1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 01 (Class XI, Semester - 1)
Module Name/Title	Subsets and Supersets - Part 2
Module Id	kemh_10102
Pre-requisites	Understands the concept of a set, Represents a set into Roster and Set builder form, Describes a set as finite, Infinite, Empty, Singleton
Objectives	After going through this lesson, the learners will be able to do the following: • Write subsets of a given set • Represent Interval-open or closed, as a subset of a given set • Write Power Set for a given set • Find the number of subsets of a given set • Find Cardinality of a set based on the number of elements in it
Keywords	Subset, Superset, Power Set, Interval, Cardinality of a set

2. Development Team

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1. Subsets of a set

Sets of things are often further subdivided. For example, owls are a particular type of bird, so every owl is also a bird. We express this in the language of sets by saying that the set of owls is a subset of the set of birds.

A set S is called a subset of another set T if every element of S is an element of T. This is written as

S ⊆ **T** (Read this as 'S is a subset of T'.)

The new symbol ⊂ means **'is a subset of'**.

Thus { owls } ⊂ { birds } because every owl is a bird.

Similarly, if A = { 2, 4, 6 } and B = { 0, 1, 2, 3, 4, 5, 6 }, then A ⊂ B, because every element of A is an element of B.

The sentence **'S is not a subset of T'** is written as

S ⊈ T

This means that at least one element of S is not an element of T. For example,

{ birds } ⊈ { flying creatures }

because an ostrich is a bird, but it does not fly.

Similarly, if A = { 0, 1, 2, 3, 4 } and B = { 2, 3, 4, 5, 6 }, then

A ⊈ B, because 0 ∈ A, but 0 ∉ B.

The set itself and the empty set are always subsets.

Any set S is a subset of itself, because every element of S is an element of S.

For example: { birds } ⊂ { birds } and

{ 1, 2, 3, 4, 5, 6 } = { 1, 2, 3, 4, 5, 6 }.

Furthermore, the empty set *Φ* is a subset of every set S,

2. Superset of a set

If A and B are two sets, and every element of set A is also an element of set B, then B is called a superset of A and we write it as **B ⊇ A.**

3. Equal Sets

If A and B are two sets, then A is called the proper subset of B if A ⊆ B but B ⊈ A

i.e., A ≠ B. The symbol '⊂' is used to denote proper subset. Symbolically, we write **A ⊂ B**.

Note:

No set is a proper subset of itself.

Null set or ∅ is a proper subset of every set.

For example: A = {p, q, r}

B = {p, q, r, s, t}

Here A is a proper subset of B as all the elements of set A are in set B and also $A \neq B$.

Some of the obvious relations among these subsets are:

$$\mathbf{N \subset Z \subset Q, Q \subset R, T \subset R, N \not\subset T.}$$

Note:

If $A \subseteq B$ and $B \subseteq A$, then $A = B$, i.e., they are equal sets.

For example, Let A = {2, 4, 6}

B = {x : x is an even natural number less than 8}

Here $A \subset B$ and $B \subset A$.

Hence, we can say A = B

4. Intervals as subsets

Let $a, b \in R$ and $a < b$. Then the set of real numbers $\{ y : a < y < b\}$ is called an **open interval** and is denoted by **(a, b)**. All the points between a and b belong to the open interval (a, b) but a, b themselves do not belong to this interval.

The interval which contains the end points also is called **closed interval** and is denoted by

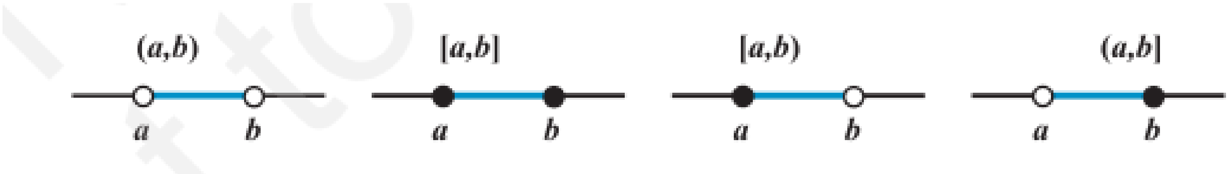
[a, b]. Thus $[a, b] = \{x : a \leq x \leq b\}$

We can also have intervals closed at one end and open at the other,

i.e., **[a, b)** = $\{x : a \leq x < b\}$ is an open interval from a to b, including a but excluding b.

(a, b] = $\{ x : a < x \leq b \}$ is an open interval from a to b including b but excluding a.

