1. Details of Module and its structure

Module Detail	
Subject Name	Mathematics
Course Name	Mathematics 01 (Class XI, Semester - 1)
Module Name/Title	Set Notations - Part 1
Module Id	kemh_10101
Pre-requisites	Group objects and numbers based on similarities and dissimilarities
Objectives	After going through this lesson, the learners will be able to do the following: • Group objects and numbers into a Set • Represent a set into Roster form or Set builder form • Compare Sets as Equal, Unequal and Disjoint • Describe a Set as finite or infinite • Understand the concepts of Empty Set and Singleton Set
Keywords	Set, Set builder form of representation, Roster form of representation, Equal Sets, Disjoint Sets, Finite Set, Infinite Set, Empty Set, Singleton Set

2. Development Team

Role	Name	Affiliation
National MOOC Coordinator (NMC)	Prof. Amarendra P. Behera	CIET, NCERT, New Delhi
Program Coordinator	Dr. Mohd. Mamur Ali	CIET, NCERT, New Delhi
Course Coordinator (CC) / PI	Dr. Til Prasad Sarma	DESM, NCERT, New Delhi
Course Co-Coordinator / Co-PI	Dr. Mohd. Mamur Ali	CIET, NCERT, New Delhi
Subject Matter Expert (SME)	Ms. Anjali Chhugani	Sanskriti School, New Delhi
Review Team	Dr. Sadhna Shrivastava	KVS, Faridabad, Haryana

Table of Contents :

1. Introduction

<u>Find the odd one out</u>

1. Rabbit, Deer, Tiger, Giraffe, Elephant.
2. 2, 3, 17, 19, 9, 23
3. a, e, i, d, o, u
4. 1, 2, 3, 4, 6, 8, 15, 24
5. Ganga, Yamuna, Nile, Narmada, Kaveri

In the above examples, the odd ones out have been highlighted. As you may observe that other than the highlighted objects, the rest of the objects share some similarities and can thus be grouped together.

For instance,

1. Except 'Tiger' all other animals are herbivores animals.
2. Except '9' all other numbers are prime numbers.
3. Except 'd' all other alphabets are English Vowels.
4. Except' 15' all other numbers are factors of 24.
5. Except 'Nile' the rest are rivers of India.

Each of the above examples is a well-defined collection of objects in the sense that we can definitely decide whether a given particular object belongs to a given collection or not.

Again the collection of five most renowned mathematicians of the world is not well-defined, because the criterion for determining a mathematician as most renowned may vary from person to person. Thus, it is not a well-defined collection.

These **well-defined collection of objects** are termed as **Sets**.

We give below a few more examples of sets used particularly in mathematics, viz.
N : the set of all natural numbers
Z : the set of all integers
Q : the set of all rational numbers
R : the set of real numbers
Z+ : the set of positive integers
Q+ : the set of positive rational numbers, and
R+ : the set of positive real numbers

2. Describing and Naming Sets

A set is just a collection of objects, but we need some new words and symbols and diagrams to be able to talk sensibly about sets.

In our ordinary language, we try to make sense of the world we live in by classifying collections of things. English has many words for such collections. For example, we speak of ‘a flock of birds’, ‘a herd of cattle’, ‘a swarm of bees’ and ‘a colony of ants’.

We do a similar thing in mathematics, and classify numbers, geometrical figures and other things into collections that we call sets. The objects in these sets are called the elements of the set.

Describing a set

- A set can be described by listing all of its elements.

  For example, S = { 1, 3, 5, 7, 9 },

  which we read as ‘S is the set whose elements are 1, 3, 5, 7 and 9’.

  The five elements of the set are separated by commas, and the list is enclosed between curly brackets.

  This representation of a Set is known as the **Roster or tabular form**.

- A set can also be described by writing a description of its elements between curly brackets.

  Thus the set S above can also be written as

  S = { odd whole numbers less than 10 },

  which we read as ‘S is the set of odd whole numbers less than 10’.

  This representation of a Set is known as the **Set Builder Form**.

  A set must be well defined. This means that our description of the elements of a set is clear and unambiguous. For example, {tall people } is not a set because people tend to disagree about what ‘tall’ means.

3. Equal sets

Two sets are called **equal** if they have exactly the same elements. Thus following the usual convention that ‘y’ is not a vowel,

{vowels in the English alphabet } = {a, e, i, o, u }

On the other hand, the sets { 1, 3, 5 } and { 1, 2, 3 } are not equal, because they have different elements. This is written as

$$\{ 1, 3, 5 \} \neq \{ 1, 2, 3 \}.$$

The order in which the elements are written between the curly brackets does not matter at all. For example,

$$\{ 1, 3, 5, 7, 9 \} = \{ 3, 9, 7, 5, 1 \} = \{ 5, 9, 1, 3, 7 \}.$$

If an element is listed more than once, it is only counted once. For example,

$$\{ a, a, b \} = \{ a, b \}.$$

The set { a, a, b } has only the two elements a and b. The second mention of a is an unnecessary repetition and can be ignored. It is normally considered poor notation to list an element more than once.

The symbols $\in$ and $\notin$

The phrases ‘is an element of’ and ‘is not an element of’ occur so often in discussing sets that the special symbols and are used for them. For example, if A = { 3, 4, 5, 6 }, then

$3 \in A$ (Read this as ‘3 is an element of the set A’.)

$8 \notin A$ (Read this as ‘8 is not an element of the set A’.)

4. Disjoint Sets

Two sets A and B are said to be disjoint, if they do not have any element in common.

For example:

A = {x : x is a prime number}

B = {x : x is a composite number}.

Clearly, A and B do not have any element in common and are disjoint sets.

