) i.e.,$\cap$ distributes over $\cup$

- **Some Properties of Complement Sets**

a) Complement laws: (i) $A \cup A' = U$

(ii) $A \cap A' = \varphi$

b) De Morgan's law: (i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

c) Law of double complementation : $(A')' = A$

d) Laws of empty set and universal set : $\varphi' = U$ and $U' = \varphi$.