On real number line, various types of intervals described above as subsets of R, are shown in the figure below:



For example, the set {x : x ∈ R, $-5 < x \leq 7$}, written in set-builder form, can be written in the form of interval as (–5, 7] and the interval [–3, 5) can be as {x : $-3 \leq x < 5$}.

Note : Here unfilled circle O indicate that point is not included and filled circle O that point is included.

5. Power Set

We have defined a set as a collection of its elements. Thus if S is a set then the collection or family of all subsets of S is called the power set of S and it is denoted by **P(S)**.

If S ={ a, b } then the power set of S is given by

P(S) = {{a}, {b}, {a, b}, *Φ*}

The null set or empty set *Φ* having no element of its own, is also an element of the power set; since, it is a subset of all sets.

The set S being a subset of itself is also as an element of the power set.

6. Number of Subsets of a given Set

If a set contains 'n' elements, then the number of subsets of the set is 2^n.

For example:

If A {1, 3, 5}, then write all the possible subsets of A. Find their numbers.

Solution:

The subset of A containing no elements is Ø

The subset of A containing one element each are {1} {3} {5}

The subset of A containing two elements each are {1, 3} {1, 5} {3, 5}

The subset of A containing three elements is {1, 3, 5)

Therefore, all possible subsets of A are { }, {1}, {3}, {5}, {1, 3}, {1,5}, {3, 5}, {1, 3, 5}

Therefore, number of all possible subsets of A is 8 which is equal 2^3.

8. Cardinality of a set

The cardinality of a set is the numbers of elements in that set.

If S is a finite set, the symbol **n(S)** or stands for the number of elements of S.

For example: If S = { 1, 3, 5, 7, 9 }, then n(S) = 5.

If A = { 1001, 1002, 1003, …, 3000 }, then n(A) = 2000.

If T = { letters in the English alphabet }, then n(T)| = 26.

The set S = { 5 } is a one-element set because n(S) = 1. It is important to distinguish between the number 5 and the set S = { 5 }:

$$5 \in S \text{ but } \{5\} \neq S.$$

Example 1: Consider the sets

$$\varphi, A=\{1,3\}, B=\{1,5,9\}, C=\{1,3,5,7,9\}$$

Insert the symbol $\subset$ or $\not\subset$ between each of the following pair of sets:

(i) $\varphi......B$

(ii) $A.....B$

(iii) $A.....C$

(iv) $B.....C$

Solution:

(i) $\varphi \subset B$, as null set is a subset of every set.

(ii) $A \not\subset B, \ as \ 3 \in A \ and \ 3 \notin B.$

(iii) $A \subset C, \ as \ 1,3 \in A$ also belongs to C.

(iv) $B \subset C$ as each element of B is also an element of C.

Example 2: Let A, B and C be three sets. If $A \in B \ and \ B \subset C$, is it true that $A \subset C$?

If not, give an example.

Solution: No.

$A = \{1\}, B = \{\{1\}, 2\} \ and \ C = \{\{1\}, 2, 3\}$. Here $A \in B \ as \ A = \{1\}$ and $B \subset C$.

But $A \not\subset C \ as \ 1 \in A \ and \ 1 \notin C$.

9. Summary

i. A set S is called a subset of another set T if every element of S is an element of T. This is written as $S \subseteq T$

ii. S is not a subset of T' is written as $S \nsubseteq T$. This means that at least one element of S is not an element of T.

iii. If A and B are two sets, and every element of set A is also an element of set B, then B is called a superset of A and we write it as $B \supseteq A$.

iv. The set itself and the empty set are always subsets.

v. If $A \subseteq B$ and $B \subseteq A$, then $A = B$, i.e., they are equal sets of a set.

vi. The set of real numbers $\{ y : a < y < b\}$ is called an open interval and is denoted by (a, b)

vii. The interval which contains the end points also is called closed interval and is denoted by [a, b]. Thus $[a, b] = \{x : a \leq x \leq b\}$

viii. $[a, b) = \{x : a \leq x < b\}$ is an open interval from a to b, including a but excluding b.

ix. $(a, b] = x : a < x \leq b \}$ is an open interval from a to b including b but excluding a.

x. If S is a set then the collection or family of all subsets of S is called the power set of S and it is denoted by P(S).

xi. If a set contains 'n' elements, then the number of subsets of the set is 2^n.

xii. The cardinality of a set is the numbers of elements in a set. If S is a finite set, the symbol n(S) or stands for the number of elements of S.