5. Finite and infinite sets

All the sets we have seen so far have been **finite** sets, meaning that we can list all their elements.

Here are few examples:

{ whole numbers between 2000 and 2005 } = { 2001, 2002, 2003, 2004 }

{ whole numbers between 2000 and 3000 } = { 2001, 2002, 2003,…, 2999 }

The three dots '…' in the second example stand for the other 995 numbers in the set. We could have listed them all, but to save space we have used dots instead. This notation can only be used if it is completely clear what it means, as in this situation.

A set can also be **infinite** – all that matters is that it is well defined. Here are two examples of infinite sets:

{ even whole numbers } = { 0, 2, 4, 6, 8, 10, …}

{ whole numbers greater than 2000 } = { 2001, 2002, 2003, 2004, …}

Both these sets are infinite because no matter how many elements we list, there are always more elements in the set that are not on our list. This time the dots '…' have a slightly different meaning, because they stand for infinitely many elements that we could not possibly list, no matter how long we tried.

Remark : If a set is not finite, then it is called an infinte set.

6. The empty set

The symbol Φ represents the empty set, which is the set that has no elements at all. There is only one empty set, because any two empty sets have exactly the same elements, so they must be equal to one another. Nothing in the whole universe is an element of Φ**.**

For example: (a) The set of whole numbers less than 0.

(b) $N = \{x : x \in N, 3 < x < 4\}$

7. Singleton Set

A set which contains only one element is called a singleton set.

For example:

- A = {x : x is neither prime nor composite}

It is a singleton set containing one element, i.e., 1.

B = {x : x is a whole number, x < 1}

This set contains only one element 0 and is a singleton set.

Example1: Write the following set in the roaster form:

$A = \{x | x \text{ } is\, a\ postive\ \text{int}\, ger\, less\, than 10\, and\ 2^x - 1 is\, an\, odd\ number\}$

Solution:

$2^x - 1$is always an odd number for all positive integral values of x. In particular $2^x - 1$ is an odd number for x = 1,2,……….,9. Thus, A = {1,2,3,4,5,6,7,8,9}

Example 2: Write the following set in the roaster form:

$C = \{x : x^2 + 7x - 8 = 0, x \in R\}$

Solution:

$$x^2 + 7x - 8 = 0$$
$$\Rightarrow (x+8)(x-1) = 0$$
$$\Rightarrow x = -8\, or\ x = 1$$

Thus, C = {-8, 1}

Example 3: Which of the following pairs of sets are equal? Justify your answer.

$A = \{0\}$, $B = \{x : x > 15 \ and \ x < 5\}$

$C = \{x : x - 5 = 0\}$, $D = \{x : x^2 = 25\}$

E= { x : x is an integral positive root of the equation $x^2 - 2x - 15 = 0$ }.

Solution:

Since $0 \in A$ and 0 does not belong to any of the sets B, C, D and E, it follows that,

$A \neq B, A \neq C, A \neq D, A \neq E.$

Since B = { }, but none of the sets are empty. Therefore $B \neq C, B \neq D \ and \ B \neq E.$

Also $C = \{5\} \ but \ -5 \in D, \ hence \ C \neq D.$

Since $E = \{5\}, C = E.$ Further, $D = \{-5, 5\} \ and \ E = \{5\}$, we find that $D \neq E$.

Thus, the only pair of equal sets is C and E.

Example 4: Given that N = {1, 2, 3,...,100}, then

(i) Write the subset A of N, whose element are odd numbers.

(ii) Write the subset B of N, whose elements are represented by x + 2, where $x \in N$

Solution: (i)

$$A = \{x : x \in N \; and \; x \; is \; odd\}$$
$$= \{1, 3, 5, 7, ..., 99\}$$

(ii)

$$B = \{y : y = x + 2, x \in N\}$$

So, for

$$1 \in N, \; y = 1 + 2 = 3$$
$$2 \in N, y = 2 + 2 = 4$$

and so on. Therefore, B = { 3, 4, 5, 6, …, 100}.

Example 5: Given that E = { 2, 4, 6, 8, 10 }. If n represents any member of E, then, write the following sets containing all numbers represented by (i) n+1 (ii) n^2 .

Solution: Given E = { 2, 4, 6, 8, 10 }

(i) Let $A = \{x : x = n + 1, n \in E\}$

Thus, for

$$2 \in E, \; x = 3$$
$$4 \in E, x = 5,$$

and so on. Therefore, A = {3, 5, 7, 9, 11}.

(ii) Let $B = \{x : x = n^2, n \in E\}$

So, for

$$2 \in E, \; x = 2^2 = 4$$
$$4 \in E, x = 4^2 = 16$$
$$6 \in E, x = 6^2 = 36,$$

and so on. Therefore, B = {4, 16, 36, 64, 100}.

8. Summary

- A set is a collection of objects, called the elements of the set.
- A set must be well defined, meaning that its elements can be described and listed without ambiguity. For example: { 1, 3, 5 } and { letters of the English alphabet }.
- A set can be represented in two ways : Roster form or Set builder form.
- Two sets are called equal if they have exactly the same elements. - The order is irrelevant. Any repetition of an element is ignored.
- Two sets A and B are said to be disjoint, if they do not have any element in common.
- If a is an element of a set S, we write $a \in S$.
- If b is not an element of a set S, we write $b \notin S$.
- A set is called finite if we can list all of its elements.
- A set is called infinite, if it is not a finite set.
- An infinite set has the property that no matter how many elements we list, there are always more elements in the set that are not on our list.
- The set with no elements is called the empty set, and is written as .
- A set which contains only one element is called a singleton set.